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# Scalar Kaluza-Klein modes in a multiply warped braneworld

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**Abstract** – The Kaluza-Klein (KK) modes of a massive scalar field on a 3-brane embedded in a six-dimensional multiply warped spacetime are determined. Due to the presence of warping along both the extra dimensions the KK mass spectrum splits into two closely spaced branches which is a distinct feature of this model compared to the five-dimensional Randall-Sundrum model. This new cluster of the KK mode spectrum is expected to have interesting phenomenological implications for the upcoming collider experiments. It is also shown that the effective scalar self-couplings of all orders are enhanced due to the presence of an extra warped dimension. Such a scenario may also be extended for even larger number of orbifolded extra dimensions.



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**Introduction.** – The Randall-Sundrum (RS) model [1] was originally proposed to resolve the hierarchy between the scale of weak and gravitational interactions,  $m_W \sim 10^2$  GeV and  $M_P \sim 10^{18}$  GeV, respectively. The RS model is based on a truncated  $AdS_5$  spacetime, bounded by two 4D Minkowski walls, often called UV (Planck) and IR (TeV) branes. The curvature in 5D induces a warped geometry on the brane which redshifts scales of order  $M_P$  at the UV brane to scales of order  $m_W$  at the IR brane. Much work has been carried out on diverse aspects of such models in the last few years. These include the attempted resolution of the hierarchy problem [1,2], questions about the localization of various types of fields on the brane [3], particle phenomenology in the context of braneworld [2] and various other cosmological consequences [4]. The actual existence of warped extra dimensions as well as a firm foundational basis for these models still remain open issues. However, it has been widely recognised that one of the key signature for extra dimensions can be obtained from the collider physics by exploring the contributions of various Kaluza-Klein (KK) modes in scattering amplitudes. Though the original RS model was formulated with only the gravity in the bulk, various other models subsequently considered the implications of the bulk standard model fields and their KK modes on the

brane. The first step in this direction was to study the role of a scalar field in the bulk.

In recent times several extensions of the RS model have been proposed with more than one extra dimension [5,6]. Most of these models consider several independent  $S^1/Z_2$  orbifolds along with four-dimensional Minkowski spacetime. Leblond *et al.* showed a set of consistency conditions for braneworld scenarios with a spatially periodic internal space in [7], from which it is derived that the necessity of a negative tension brane appears in five dimensions only and this is not the prerequisite at higher-dimensional generalizations [7]. It is also apparent from the multigravity scenario discussed in [8] that the radion stabilization problem and the presence of negative tension brane is an artifact of five-dimensional spacetime. The non-trivial curvature of the internal space in case of two or more extra dimensions provides the necessary bounce configuration of the warp factor without the need of any negative tension brane [5].

An interesting model has been proposed in an alternative scenario in [9] where the warped compact dimensions get further warped by a series of successive warping leading to *multiply warped spacetime* with various  $p$ -branes sitting at the different orbifold fixed points satisfying appropriate boundary conditions. In this scenario the lower-dimensional branes including the standard model 3-brane exist at the intersection edges of the higher-dimensional branes. The resulting geometry of the multiply warped  $D$ -dimensional spacetime is given by  $M^{1,D-1} \rightarrow \{[M^{1,3} \times S^1/Z_2] \times S^1/Z_2\} \times \dots$ , with

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$(D-4)$  such warped directions. It has been argued that this multiply warped spacetime gives rise to interesting phenomenology and offers a possible explanation of the small mass splitting among the standard model fermions [9,10]. One of the interesting characteristics of such a model is the bulk coordinate dependence of the higher-dimensional brane tensions. Such a coordinate-dependent brane tension is shown to be equivalent to a scalar field distribution on the higher-dimensional brane which constitutes the bulk for the 3-branes located at the intersection edges of these higher-dimensional branes. This emerges naturally from the requirement of the orbifolded boundary conditions along the two compact directions [9]. Such a scalar field distribution may have several interesting phenomenological significance for the TeV brane physics [9].

In this work we study the Kaluza-Klein modes of a 6-dimensional bulk scalar in the multiply warped spacetime. A higher-dimensional scalar, after compactification appears with its zero mode and various KK modes in the effective  $(3+1)$ -dimensional theory. We undertake to study both massless and massive modes for a bulk scalar as well as their effective self-coupling on the visible brane.

The present article is organized as follows. In the second section, we briefly discuss about the multiply warped six-dimensional brane model. Then, in the third section we focus on the bulk scalar and find out its equation of motion and the Kaluza Klein modes. In the fourth section we consider a bulk scalar field with a general self-coupling. The effective values of the scalar couplings at different order on the standard model brane are determined. We conclude with discussions in the fifth section.

