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Semileptonic decays of pseudoscalar mesons to the scalar f_0 meson

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Abstract – The transition form factors of $D_s \to f_0\ell\nu$, $D \to f_0\ell\nu$ and $B_u \to f_0\ell\nu$ decays are calculated within the 3-point QCD sum rule method, assuming that f_0 is a quark-antiquark state with a mixture of strange and light quarks. Having obtained the expressions of the transition form factors, the branching ratios of these decays are calculated. The experimental measurement of the branching ratios of these decays can provide a direct estimation of the mixing angle.

Introduction. – The inner structure of the scalar mesons in terms of quarks is still an open question in particle physics and it is the subject of intense and continuous theoretical and experimental investigations for establishing their nature (for a review, see [1]). There are numerous scenarios for the classification of the scalar mesons. The established 0^{++} mesons are divided into two groups: 1) near and above 1 GeV, and 2) in the region 1.3 GeV–1.5 GeV. The first-group scalar mesons form an $SU(3)$ nonet, which contains two isosinglets, an isotriplet and two strange isodoublets. In the quark model, the flavor structure of these scalar mesons would be

$$\begin{aligned}
\sigma &= \cos\theta(\bar{n}n) - \sin\theta(\bar{s}s),\\
f_0 &= \cos\theta(\bar{s}s) + \sin\theta(\bar{n}n),\\
a_0^0 &= \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d), \qquad a_0^+ = u\bar{d}, \qquad a_0^- = \bar{d}u,\\
\kappa^+ &= \bar{s}u, \qquad \bar{\kappa}^0 = \bar{d}s, \qquad \kappa^- = \bar{u}s, \qquad \kappa^0 = \bar{s}d,
\end{aligned}$$

where $\bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$, and θ is the mixing angle. Here we take into account the fact that between isoscalars $\bar{s}s$ and $\bar{u}u + \bar{d}d$ there is mixing, which follows from experiments. Indeed the observation

$$\Gamma(J/\psi \to f_0\omega) \simeq \frac{1}{2}\Gamma(J/\psi \to f_0\phi)$$

indicates that the quark content of $f_0(980)$ (hereafter we shall denote the $f_0(980)$ meson as f_0) is not a pure $\bar{s}s$ state, but should have non-strange parts too [2]. Secondly, if f_0 is a pure $\bar{s}s$ state, then there is no phase space for $f_0 \to KK^0$, and hence the OZI-suppressed $f_0 \to \pi\pi$ mode becomes favorable. But the decay width of f_0 is dominated by $f_0 \to \pi\pi$ which leads to the conclusion that in f_0 there should be $\bar{n}n$ parts as well. Therefore f_0 should be a mixture of $\bar{s}s$ and $\bar{n}n$, as is presented in eq. (1). The analysis of the experimental data shows that the mixing angle θ lies in the range $25° < \theta < 40°$ or $140° < \theta < 165°$ [3].

Although there is another scenario where mesons below or about 1 GeV are described as a four-quark state (see for example [4]), in this work we restrict ourselves to considering the $\bar{q}q$ description for the f_0 meson, but taking into account the mixing between $\bar{s}s$ and $\bar{n}n$. In the present work we study the semileptonic decays $B^+ \to f_0\ell^+\nu$, $D^+_{d,s} \to f_0\ell^+\nu$ in order to get information about the quark content of f_0.

From the theoretical point of view, the investigation of the semileptonic decays is simpler compared to that of hadronic decays, because leptons do not participate in strong interactions. The experimental study of weak semileptonic decays of heavy flavored mesons is very important for the more accurate determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, their leptonic decay constants, etc.

The precise determination of the CKM matrix elements depends crucially on the possibility of controlling long-distance interaction effects. So, in the study of the exclusive semileptonic decays the main problem is the calculation of the transition form factors, which involves the long-distance QCD dynamics, belonging to the non-perturbative sector of QCD. For this reason, in the calculation of the transition form factors some kind of non-perturbative approach is needed. Among all non-perturbative approaches the QCD sum rules method [5]

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is more powerful, since it is based on the first principles of QCD. About the most recent status of QCD sum rules, the interested readers are advised to consult [6].

Semileptonic decays $D\to\bar{K}^0 e\bar{\nu}_e$ [7], $D^+\to K(K^{0*})e^+\nu_e$ [8], $D\to\pi e\bar{\nu}_e$ [9], $D\to\rho e\bar{\nu}_e$ [10], $B\to D(D^*)\ell\bar{\nu}_\ell$ [11] and $D\to\phi\ell\bar{\nu}_\ell$ [12] are all studied in the framework of the 3-point QCD sum rules method. Recently, the $B_s\to f_0\ell^+\ell^-$ and $\mathcal{D}_s\to f_0 e^+\nu_e$ decays are analysed within the light cone QCD sum rules method in [13].

