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Aspects of diffeomorphism and conformal invariance in classical Liouville theory

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Abstract – The interplay between the diffeomorphism and conformal symmetries (a feature common in quantum field theories) is shown to be exhibited for the case of black holes in two-dimensional classical Liouville theory. We show that although the theory is conformally invariant in the near-horizon limit, there is a breaking of the diffeomorphism symmetry at the classical level. On the other hand, in the region away from the horizon, the conformal symmetry of the theory gets broken with the diffeomorphism symmetry remaining intact.

It is a well-known fact from general quantum-field–theoretic considerations [1] that in two dimensions there is an interplay between the diffeomorphism and the trace anomaly of the energy-momentum tensor in the sense that it is possible to remove one of them but not both. This is because for the $D=2$ case, there is no regularization which simultaneously preserves conformal as well as diffeomorphism symmetries [1–4].

Interestingly, we find that such a situation arises in a classical Liouville theory [5] used to compute the black-hole entropy owing to the presence of a nontrivial central charge in the theory [6–11]. Further, Hawking effect was also studied by using the boundary Liouville model [12]. In classical Liouville theory (which is relevant for black-hole physics) the energy-momentum tensor is covariantly conserved but has a nontrivial trace anomaly. However, it turns out that if we consider terms up to leading order in the expansion of the metric, near the horizon, the trace anomaly vanishes and hence the theory becomes conformally invariant (which explains its utility for studies on black-hole physics), but the energy-momentum tensor fails to remain covariantly conserved. Hence, the interplay between the diffeomorphism and the trace anomaly of the energy-momentum tensor gets exhibited at the classical level.

To begin with, we consider a four-dimensional spherically symmetric metric of the form

$$ds^2=\gamma_{ab}(x^0,x^1)\,dx^a\,dx^b+r^2(x^0,x^1)\,d\Omega^2, \qquad (1)$$

where $\gamma_{ab}(x^0,x^1)$ is the metric on an effective 2D spacetime M^2 with coordinates (x^0,x^1). We now start from the four-dimensional Einstein-Hilbert action defined by

$$S_{EH}=-\frac{1}{16\pi G}\int_{M^4}d^4x\sqrt{-g}R_{(4)}. \qquad (2)$$

Considering the above action on the class of spherically symmetric metrics (1), we obtain an effective two-dimensional theory described by the action

$$S=-\int_{M^2}d^2x\sqrt{-\gamma}\left(\frac{1}{2}(\nabla\Phi)^2+\frac{1}{4}\Phi^2R_{(2)}+\frac{1}{2G}\right), \qquad (3)$$

where $\Phi=rG^{-1/2}$ and $R_{(2)}$ is the two-dimensional scalar curvature[1]. This action represents dilaton gravity in two dimensions with r playing the role of dilaton field.

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[1] From now onwards we shall suppress the suffix on R but it is understood as 2D curvature scalar.

The above action can now be transformed to a form similar to that of the Liouville theory [13]

$$S_L = -\int_{M^2} \mathrm{d}^2x\sqrt{-\bar{\gamma}}\left(\frac{1}{2}(\bar{\nabla}\phi)^2 + \frac{1}{4}q\Phi_h\phi\bar{R} + U(\phi)\right) \quad (4)$$

with the aid of the following transformations:

$$\gamma_{ab} = \left(\frac{\phi_h}{\phi}\right)^{\frac{1}{2}} e^{\frac{2\phi}{q\phi_h}}\bar{\gamma}_{ab}, \quad \phi = \frac{\Phi^2}{q\Phi_h}, \quad (5)$$

where $\Phi_h = r_h G^{-1/2}$ is the classical value of the field Φ at the horizon and $U(\phi) = \frac{1}{2G}(\frac{\phi_h}{\phi})^{1/2}e^{\frac{2\phi}{q\phi_h}}$. In (4), $\bar{\nabla}$ is the covariant derivative compatible with the metric $\bar{\gamma}_{ab}$ while $\bar{R}$ is the corresponding curvature scalar. Varying the above action with respect to the scalar field ϕ yields the equation of motion for ϕ

$$\bar{\nabla}^a\bar{\nabla}_a\phi = \frac{q\Phi_h}{4}\bar{R} + \frac{\mathrm{d}U}{\mathrm{d}\phi}. \quad (6)$$