**Model.** – The spacetime that we are interested in is a doubly warped and compactified six-dimensional one with a  $Z_2$  orbifolding in each of the extra dimensions [9]. We use the following notations for our discussion. The non-compact directions are denoted by  $x^\mu$  ( $\mu=0, \dots, 3$ ) and the orbifolded compact directions are represented by the angular coordinates  $y$  and  $z$ , respectively, with  $R_y$  and  $r_z$  as respective moduli. The corresponding metric for the six-dimensional spacetime is given by

$$ds_{(6)}^2 = b^2(z)[a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2, \quad (1)$$

where  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ . The warp factors due to the extra dimensions  $y$  and  $z$  are given by the functions  $a(y)$  and  $b(z)$ , respectively. Since orbifolding requires, in general, a localized concentration of energy, the four branes are considered to be located at the orbifold fixed points, namely  $y=0, \pi$  and  $z=0, \pi$ . The braneworld model is thus constructed from the following action (2) which consists of a negative bulk cosmological constant and coordinate-dependent brane tensions. The total

bulk-brane action is thus given by

$$\begin{aligned} S &= S_6 + S_5 + S_4, \\ S_6 &= \int d^4x dy dz \sqrt{-g_6} (R_6 - \Lambda), \\ S_5 &= \int d^4x dy dz [V_1 \delta(y) + V_2 \delta(y - \pi)] \\ &\quad + \int d^4x dy dz [V_3 \delta(z) + V_4 \delta(z - \pi)], \\ S_4 &= \int d^4x dy dz \sqrt{-g_{vis}} [\mathcal{L} - \hat{V}], \end{aligned} \quad (2)$$

where  $\Lambda$  is the bulk cosmological constant which is necessarily negative. In general, the brane potential terms (brane tensions) are  $V_{1,2} = V_{1,2}(z)$ , whereas  $V_{3,4} = V_{3,4}(y)$ . The contributions due to possible 3-branes located at  $(y, z) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$  are indicated by the term  $S_4$ . The solution of the Einstein equation for the metric (1) in this action (2) leads to the warp factors in the following form:

$$\begin{aligned} a(y) &= e^{-c|y|}, \quad c = \frac{R_y k}{r_z \cosh(k\pi)}, \\ b(z) &= \frac{\cosh(kz)}{\cosh(k\pi)}, \quad k = r_z \sqrt{\frac{-\Lambda}{M^4}}. \end{aligned} \quad (3)$$

Note that the solutions are  $Z_2$  symmetric about  $y$  and  $z$  directions. One can obtain the brane tensions by considering the boundary terms. The brane tensions at the two boundaries  $y=0$  and  $y=\pi$  are given by

$$V_1(z) = -V_2(z) = 8M^2 \sqrt{\frac{-\Lambda}{10}} \text{sech}(kz). \quad (4)$$

In other words, the two 4-branes sitting at  $y=0$  and  $y=\pi$  have  $z$ -dependent tensions. Similarly, the boundary condition for the infinitesimal interval across  $z=0$  and  $z=\pi$  leads to

$$\begin{aligned} V_3(y) &= 0, \\ V_4(y) &= -\frac{8M^4 k}{r_z} \tanh(k\pi). \end{aligned} \quad (5)$$

Note that, in this case the brane tensions are constants unlike the previous case, but quite similar to the case for the original RS model. The fact of  $g_{yy}$  being a non-trivial function of  $y$ , however, made it mandatory that the two hypersurfaces accounting for the  $y$  orbifolding must have a  $z$ -dependent energy density. This is, in fact, the most interesting part of the model that we are currently interested in. If there exists no other brane with a natural energy scale lower than ours, we must identify the SM brane with the one at  $y=\pi, z=0$ .

In this model the solution of the hierarchy problem (*i.e.* the mass rescaling due to warping) demands that unless there is a large hierarchy between the moduli  $r_z$  and  $R_y$ , either of  $c$  and  $k$  must be small implying large warping in one direction and small in the other. This particular feature of this model is revealed from the relation  $c = \frac{R_y k}{r_z \cosh(k\pi)}$ , which implies that for  $R_y \sim r_z$  a large hierarchy in the  $y$ -direction (a situation very close in spirit with RS)

necessarily leads to a relatively small  $k \sim \mathcal{O}(1)$  and hence little warping in the  $z$ -direction.

In summary we are dealing with a braneworld which is doubly warped and the warping is large along one direction and small in the other. We now address the nature of the Kaluza-Klein modes of a six-dimensional bulk scalar field in such a braneworld.