In this work we study the semileptonic $B_u\to f_0\ell^+\nu_\ell$ and $D_{s(d)}\to f_0\ell^+\nu_\ell$ decays in the 3-point QCD sum rules method. The paper is organized as follows: in the second section, we derive the sum rules for the form factors, responsible for the pseudoscalar to scalar meson transition. The third section is devoted to the numerical analysis of the transition form factors and discussion and contains our conclusions.

Pseudoscalar-scalar meson transition form factors from QCD sum rules. – For calculating the pseudoscalar-scalar meson transition form factors in QCD sum rules, the leptonic decay constant of scalar mesons is needed. Obviously, the semileptonic decay rate should depend critically on the coupling of resonances to the quark current.

Remember that for the mixing scheme in the flavor basis [14,15] σ and f_0 states can be written as a combination of $|\bar{n}n\rangle=(\bar{u}u+\bar{d}d)/\sqrt{2}$ and $|\bar{s}s\rangle$ states as follows:

$$\begin{aligned}|\sigma\rangle &= \cos\theta_q|\bar{n}n\rangle-\sin\theta_s|\bar{s}s\rangle,\\ |f_0\rangle &= \sin\theta_q|\bar{n}n\rangle+\cos\theta_s|\bar{s}s\rangle.\end{aligned}\qquad(1)$$

It is shown in [14] that in this scheme a single mixing angle is required, since $|\theta_s-\theta_q|/|\theta_s+\theta_q|\ll 1$, and this is confirmed from QCD sum rules calculation [15]. For this reason one can assume that $\theta_q=\theta_s=\theta$. In QCD sum rules we deal with interpolating currents, and for this reason we choose the interpolating current of the scalar f_0 meson in the following form:

$$J_{f_0}=\cos\theta\,\bar{s}s+\sin\theta\frac{1}{\sqrt{2}}(\bar{u}u+\bar{d}d).\qquad(2)$$

The coupling constant of the f_0 meson to the current (1) can be parametrized as

$$\langle 0\,|J_{f_0}|\,f_0\rangle=\lambda_{f_0}.\qquad(3)$$

The coupling constant λ_{f_0} is calculated in [16], using the two-point correlation function (the same correlation function is studied in [17] for the case $\theta=0$, and it is obtained that $\lambda_{f_0}=0.18\pm0.015\,\text{GeV}$), where the interpolating current for the f_0 meson is taken in the form as presented in eq. (2), and which predicts that $\lambda_{f_0}=(0.19\pm0.02)\,\text{GeV}^2$. In further numerical calculations, we will use this value of λ_{f_0}.

Having calculated the value of λ_{f_0}, our concern now is to determine the pseudoscalar *D(B)*-scalar f_0 transition form factors. Pseudoscalar-scalar transition form factors are defined via the matrix element of the weak current sandwiched between initial and final meson states $\langle f_0(p')\,|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|\,P(p)\rangle$, where q_1 and q_2 are the relevant quarks, P and f_0 are the pseudoscalar and scalar f_0 meson states, respectively. It follows from parity conservation in strong interactions that only the axial part of the weak current gives non-zero contribution to this matrix element, and imposing Lorentz invariance, it can be written in terms of the form factors as follows:

$$\langle f_0(p')\,|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|\,P(p)\rangle=-i\mathcal{A}[f_+(p+p')_\mu+f_-q_\mu],\qquad(4)$$

where $q_\mu=p_1-p_2$, $f_+(q^2)$, $f_-(q^2)$ are the transition form factors, and

$$\mathcal{A}=\begin{cases}\cos\theta \text{ for } D_s\to f_0,\\ \dfrac{\sin\theta}{\sqrt{2}} \text{ for } D\to f_0, \text{ and } B_u\to f_0.\end{cases}$$

For the evaluation of these form factors in the QCD sum rule, we consider the following 3-point correlation function:

$$\begin{aligned}\Pi_\mu(p^2,p'^2,q^2) &= -\int \mathrm{d}^4x\,\mathrm{d}^4y\,e^{i(p'y-px)}\\ &\times\left\langle 0\left|T\left\{J_{f_0}(y)J_\mu^A(0)J_P(x)\right\}\right|0\right\rangle,\end{aligned}\qquad(5)$$

where $J_\mu^A=\bar{q}_2\gamma_\mu\gamma_5 q_1$ and $J_P=\bar{q}_1\gamma_5 q_2$ are the interpolating currents of scalar and pseudoscalar mesons, and weak axial currents, and J_{f_0} is the interpolating current of the f_0 meson given in eq. (2), respectively. It should be noted here that, $q_3=u$, $q_2=u$ and $q_1=b$ for the $B_u\to f_0$ transition; and $q_3=s(d)$, $q_2=s(d)$ and $q_1=c$ for the $D_{s(d)}\to f_0$ transition, respectively.