Similarly, varying the action with respect to the metric $\bar{\gamma}_{ab}$, we obtain the constraints

$$\begin{aligned} T_{ab} \equiv\; & \frac{1}{2}\partial_a\phi\partial_b\phi - \frac{1}{4}\bar{\gamma}_{ab}(\bar{\nabla}\phi)^2 \\ & + \frac{1}{4}q\Phi_h(\bar{\gamma}_{ab}\bar{\nabla}^c\bar{\nabla}_c\phi - \bar{\nabla}_a\bar{\nabla}_b\phi) - \frac{1}{2}\bar{\gamma}_{ab}U(\phi) = 0. \end{aligned} \quad (7)$$

The theory of the scalar field ϕ described by the action (4) is not conformal in general. This can be easily seen by contracting (7) with metric $\bar{\gamma}^{ab}$ to obtain

$$T^a{}_a = \frac{q\Phi_h}{4}\bar{\nabla}^a\bar{\nabla}_a\phi - U(\phi). \quad (8)$$

By substituting (6) in the above equation we obtain on shell expression for the trace of the energy-momentum tensor

$$T^a{}_a = \left(\frac{q\Phi_h}{4}\right)^2\bar{R} + \frac{q\Phi_h}{4}\frac{dU}{d\phi} - U(\phi). \quad (9)$$

Thus, we have a nonvanishing trace of the energy-momentum tensor, leading to the breaking of the conformal symmetry in the classical Liouville theory. We would like to point out that such a violation in the conformal invariance generally occurs when we quantize the theory on the curved background.

We now take the divergence of (7) and obtain

$$\bar{\nabla}_a T^a{}_b = \frac{1}{2}\left[\bar{\nabla}^c\bar{\nabla}_c\phi\bar{\gamma}_{be} - \frac{q\Phi_h}{4}\bar{R}\bar{\gamma}_{be}\right]\bar{\nabla}^e\phi - \frac{1}{2}\partial_b U(\phi). \quad (10)$$

Substituting the equations of motion for ϕ (6) in (10), leads to the conservation of the energy-momentum tensor

$$\bar{\nabla}_a T^a{}_b = 0. \quad (11)$$

Hence, we find that although there is a breaking of the classical conformal invariance as the energy-momentum tensor has a nonvanishing trace (9), the diffeomorphism symmetry remains intact. This is a feature which has been observed earlier only in a quantum theory.

Now we consider the near-horizon behavior of the theory defined by (4). For simplicity we consider the metric

$$\mathrm{d}s^2 = \bar{\gamma}_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b = -g(x)\,\mathrm{d}t^2 + \frac{1}{g(x)}\,\mathrm{d}x^2, \quad (12)$$

where $g(x = x_h) = 0$ defines the location of the event horizon x_h in Schwarzschild coordinates (t, x). Since the metric coefficient $g(x)$ is a well-behaved function, we can expand it about $x = x_h$

$$\begin{aligned} g(x) \;=\; & g'(x_h)(x - x_h) + \frac{1}{2!}(x - x_h)^2 g''(x_h) \\ & + \frac{1}{3!}(x - x_h)^3 g'''(x_h) + \cdots . \end{aligned} \quad (13)$$

In the vicinity of the horizon, we keep terms proportional to $(x - x_h)$ only. With this approximation, the above function (13) becomes

$$g(x) \approx g'(x_h)(x - x_h) = \frac{2}{\beta_H}(x - x_h), \quad (14)$$

where β_H is related to the Hawking temperature [6]. Now we would like to see how the metric function vanishes in the region near to the horizon. For that it is convenient to introduce coordinate z defined as

$$z = \int^x \frac{\mathrm{d}x}{g(x)}. \quad (15)$$

Substituting (14) in (15) we get

$$g(x) \equiv g(z) = \frac{2}{\beta_H}e^{\frac{2z}{\beta_H}}. \quad (16)$$

Note that in the (t, z)-coordinates, large negative z corresponds to the near-horizon limit. We now consider the equation of motion for scalar field (6) in the vicinity of the horizon. Expressing (6) in (t, z)-coordinates we get

$$-\partial_t^2\phi + \partial_z^2\phi = g(z)\left[\frac{q\Phi_h}{4}\bar{R} + U'(\phi)\right]. \quad (17)$$