**Scalar modes.** – We start with a six-dimensional free massive scalar,  $\Phi(x_\mu, y, z)$ , of mass  $m_\phi$ . The action for the scalar is

$$S_\Phi = \frac{1}{2} \int d^4x \int_{-\pi}^{\pi} dy \int_{-\pi}^{\pi} dz [-g^{MN} \partial_M \Phi \partial_N \Phi - m_\phi^2 \Phi^2], \quad (6)$$

where  $g_{MN}$  is the six dimensional metric and  $m_\phi$  is of the order of the Planck mass. We perform a Kaluza-Klein decomposition of the scalar as a sum over the modes.

$$\Phi(x^\mu, y, z) = \sum_{np} \frac{\phi_{np}(x^\mu) \xi_n(y) \chi_p(z)}{\sqrt{R_y r_z}} \quad (7)$$

Substituting this in the six-dimensional action one arrives at a four-dimensional action for a free massive scalar field with a canonical kinetic term in the scalar field action provided the following normalization conditions are satisfied:

$$\int a^2(y) \xi_{n_1}(y) \xi_{n_2}(y) dy = \delta_{n_1 n_2}, \quad (8)$$

$$\int b^3(z) \chi_{p_1}(z) \chi_{p_2}(z) dz = \delta_{p_1 p_2}. \quad (9)$$

Under this compactification the four-dimensional effective action of the scalar field becomes

$$S_{4D}^{eff}(\phi) = -\frac{1}{2} \sum_{np} \int (\partial^\mu \phi_{np} \partial_\mu \phi_{np} + m_{np}^2 \phi_{np}^2) d^4x \quad (10)$$

As in usual Kaluza-Klein (KK) compactifications, the bulk field  $\Phi(x^\mu, y, z)$  manifests itself to a four-dimensional observer as an infinite “tower” of scalar modes  $\phi_{np}(x)$  with mass  $m_{np}$ . Note that, the KK mass term carries two indices because of the two compact warped extra dimensions. This in turn means that the usual five-dimensional massive tower here further splits into a further sub-tower because of the additional warped dimension. These extra modes naturally will have their contributions in the particle collider experiments and are expected to produce enhanced signature for the extra dimensions.

From the action we find the following eigenvalue equations for the  $y$  and  $z$  part of the scalar field. The equation for the “ $y$ ” dependence of the scalar is given by

$$\frac{1}{R_y^2} \frac{d}{dy} \left( a^4 \frac{d\xi_n(y)}{dy} \right) - m_p^2 a^2 \xi_n(y) = -m_{np}^2 a^4 \xi_n(y), \quad (11)$$

while the equation corresponding to the “ $z$ ” dependence turns out to be

$$\frac{1}{r_z^2} \frac{d}{dz} \left( b^5 \frac{d\chi_p(z)}{dz} \right) - m_\phi^2 b^5 \chi_p(z) = -m_{np}^2 b^3 \chi_p(z). \quad (12)$$

The mass tower due to the extra dimension along the  $z$ -direction is given by  $m_p$  whereas the four-dimensional KK mass tower is represented by  $m_{np}$ . It is interesting to note from the above equations that like in the 5-dimensional case, the bulk scalar mass appears in the equation for  $\chi_p(z)$  to determine the parameter  $m_p$ , whereas  $m_p$  enters into the equation for  $\xi_n(y)$  and determines  $m_{np}^2$ , *i.e.* various KK mode masses in terms of the two KK numbers  $p$  and  $n$ . Therefore to obtain the KK mass spectrum we first determine the tower denoted by  $m_p$  and then using it as the input to eq. (11) we finally achieve to obtain the desired mass spectrum in the 3-dimensional visible brane. Considering the allowed domain for the values of  $k$  we consider the following approximated form of the warp factor:  $b(z) \sim e^{-k(\pi-z)} = e^{-k\bar{z}}$ . Now redefining the variable as  $z_p = \frac{m_p}{k'} e^{k\bar{z}}$  where  $k' = \frac{k}{r_z}$  and the function as  $\tilde{\chi}_p(z) = e^{-\frac{5}{2}k\bar{z}} \chi_p(z)$ , eq. (12) can be recast as

$$z_p^2 \frac{d^2 \tilde{\chi}_p}{dz_p^2} + z_p \frac{d\tilde{\chi}_p}{dz_p} + (z_p^2 - \nu_\phi^2) \tilde{\chi}_p = 0, \quad (13)$$

where  $\nu_\phi^2 = \left( \frac{m_\phi^2}{k'^2} + \frac{25}{4} \right)$ ,  $m_\phi$  being the bulk mass of the scalar field. The solutions of the above equation are the Bessel functions of order  $\nu_\phi$

$$\chi_p(z) = \frac{1}{N_p} e^{\frac{5}{2}k\bar{z}} \left[ J_{\nu_\phi} \left( \frac{m_p}{k'} e^{k\bar{z}} \right) + b_p Y_{\nu_\phi} \left( \frac{m_p}{k'} e^{k\bar{z}} \right) \right], \quad (14)$$

where  $N_p$  is the normalization constant and  $b_p$  is an arbitrary constant.