The decomposition of the correlation function (4) into the Lorentz structures, obviously, has the form

$$\Pi_\mu=\Pi_+(p+p')_\mu+\Pi_-(p-p')_\mu.\qquad(6)$$

For the amplitudes Π_+ and Π_-, we have the following dispersion relation:

$$\begin{aligned}\Pi_\pm(p^2,p'^2,Q^2) &= -\frac{1}{(2\pi)^2}\int\frac{\rho_\pm(s,s',Q^2)\,\mathrm{d}s\,\mathrm{d}s'}{(s-p^2)(s'-p'^2)}\\ &+\text{subtraction terms},\end{aligned}\qquad(7)$$

where $\rho_\pm$ is the corresponding spectral density and $Q^2=-q^2>0$. According to the QCD sum rules approach, the correlation function is calculated by the operator product expansion (OPE) at large Euclidean momenta p^2 and p'^2 on the one hand, and on the other hand it is calculated by inserting a complete set of intermediate states having the same quantum numbers with the currents J_{f_0} and J_P.

The phenomenological part of (4) is obtained by saturating correlator with the lowest pseudoscalar (in our case B_u, D_s or D mesons) and scalar f_0 mesons, yielding

$$\begin{aligned}\Pi_\mu &= \frac{\langle 0\,|J_{f_0}|\,f_0(p')\rangle\left\langle f_0(p')\left|J_\mu^A(0)\right|P(p)\right\rangle\langle P(p)\,|J_P(x)|\,0\rangle}{(m_{f_0}^2-p'^2)(m_P^2-p^2)}\\ &+\text{excited states}.\end{aligned}\qquad(8)$$

The matrix elements in eq. (8) are defined as

$$\langle 0\,|J_{f_0}|\,f_0(p')\rangle = \lambda_{f_0},$$
$$\langle P\,|J_P|\,0\rangle = -i\frac{m_P^2 f_P}{m_1+m_2}, \qquad (9)$$

where λ_{f_0} and f_P are the leptonic decay constants of scalar and pseudoscalar mesons, and m_{f_0} and m_P are their masses, respectively. Note that the leptonic decay constant λ_{f_0} in eq. (9) is scale dependent for which we choose the scale to be $\mu = 1\,\mathrm{GeV}^2$, and

$$m_1 = \begin{cases} m_b, & \text{for } B_u \to f_0\ell\nu, \\ m_c, & \text{for } D_s \to f_0\ell\nu, \quad D \to f_0\ell\nu, \end{cases}$$

$$m_2 = \begin{cases} m_u, & \text{for } B_u \to f_0\ell\nu, \quad D \to f_0\ell\nu, \\ m_s, & \text{for } D_s \to f_0\ell\nu. \end{cases}$$

Using eqs. (4), (6), (8) and (9), for the invariant structures we get

$$\Pi_\pm = -\frac{f_P m_P^2}{m_1+m_2}\,\frac{\mathcal{A}\lambda_{f_0} f_\pm}{(m_{f_0}^2 - p'^2)(m_P^2 - p^2)}\,. \qquad (10)$$

From the QCD side, the correlation function can be calculated with the help of the OPE at short distance, and in this work we will consider operators up to dimension six. The theoretical part of the correlator for $B_s \to D_{s_0}(2317)\ell\nu$ is calculated in [18], and in the present work, for the theoretical part of the corresponding sum rules, we will use the results of this work.