In the region near to the horizon, in view of (16), the right-hand side of above equation vanishes exponentially. Hence, in the vicinity of the horizon we have

$$-\partial_t^2\phi + \partial_z^2\phi = 0. \quad (18)$$

Therefore, in the near-horizon region Liouville theory gets effectively described by free massless scalar field. Noting that the trace of the energy-momentum tensor (8) can be written in the form (dropping the interaction term $U(\phi)$ as it is not important for our present purpose)

$$T^a{}_a = \frac{1}{g(x)}(-T_{00} + T_{zz}) = -\frac{q\Phi_h}{4}\left[\frac{1}{g(x)}(\partial_t^2\phi - \partial_z^2\phi)\right], \quad (19)$$

we find that in the near-horizon limit, using the near-horizon equation of motion (18), the trace of the energy-momentum tensor as defined in [6] becomes

$$-T_{00}+T_{zz}=0. \tag{20}$$

This indicates the fact that the near-horizon theory is conformally invariant. However, one can rewrite the right-hand side of (19) in terms of $\bar{R}$ using (17). This simply reproduces the exact result for the trace of the energy-momentum tensor (9). The near-horizon result is trivially obtained to be

$$T^a{}_a=\left(\frac{q\Phi_h}{4}\right)^2\bar{R}(x_h)=-\left(\frac{q\Phi_h}{4}\right)^2 g''(x_h). \tag{21}$$

This shows that the above way of computing the trace near the horizon leads to a nonvanishing result which is incompatible with (20). To make the trace $T^a{}_a$ compatible with (20), we shall use the equation of motion (18) describing free massless scalar field as the theory near the horizon has conformal invariance. This leads to $T^a{}_a=0$, since the term in the braces in (19) vanishes as a consequence of (18). Hence, in the near-horizon limit, we take the equation of motion to be (18) with $g(x)$ vanishingly small but not zero. We shall consistently apply this approximation in the subsequent near-horizon analysis.

Now we show that, though we are able to keep the conformal invariance intact, the near-horizon theory does not preserve the diffeomorphism invariance. For that, we write the energy-momentum tensor (7) in the (t,z)-coordinates as

$$T_{00}(z)=\frac{1}{4}\left(\dot{\phi}^2+(\partial_z\phi)^2\right)-\frac{q\Phi_h}{4}\left(\partial_z^2\phi-\frac{g'(x)}{2}\partial_z\phi\right), \tag{22}$$

$$T_{0z}(z)=\frac{1}{2}\dot{\phi}\;\partial_z\phi-\frac{q\Phi_h}{4}\left(\partial_z\dot{\phi}-\frac{g'(x)}{2}\dot{\phi}\right), \tag{23}$$

$$T_{zz}(z)=\frac{1}{4}\left(\dot{\phi}^2+(\partial_z\phi)^2\right)+\frac{q\Phi_h}{4}\left(-\ddot{\phi}+\frac{g'(x)}{2}\partial_z\phi\right), \tag{24}$$

where the "dot" represents the derivative with respect to time and the "prime" represents the derivative with respect to x. Note that once again we do not consider the interaction term $U(\phi)$ since it is not important for our purpose. Now, let us consider the near-horizon approximation of the above equations. First, we note that T_{00}, T_{0z} and T_{zz} contain a term proportional to a derivative of the metric function. Therefore, it will be inappropriate to substitute the form of the metric function to first order in $(x-x_h)$ (see (14)) in (22)–(24), rather, we have to take into account the next-order term in the metric expansion (13). In other words, we put

$$g'(x)=\frac{2}{\beta_H}+g''(x_h)(x-x_h) \tag{25}$$

in the expressions for T_{00}, T_{0z} and T_{zz}. Then we have

$$\begin{aligned}T_{00}(z)\;=\;&\frac{1}{4}\left(\dot{\phi}^2+(\partial_z\phi)^2\right)\\&-\frac{q\Phi_h}{4}\left(\partial_z^2\phi-\frac{1}{2}\left[\frac{2}{\beta_H}+g''(x_h)(x-x_h)\right]\partial_z\phi\right),\end{aligned} \tag{26}$$

$$\begin{aligned}T_{0z}(z)\;=\;&\frac{1}{2}\dot{\phi}\;\partial_z\phi\\&-\frac{q\Phi_h}{4}\left(\partial_z\dot{\phi}-\frac{1}{2}\left[\frac{2}{\beta_H}+g''(x_h)(x-x_h)\right]\dot{\phi}\right),\end{aligned} \tag{27}$$

$$\begin{aligned}T_{zz}(z)\;=\;&\frac{1}{4}\left(\dot{\phi}^2+(\partial_z\phi)^2\right)\\&+\frac{q\Phi_h}{4}\left(-\ddot{\phi}+\frac{1}{2}\left[\frac{2}{\beta_H}+g''(x_h)(x-x_h)\right]\partial_z\phi\right).\end{aligned} \tag{28}$$

Further justification for this approximation will become clear in the subsequent discussion.