For the spectral densities we have

$$\rho_+ = \frac{\mathcal{A}N_c}{4\lambda^{1/2}(s,s',Q^2)}[(\Delta'+\Delta)(1+A+B) + (m_1^2+2m_1m_2+Q^2)(A+B)], \qquad (11)$$

$$\rho_- = \frac{\mathcal{A}N_c}{4\lambda^{1/2}(s,s',Q^2)}[(\Delta'+\Delta+m_1^2+2m_1m_2+Q^2) \times(A-B)+\Delta'-\Delta-2m_1m_2], \qquad (12)$$

where $N_c = 3$, $\Delta = s - m_1^2$, $\Delta' = s' - m_2^2$, $\lambda(s,s',Q^2) = (s+s'+Q^2)^2 - 4\,ss'$, and

$$A = \frac{1}{\lambda(s,s',Q^2)}[-(s+s'+Q^2)\Delta' + 2\,s'\Delta],$$
$$B = \frac{1}{\lambda(s,s',Q^2)}[-(s+s'+Q^2)\Delta + 2\,s\Delta'].$$

For the decays under consideration, m_2 is $m_u(m_d)$ or m_s, and therefore, to take into account $SU(3)$-violating effects, here and in all following calculations, we will retain terms that are linear with m_2, and neglect the terms higher order in m_2.

For power corrections (PC) we get

see eq. (13) *on the next page*

see eq. (14) *on the next page*

where $r = p^2 - m_1^2$ and $r' = p'^2$. Note that the $D_s \to f_0\ell^+\nu_\ell$ and $D \to f_0\ell^+\nu_\ell$ decays which are considered in [16] differ from our results in three aspects:

- Our result on the spectral density is two times smaller compared to that given in [16]. Since it is known that the main contribution to the sum rules comes from the spectral density, it is indispensable that our results on the form factors differ from those predicted in [16].
- In [16], part of those diagrams which are proportional to m_s are not taken into account (in our case they correspond to the terms proportional to $m_2 m_0^2\langle\bar{q}_2 q_2\rangle$).
- Sum rules for the form factor f_- are totally absent in [16], which could be essential for the $B_u \to f_0\tau\nu_\tau$ decay.

Contributions of higher states in the physical part of the sum rules are taken into account with the help of the hadron-quark duality, *i.e.*, corresponding spectral density for higher states is equal to the perturbative spectral density for s_0 and s_0' starting from $s > s_0$ and $s' > s_0'$, where s and s' are the continuum thresholds in the corresponding channels.

Equating the two representations for the invariant structures $\Pi_\pm$, and applying the double Borel transformation on the variables p^2 and p'^2 ($p^2 \to M^2$, $p'^2 \to M'^2$) in order to suppress the higher states and continuum contributions, we get the following sum rules for the form factors f_+ and f_-:

$$\lambda_{f_0} f_\pm(q^2) e^{-m_{f_0}^2/M^2} = -\frac{m_1+m_2}{f_P m_P^2} e^{m_P^2/M^2} \times\left\{\int \mathrm{d}s\,\mathrm{d}s'\rho'_\pm(s,s',Q^2)e^{-s/M^2-s'/M'^2} + \mathcal{B}_{M^2}\mathcal{B}_{M'^2}\Pi'^{PC}_\pm\right\}, \qquad (15)$$

where the prime on $\rho_\pm$ and $\Pi^{PC}_\pm$ refers to eqs. (11)–(14) without the multiplying factor $\mathcal{A}$. The double Borel transformation for the quantity $1/r^n r'^m$ is defined as

$$\mathcal{B}_{M^2}\mathcal{B}_{M'^2}\frac{1}{r^n r'^m} = (-1)^{n+m}\frac{(M^2)^{n-1}}{\Gamma(n)}\frac{(M'^2)^{m-1}}{\Gamma(m)} \times e^{-m_1^2/M^2}e^{-m_2^2/M'^2}. \qquad (16)$$

The integration region for the perturbative contribution is determined from the following inequalities:

$$-1 \leqslant \frac{2ss' + (m_1^2 - s)(s+s'+Q^2)}{\lambda^{1/2}(s,s',Q^2)(m_1^2-s)} \leqslant 1. \qquad (17)$$

$$\Pi_+^{PC} = \mathcal{A}\Bigg\{\frac{1}{2}\langle\bar{q}_2 q_2\rangle\frac{m_1-m_2}{rr'}+\frac{1}{4}m_2\langle\bar{q}_2 q_2\rangle\left(\frac{m_1^2}{r^2r'}-\frac{2}{rr'}\right)-\frac{1}{12}m_0^2\langle\bar{q}_2 q_2\rangle$$
$$\times\left[\frac{3m_1^2(m_1-m_2)}{r^3r'}+\frac{2(m_1-2m_2)}{rr'^2}+\frac{2(2m_1-m_2)}{r^2r'}+\frac{m_1(2m_1^2+m_1m_2+2Q^2)-2m_2(m_1^2+Q^2)}{r^2r'^2}\right]$$
$$+\frac{4}{81}\pi\alpha_s\langle\bar{q}_2 q_2\rangle^2\Bigg[-\frac{12m_1^3(m_1-m_2)}{r^4r'}+\frac{8m_1m_2(m_1^2+Q^2)}{r^2r'^3}+\frac{56m_1m_2}{rr'^3}$$
$$-\frac{4m_1^2(2m_1^2+m_1m_2+2Q^2)-8m_1m_2(m_1^2+Q^2)}{r^3r'^2}-\frac{8m_1(8m_1-7m_2)}{r^3r'}+\frac{48}{rr'^2}+\frac{48}{r^2r'}-\frac{4(5m_1^2-20m_1m_2-2Q^2)}{r^2r'^2}\Bigg]$$
$$+\frac{1}{9}m_0^2m_2\langle\bar{q}_2 q_2\rangle^2\left[-\frac{m_1^2(m_1^2+Q^2)}{r^3r'^2}+\frac{5m_1^2+4Q^2}{r^2r'^2}+\frac{6m_1^4}{r^4r'}+\frac{10m_1^2}{r^3r'}\right]\Bigg\}, \quad (13)$$

$$\Pi_-^{PC} = \mathcal{A}\Bigg\{-\frac{1}{2}\langle\bar{q}_2 q_2\rangle\frac{m_1+m_2}{rr'}+\frac{1}{4}m_1m_2\langle\bar{q}_2 q_2\rangle\left(-\frac{m_1}{r^2r'}\right)+\frac{1}{12}m_0^2\langle\bar{q}_2 q_2\rangle$$
$$\times\left[\frac{3m_1^2(m_1+m_2)}{r^3r'}+\frac{2(m_1+3m_2)}{rr'^2}+\frac{2(3m_1+m_2)}{r^2r'}+\frac{m_1(2m_1^2+m_1m_2+2Q^2)+2m_2(m_1^2+Q^2)}{r^2r'^2}\right]$$
$$+\frac{1}{81}\pi\alpha_s\langle\bar{q}_2 q_2\rangle^2\Bigg[\frac{12m_1^3(m_1+m_2)}{r^4r'}-\frac{8m_1m_2(m_1^2+Q^2)}{r^2r'^3}-\frac{56m_1m_2}{rr'^3}$$
$$+\frac{4m_1^2(2m_1^2+m_1m_2+2Q^2)+8m_1m_2(m_1^2+Q^2)}{r^3r'^2}+\frac{8m_1(9m_1+7m_2)}{r^3r'}+\frac{28m_1^2}{r^2r'^2}+\frac{8}{rr'^2}-\frac{8}{r^2r'}\Bigg]$$
$$+\frac{1}{9}m_0^2m_2\langle\bar{q}_2 q_2\rangle^2\left[\frac{m_1^2(m_1^2+Q^2)}{r^3r'^2}-\frac{m_1^2}{r^2r'^2}-\frac{6m_1^4}{r^4r'}+\frac{4}{rr'^2}-\frac{4}{r^2r'}-\frac{24m_1^2}{r^3r'}\right]\Bigg\}, \quad (14)$$

Numerical analysis. – In this section we present our results for the form factors $f_+(q^2)$ and $f_-(q^2)$ for the decays under consideration. The main input parameters for the sum rules are the Borel parameters M^2 and M'^2 and the continuum thresholds s_0 and s_0'. The values of other parameters needed are: $m_b=(4.7\pm0.1)\,\mathrm{GeV}$ [6], $m_c=1.4\,\mathrm{GeV}$, $\langle\bar{u}u\rangle|_{\mu=1\,\mathrm{GeV}}=-(1.65\pm0.15)\times10^{-2}\,\mathrm{GeV}^3$, $\langle\bar{s}s\rangle=0.8\times\langle\bar{u}u\rangle$ [19], $m_s(\mu=2\,\mathrm{GeV})=(102\pm8)\,\mathrm{MeV}$ for $\Lambda=381\pm16\,\mathrm{MeV}$ [20]. The values of the leptonic decay constants of B_u, D_s and D mesons are determined from the analysis of the corresponding two-point correlators: $f_{B_u}=(0.14\pm0.01)\,\mathrm{GeV}$ [21], $f_{D_s}=(0.22\pm0.02)\,\mathrm{GeV}$ [22] and $f_D=(0.17\pm0.02)\,\mathrm{GeV}$ [6,20,22]. For the continuum thresholds we take the values $s_0^{B_u}=(33\pm2)\,\mathrm{GeV}^2$, $s_0^{D_s}=(7.7\pm1.1)\,\mathrm{GeV}^2$, $s_0^D=(6\pm0.2)\,\mathrm{GeV}^2$ and $s_0'=1.6\pm0.1)\,\mathrm{GeV}^2$ which is determined from 2-point sum rules analysis [6,14,23]. Using more recent experimental data, the following values for the leptonic decay constants f_D, f_{D_s} and f_B are obtained in [24]:

$$f_{D_s}=(0.257\pm0.061)\,\mathrm{MeV},$$
$$f_D=(0.207\pm0.009)\,\mathrm{MeV},$$
$$f_B=(0.193\pm0.011)\,\mathrm{MeV}. \quad (18)$$