We move on to compute the covariant divergence of the energy-momentum tensor (26)–(28). This can be written as

$$\bar{\nabla}_a T^a{}_b=\Lambda^{ac}\bar{\nabla}_a T_{cb}, \tag{29}$$

where, Λ_{ab} is the metric in the (t,z)-coordinates[2]. For $b=t$, we obtain

$$\bar{\nabla}_a T^a{}_t=-\frac{1}{g(x)}\left[\frac{1}{2}\dot{\phi}(\ddot{\phi}-\partial_z^2\phi)-\frac{q\Phi_h}{8}g(x)g''(x_h)\dot{\phi}\right]. \tag{30}$$

It is worth mentioning now that rewriting the first term in (30) in terms of $\bar{R}$ using the exact equation (17) yields a structure that precisely cancels the second term in (30) thereby leading to a vanishing covariant divergence of the energy-momentum tensor. This way, however, the effects of the near-horizon approximation are bypassed and the exact result is expectedly reproduced. This is just the analogue of computing the trace (21) by using the exact equation of motion (17). In order to systematically implement the near-horizon approximation that would be consistent with getting a vanishing trace (20) [6], we adopt the previous interpretation, that is take the equation of motion as (18) with $g(x)$ vanishingly small but not zero. This leads to

$$\bar{\nabla}_a T^a{}_t=\frac{q\Phi_h}{8}g''(x_h)\dot{\phi}. \tag{31}$$

For $b=z$, we obtain (after using the near-horizon equation of motion (18)):

$$\bar{\nabla}_a T^a{}_z=\frac{q\Phi_h}{8}g''(x_h)\partial_z\phi. \tag{32}$$

The above equations can be compactly written as

$$\bar{\nabla}_a T^a{}_b=\frac{q\Phi_h}{8}g''(x_h)\partial_b\phi. \tag{33}$$

It is important to observe that the near-horizon equation of motion for ϕ (18) (which is different from the equation of motion satisfied by ϕ away from the horizon (6)) plays an important role in the derivation of (33). We therefore

[2]Note that in the (t,z)-coordinates, the metric (12) reads $\mathrm{d}s^2=-g(x)\,\mathrm{d}t^2+g(x)\,\mathrm{d}z^2$. Hence, $\Lambda_{tt}=-g(x)$ and $\Lambda_{zz}=g(x)$.

conclude that the classical Liouville theory, when considered in the region near to the horizon, respects conformal symmetry but it does not preserve the diffeomorphism invariance.

This is a new result in our paper. We find that though we are able to keep the conformal invariance intact, the near-horizon theory does not preserve the diffeomorphism invariance thereby clearly pointing out the interplay between the diffeomorphism and conformal invariance exhibited in the case of black holes in the two-dimensional classical Liouville theory. To put this result in a proper perspective, we recall the findings of [6] where it was shown that the conformal symmetry near the horizon of the black hole leads to a Virasoro algebra among the Fourier transform of specific combinations of the components of the energy-momentum tensor. Here, we show that the Virasoro algebra which is a reflection of conformal symmetry near the horizon of the black hole can also be understood as the breaking of diffeomorphism symmetry near the black-hole horizon.

This observation is similar to that of quantum anomalous theories. Indeed, when we quantize the scalar field theory on general curved background, the trace of $\langle T_{ab}\rangle$ turns out to be nonzero. In particular, for the nonchiral theory in $1+1$ dimensions, $\langle T^a{}_a\rangle$ is proportional to the curvature scalar. However, the regularization adopted to compute the trace anomaly preserves the diffeomorphism invariance. One may adopt a different type of regularization which spoils the diffeomorphism symmetry but keeps conformal symmetry intact. It turns out that in $D=2$ dimensions there is no regularization prescription which preserves the conformal as well as diffeomorphism invariance simultaneously [1,2].

As a side remark we mention that it might be possible to write an improved stress tensor from (33) that is conserved,

$$\hat{T}^a{}_b = T^a{}_b - \frac{q\Phi_h}{8} g''(x_h)\delta^a_b\phi. \tag{34}$$

However, as is easily observed, this tensor is no longer traceless, since

$$\hat{T}^a{}_a = \frac{q\Phi_h}{4}\bar{R}\phi. \tag{35}$$

Consequently, simultaneous imposition of both Ward identities is not feasible. This shift of anomaly by using counterterms is more akin to what is done in quantum field theory. A similar phenomenon for the Liouville theory was also observed, though in a different context [14].