The Borel parameters M^2 and M'^2 are the auxiliary parameters and therefore the physical quantities should be independent of them. For this reason we need to find the working regions of M^2 and M'^2 where form factors are practically independent of them.

In obtaining the working regions of M^2 and M'^2 the following two conditions should be satisfied:

- the continuum contribution should be small, and,
- power corrections should be convergent.

Our numerical analysis shows that both conditions are satisfied in the region $10\,\mathrm{GeV}^2\leqslant M^2\leqslant20\,\mathrm{GeV}^2$ for $B_u\to f_0\ell\bar{\nu}_\ell$, $4\,\mathrm{GeV}^2\leqslant M^2\leqslant8\,\mathrm{GeV}^2$ for $D_s(D)\to f_0\ell\bar{\nu}_\ell$, and $1.2\,\mathrm{GeV}^2\leqslant M'^2\leqslant2\,\mathrm{GeV}^2$ for all channels.

Varying the input parameters s_0, s_0', f_0, f_{D_s}, f_B and f_D in the respective regions as mentioned in the text, we get the following results for the form factors at $q^2=0$:

$$f_+^{B_u}(0)=0.59\pm0.18\ (0.42\pm0.13),$$
$$f_-^{B_u}(0)=-0.58\pm0.18\ (-0.41\pm0.13),$$
$$f_+^{D}(0)=0.68\pm0.23\ (0.57\pm0.19),$$
$$f_+^{D_s}(0)=0.48\pm0.23\ (0.28\pm0.14), \quad (19)$$

where the errors come from the uncertainties in the variation of the Borel parameters M^2 and M'^2 and

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} &= \frac{1}{192\pi^3 m_P^3} G^2 \left|V_{ij}\right|^2 \lambda^{1/2}(m_P^2, m_{f_0}^2, q^2) \left(\frac{q^2 - m_\ell^2}{q^2}\right)^2 \\ &\times \left\{ -\frac{(2q^2 + m_\ell^2)}{2} \left[\left|f_+(q^2)\right|^2 (2m_P^2 + 2m_{f_0}^2 - q^2) + 2(m_P^2 - m_{f_0}^2)\,\mathrm{Re}\left[f_+(q^2) f_-^*(q^2)\right] + \left|f_-(q^2)\right|^2 q^2 \right] \right. \\ &+ \left. \frac{(q^2 + 2m_\ell^2)}{q^2} \left[\left|f_+(q^2)\right|^2 (m_P^2 - m_{f_0}^2)^2 + 2(m_P^2 - m_{f_0}^2) q^2\,\mathrm{Re}\left[f_+(q^2) f_-^*(q^2)\right] + \left|f_-(q^2)\right|^2 q^4 \right] \right\} |\mathcal{A}|^2, \quad (21)\end{aligned}$$

Table 1: Form factors for the $D_s \to f_0 \ell \bar{\nu}_\ell$, $D \to f_0 \ell \bar{\nu}_\ell$ and $B_u \to f_0 \ell \bar{\nu}_\ell$ decays in a four-parameter fit.

	$f_+(0)$	$f_-(0)$	a	b	c	d
D_s	0.48		0.81	−0.18	0.19	0.86
D	0.68		0.82	−0.40	0.21	−1.00
B_u	0.59		0.51	−0.21	−0.47	−0.95
B_u		−0.58	0.46	−0.30	−0.84	−1.36

continuum thresholds s_0 and s_0', as well as from the uncertainties in the determination of the input parameters entering into the sum rules. Note that we present the form factor f_- only for the $B_u \to f_0 \tau \bar{\nu}_\tau$ decay, because this form factor can give considerable contribution to this decay, because using the Dirac equation one can see that f_- is multiplied to the lepton mass. The values of the form factors presented in the paranthesis are obtained by using the values of the leptonic decay constants presented in eq. (18).

In estimating the width of $P \to f_0 \ell \bar{\nu}_\ell$ decay, we need to know the q^2-dependence of the form factors $f_+(q^2)$ and $f_-(q^2)$ in the whole kinematical region $m_\ell^2 \leqslant q^2 \leqslant (m_P - m_{f_0})^2$. The q^2-dependence of the form factors can be calculated from QCD sum rules (see [8,9]). Unfortunately QCD sum rules cannot reliably predict the q^2-dependence of the form factors in the full kinematical region. The QCD sum rules can reliably predict the q^2-dependence of the form factors in the region approximately $1\,\mathrm{GeV}^2$ below the perturbative cut. In order to extend the dependence of the form factors on q^2 to the full kinematical region, we look for such a parametrization of the form factors where they coincide with the sum rules prediction in the above-mentioned region. Our numerical calculations shows that the best parametrization of the form factors with respect to q^2 is as follows:

$$f_\pm^P(q^2) = \frac{f_\pm^P(0)}{1 - a_P \hat{q} + b_P \hat{q}^2 - c_P \hat{q}^3 + d_P \hat{q}^4}, \quad (20)$$

where $P = B_u, D_s, D$ and $\hat{q} = q^2/m_P^2$. The values of the parameters $f_P(0)$, a_P, b_P, c_P and d_P are given in table 1, where the central values of the parameters in eq. (19) are presented.

Using the parametrization of eq. (3), for the $P \to f_0 \ell \bar{\nu}_\ell$ differential decay width, we get

see eq. (21) *above*

where

$$V_{ij} = \begin{cases} |V_{ub}| = (3.96 \pm 0.36) \times 10^{-3}, & \text{for } B_u \to f_0 \ell \bar{\nu}_\ell, \\ |V_{cs}| = 1.04 \pm 0.06, & \text{for } D_s \to f_0 \ell \bar{\nu}_\ell, \quad [2] \\ |V_{cd}| = 0.23 \pm 0.011, & \text{for } D \to f_0 \ell \bar{\nu}_\ell. \end{cases}$$

Taking into account the q^2-dependence of the form factors f_+ and f_-, performing integration over q^2 and using the lifetimes of B_u, D_s and D mesons, we get the following values for the branching ratios:

$$\begin{aligned}\mathcal{B}(B_u \to f_0 \tau \bar{\nu}_\tau) &= [(1.26 \pm 0.5) \times 10^{-4}] \sin^2\theta/2, \\ \mathcal{B}(B_u \to f_0 \mu \bar{\nu}_\mu) &= [(3.63 \pm 1.4) \times 10^{-4}] \sin^2\theta/2, \\ \mathcal{B}(B_u \to f_0 e \bar{\nu}_e) &= [(3.64 \pm 1.4) \times 10^{-4}] \sin^2\theta/2, \\ \mathcal{B}(D_s \to f_0 \mu \bar{\nu}_\mu) &= [(4.42 \pm 2.0) \times 10^{-3}] \cos^2\theta, \\ \mathcal{B}(D_s \to f_0 e \bar{\nu}_e) &= [(4.69 \pm 2.2) \times 10^{-3}] \cos^2\theta, \\ \mathcal{B}(D \to f_0 \mu \bar{\nu}_\mu) &= [(6.87 \pm 2.8) \times 10^{-4}] \sin^2\theta/2, \\ \mathcal{B}(D \to f_0 e \bar{\nu}_e) &= [(7.30 \pm 3.1) \times 10^{-4}] \sin^2\theta/2. \quad (22)\end{aligned}$$

The predictions for the branching ratio in eq. (22) are the main results of this work and they are independent of any mixing scheme. If we use the mixing scheme in the flavor basis, we see that the branching ratios

$$R_1 = \frac{\mathcal{B}(D \to f_0 \ell \bar{\nu}_\ell)}{\mathcal{B}(D_s \to f_0 \ell \bar{\nu}_\ell)}, \quad (23)$$

$$R_2 = \frac{\mathcal{B}(B_u \to f_0 \ell \bar{\nu}_\ell)}{\mathcal{B}(D_s \to f_0 \ell \bar{\nu}_\ell)}, \quad (24)$$

are directly related to the mixing angle θ.

On the other hand, as far as the flavor structure of f_0, as given in eq. (11), is concerned the ratio

$$R_3 = \frac{\mathcal{B}(B_u \to f_0 \ell \bar{\nu}_\ell)}{\mathcal{B}(D \to f_0 \ell \bar{\nu}_\ell)}, \quad (25)$$

is independent of the mixing angle θ. Therefore, the experimental measurement of the branching ratios of $B_u \to f_0 \ell \bar{\nu}_\ell$, $D_s \to f_0 \ell \bar{\nu}_\ell$ and $D \to f_0 \ell \bar{\nu}_\ell$ decays can give direct information about the mixing angle θ, as well as, about the flavor structure of the f_0 meson.