The fact that in the region near the horizon, the energy-momentum tensor is not covariantly conserved (33) can also be inferred by computing the classical Poisson algebra among the various light-cone components of the energy-momentum tensor. It turns out that the Poisson algebra does not close which in general, is related to the breaking of either diffeomorphism or conformal invariance. This in fact justifies our approximation of keeping terms up to first order in $(x-x_h)$ in $g'(x)$ (25) in the evaluation of the covariant divergence of the energy-momentum tensor. It is important to note that if we compute $g'(x)$ from the near-horizon expansion of $g(x)$ (14), it would finally lead to the conservation of the energy-momentum tensor. This would be in direct clash with the non-closure of the Poisson algebra.

We begin our analysis of the classical Poisson algebra by writing (26)–(28) in the light-cone coordinates:

$$\begin{aligned} x^+ &= \frac{1}{\sqrt{2}}(t+z),\\ x^- &= \frac{1}{\sqrt{2}}(t-z). \end{aligned} \tag{36}$$

Then we have

$$\begin{aligned} T_{++} &= T_{00}+T_{0z}\\ &= \frac{1}{4}\left(\dot{\phi}+\partial_z\phi\right)^2 - \frac{q\Phi_h}{4}\Bigg(\partial_z^2\phi+\partial_z\dot{\phi}\\ &\quad -\frac{1}{2}\left[\frac{2}{\beta_H}+\frac{g''(x_h)}{2}(x-x_h)\right](\partial_z\phi+\dot{\phi})\Bigg), \end{aligned} \tag{37}$$

$$\begin{aligned} T_{--} &= T_{00}-T_{0z}\\ &= \frac{1}{4}\left(\dot{\phi}-\partial_z\phi\right)^2 - \frac{q\Phi_h}{4}\Bigg(\partial_z^2\phi-\partial_z\dot{\phi}\\ &\quad -\frac{1}{2}\left[\frac{2}{\beta_H}+\frac{g''(x_h)}{2}(x-x_h)\right](\partial_z\phi-\dot{\phi})\Bigg). \end{aligned} \tag{38}$$

Next, we give the basic Poisson brackets among the canonical variables [6]

$$\{\phi(z),\dot{\phi}(z')\} = \delta(z-z'), \tag{39}$$

$$\{\phi(z),\phi(z')\} = 0, \tag{40}$$

$$\{\dot{\phi}(z),\dot{\phi}(z')\} = 0. \tag{41}$$

The Poisson algebra between $T_{++}(z)$ and $T_{++}(z')$ therefore reads

$$\begin{aligned} \{T_{++}(z),T_{++}(z')\} &= (T_{++}(z)+T_{++}(z'))\,\partial_z\delta(z-z')\\ &+\frac{q^2\Phi_h^2}{8}\Bigg[-\partial_z^3\delta(z-z')+\frac{g'(x)g'(x')}{4}\partial_z\delta(z-z')\\ &+\frac{1}{2}[g'(x)-g'(x')]\partial_z^2\delta(z-z')\Bigg] \end{aligned} \tag{42}$$

with $g'(x)$ given by (25). Note that here, we have only given the Poisson bracket among T_{++} components of the EM tensor because the algebra between $T_{--}(z)$ and $T_{--}(z')$ is identical in structure, while the Poisson bracket among T_{++} and T_{--} is zero. Now neglecting terms proportional to $(x-x_h)$ in the above algebra leads to

$$\begin{aligned} \{T_{++}(z),T_{++}(z')\} &= (T_{++}(z)+T_{++}(z'))\,\partial_z\delta(z-z')\\ &+\frac{q^2\Phi_h^2}{8}\left[-\partial_z^3\delta(z-z')+\frac{1}{\beta_h^2}\partial_z\delta(z-z')\right]. \end{aligned} \tag{43}$$

This algebra would have been the same if we had substituted $g'(x)=\frac{2}{\beta_H}$ in (22)–(24). However, in that case we would not have found any violation in the conservation of the energy-momentum tensor (33) which would be inconsistent with the fact that the Poisson algebra (43) does not close.

For the sake of completeness, we now move on to investigate the Virasoro algebra. The Virasoro generator is defined as [6]

$$L_n = \frac{L}{2\pi}\int_{L/2}^{L/2} \mathrm{d}z\ T_{++}(z)e^{2i\pi nz/L}. \tag{44}$$

From the algebra given in (43), we compute the algebra between the Virasoro generators which yields

$$\{L_n, L_m\} = (n-m)L_{n+m} + \frac{c}{12}n\left(n^2 + \left(\frac{L}{2\pi\beta_h}\right)^2\right)\delta_{n+m,0}. \tag{45}$$

This is the expression for the classical Virasoro algebra with the central charge $c = 3\pi q^2\Phi_h^2$ [6].

Discussions. – In this article we have studied thoroughly the interplay between the diffeomorphism and conformal symmetries for black holes in the $D=2$ classical Liouville theory. The energy-momentum tensor derived from the $D=2$ classical Liouville action, in general, is covariantly conserved. However, the trace of the energy-momentum tensor becomes nonzero leading to violation of the conformal invariance.

In the region near to the horizon, the Liouville theory shows interesting behavior. In the vicinity of the horizon we expand the metric function about the horizon and keep only the leading-order terms. In this approximation, the equations of motion for the Liouville field takes the form of $D=2$ free, massless Klein-Gordon equation and eventually makes the energy-momentum tensor traceless. Hence, in the vicinity of the horizon the Liouville theory is conformally invariant. However, we observe that, in the near-horizon limit the energy-momentum tensor is not covariantly conserved. In fact, we showed that the covariant divergence of the energy-momentum tensor is proportional to the curvature scalar. This fact is quite well known in the context of the quantization of fields on the general curved background. Classically, the energy-momentum tensor, of the field theory under consideration, is traceless and also covariantly conserved. However, during the process of quantization, either there is a trace or diffeomorphism anomaly, depending upon the choice of the regularization. It is impossible to preserve both, the conformal as well as the general coordinate (diffeomorphism) invariance [1,2]. In this paper, for the classical Liouville theory, we have a similar kind of behavior of the energy-momentum tensor. Away from the horizon, the classical energy-momentum tensor is covariantly conserved signalling the presence of diffeomorphism invariance but the conformal symmetry is lost due to a nonvanishing trace. Near the horizon, on the contrary, the diffeomorphism symmetry is broken but the conformal symmetry is intact. This interplay is a new finding in the context of the classical Liouville theory in the near-horizon approximation.

Another feature in our paper is that the Virasoro algebra which is a reflection of the conformal invariance near the horizon of the black hole can also be understood as the breaking of the diffeomorphism invariance near the black-hole horizon.

Here we would like to mention that similar results, though in a different context, were discussed in [14]. To be more specific counterterms were added in the parent $D=2$ Liouville action in order to restore Weyl invariance which however, breaks diffeomorphism symmetry. The point is that counterterms, taken in [14], are a manifestation of regularization ambiguities. However, the issue of regularization is meaningful only in the quantum field theory. In our example no counterterms are necessary. The interplay of the symmetries occurs very naturally with the near-horizon results displaying one feature while those away from the horizon display another feature. The whole analysis, either at the technical or conceptual level, is completely classical.

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REFERENCES

[1] Polyakov A., *Gauge Fields and Strings* (Harwood, Chur, Switzerland) 1987.

[2] Green M., Schwartz J. and Witten E., *Superstring Theory,* Vol. **1** (Cambridge University Press) 1987.

[3] Bertlmann R., *Anomalies in Quantum Field Theory* (Oxford Sciences, Oxford) 2000.

[4] Alvarez-Gaume L. and Witten E., *Nucl. Phys. B*, **234** (1984) 269.

[5] Coussaert O., Henneaux M. and van Driel P., *Class. Quantum Grav.*, **12** (1995) 2961.

[6] Solodukhin S., *Phys. Lett. B*, **454** (1999) 213 (hep-th/9812056).

[7] Cvitan M., Pallua S. and Prester P., *Phys. Lett. B*, **546** (2002) 119 (hep-th/0207265).

[8] Kang G., Koga J. and Park M., *Phys. Rev. D*, **70** (2004) 024005 (hep-th/0402113).

[9] Pinamonti N. and Vanzo L., *Phys. Rev. D*, **69** (2004) 084012 (hep-th/0312065).

[10] Cvitan M. and Pallua S., *Phys. Rev. D*, **71** (2005) 104032 (hep-th/0412180).

[11] Chen Y., *Class. Quantum Grav.*, **21** (2004) 1153 (hep-th/0310234).

[12] Solodukhin S., *Phys. Rev. Lett.*, **92** (2004) 061302 (hep-th/0310012).

[13] Russo J. G., *Phys. Lett. B*, **359** (1995) 69.

[14] Jackiw R., *Theor. Math. Phys.*, **148** (2006) 941 (hep-th/0511065).