It should be remarked here that, the first experimental result on the semileptonic $D_s \to f_0 \ell \nu$ decay is already announced by the CLEO Collaboration [25]. Using our result on the $D_s \to f_0 \ell \nu$ decay and comparing it with the measured value of the branching ratio [25], the mixing angle is estimated to have the value $\cos^2\theta = 0.98^{+0.02}_{-0.21}$. Further improvements on the planned experiments, as well as on the theoretical studies, can give valuable information about the mixing angle θ.

In conclusion, we study the semileptonic decay of pseudoscalar mesons to the scalar f_0 meson. The transition form factors are calculated using the 3-point QCD sum rule analysis and then we estimate the corresponding branching ratios. Experimental data about the branching ratios of semileptonic decays $B_u \to f_0 \ell \bar{\nu}_\ell$, $D_s \to f_0 \ell \bar{\nu}_\ell$ and $D \to f_0 \ell \bar{\nu}_\ell$ would provide a direct estimation of the mixing angle θ.

REFERENCES

[1] Spanier S. and Törnqvist N. A., *Phys. Lett. B*, **592** (2004) 1.
[2] Amsler C. *et al.*, *Phys. Lett. B*, **667** (2008) 1.
[3] Cheng H. Y., *Phys. Rev. D*, **67** (2004) 034024; Wang W., Shen Y. L., Li Y. and Lü C. D., *Phys. Rev. D*, **74** (2006) 114010.
[4] Alford M. G. and Jaffe R. L., *Nucl. Phys. B*, **578** (2000) 367.
[5] Shifman M. A., Vainshtein A. I. and Zakharov V. I., *Nucl. Phys. B*, **147** (1979) 385.
[6] Colangelo P. and Khodjamirian A., *At the Frontier of Particle Physics/Handbook of QCD*, edited by Shifman M., Vol. **3** (World Scientific, Singapore) 2001, p. 1495.
[7] Aliev T. M., Eletsky V. L. and Kogan Ya. I., *Sov. J. Nucl. Phys.*, **40** (1984) 527.
[8] Ball P., Braun V. M. and Dosch H. G., *Phys. Rev. D*, **44** (1991) 3567.
[9] Ball P., *Phys. Rev. D*, **48** (1993) 3190.
[10] Ovchinnikov A. A. and Slobodenyuk V. A., *Z. Phys. C*, **44** (1989) 433; Baier V. N. and Grozin A., *Z. Phys. C*, **47** (1990) 669.
[11] Ovchinnikov A. A., *Sov. J. Nucl. Phys.*, **50** (1989) 519.
[12] Du D. S., Li J. W. and Yang M. Z., *Eur. Phys. J. C*, **37** (2004) 173.
[13] Colangelo P., de Fazio F. and Wang Wei, *Phys. Rev. D*, **81** (2010) 074001.
[14] Feldmann T., Kroll P. and Stech B., *Phys. Rev. D*, **58** (1998) 114006.
[15] Feldmann T., *Int. J. Mod. Phys. A*, **15** (2000) 159.
[16] Bediaga I., Navarra F. S. and Nielsen M., *Phys. Lett. B*, **579** (2004) 59.
[17] De Fazio F. and Pennington M. R., *Phys. Lett. B*, **521** (2001) 15.
[18] Aliev T. M. and Savci M., *Phys. Rev. D*, **73** (2006) 114010.
[19] Ioffe B. L., *Prog. Part. Nucl. Phys.*, **56** (2006) 232.
[20] Dominguez C. A., Nasrallah N. F., Röntsch R. and Schilcher K., *Nucl. Phys. Proc. Suppl.*, **186** (2009) 133.
[21] Aliev T. M. and Eletsky V. L., *Sov. J. Nucl. Phys.*, **38** (1983) 936.
[22] Bediaga I. and Nielsen M., *Phys. Rev. D*, **68** (2003) 036001.
[23] Elias V., Fariborz A. H., Shi F. and Steele T. G., *Nucl. Phys. A*, **633** (1998) 279.
[24] Rosner J. L. and Stone S., arXiv: hep-ex/1002.1655 (2010).
[25] CLEO Collaboration (Ecklund K. M. *et al.*), *Phys. Rev. D*, **80** (2009) 052009.