Following the condition that the left-hand side of eq. (13) is self-adjoint, the first-order derivative of  $\chi_p(z)$  must be continuous at the orbifold fixed points  $z=0$  and  $z=\pi$ . This leads us to the spectrum for  $m_p$ . From the condition of self-adjointness, we approximately get a condition

$$\frac{5}{2} J_{\nu_\phi}(x_{p\nu_\phi}) + x_{p\nu_\phi} J'_{\nu_\phi}(x_{p\nu_\phi}) = 0, \quad (15)$$

where  $x_{p\nu_\phi} = \frac{m_p}{k'} e^{k\pi}$ .

After obtaining the  $z$ -dependent part of the scalar KK modes and the corresponding spectrum we solve the  $y$ -dependent part of the modes and finally arrive at the full spectrum of the KK modes from the point of view of a 3-brane observer sitting at the visible brane.

The solution of eq. (11) yields  $\xi_n(y)$  as Bessel function of order  $\nu_p = \sqrt{4 + \frac{m_p^2}{k'^2}}$  multiplied by a growing exponential factor as

$$\xi_n(y) = \frac{1}{N_n} e^{2c|y|} \left[ J_{\nu_p} \left( \frac{m_{np}}{k'} e^{c|y|} \right) + b_n Y_{\nu_p} \left( \frac{m_{np}}{k'} e^{c|y|} \right) \right]. \quad (16)$$

Here  $N_n$  is the normalization constant and  $b_n$  is an arbitrary constant.

Once again following the condition that the left-hand side of eq. (11) is self-adjoint, the first-order derivative of  $\xi_n(y)$  must be continuous at the orbifold fixed points.

Table 1: The KK mode masses  $m_{np}$  (in TeV) for different values of  $n$  and  $p$ .

	$m_\phi = 0.01k'$				$m_\phi = k'$			
$\nu_p$	$\nu_1$	$\nu_2$	$\nu_3$	$\nu_4$	$\nu_1$	$\nu_2$	$\nu_3$	$\nu_4$
$m_{1p}$	5.04	6.61	8.26	9.99	5.11	6.69	8.34	10.05
$m_{2p}$	8.44	10.26	12.14	13.98	8.53	10.36	12.23	14.15
$m_{3p}$	11.69	13.62	15.61	17.56	11.79	13.72	15.71	17.73
$m_{4p}$	14.89	16.89	18.95	20.97	14.99	17.00	19.06	21.15

This gives us the equations from which we obtain the mass spectrum,  $m_{np}$ . Applying the above condition in eq. (16) at the location of the visible brane at  $y = \pi$  and using  $e^{c\pi} \gg 1$  we obtain the following transcendental equation:

$$2J_{\nu_p}(x_{np}) + x_{np} J'_{\nu_p}(x_{np}) = 0, \quad (17)$$

where  $x_{np} = m_{np}e^{c\pi}/k'$ . The mass spectrum  $m_{np}$  thus is obtained from the solution of the above transcendental equation.

In the five-dimensional case the order of the Bessel function depends on the mass of the scalar field. Therefore for a given bulk scalar the KK modes are given by the wave functions (Bessel functions) which are of a *fixed order*. However, in the six-dimensional case we can have several orders of the Bessel function representing the KK modes because the order  $\nu_p$  can take up different values for different values  $p$ , *i.e.* for different values of  $m_p$ . Therefore for each mass splitting due to  $z$ -direction we will obtain a spectrum of KK modes. Hence, we obtain extra splitting in the spectrum over the usual 5-dimensional scenario. In table 1 we show explicitly the KK modes for two given values of bulk scalar mass.

Once again, like the 5-dimensional case, the scalar KK mode masses are suppressed by the warp factor. Taking  $m_\phi$  of the order of the Planck scale we find that the light KK modes have masses in the range of TeV. The exponential suppression can be understood from the equation of  $\xi_n(y)$  which shows that the modes are peaked near the visible brane at  $\{y = \pi, z = 0\}$  [10]. However in this case a much larger number of KK modes of mass  $\sim$  TeV appear in comparison to its 5-dimensional counterpart and are expected to produce additional contributions to various processes involving KK modes of the bulk fields in the forthcoming collider experiments at TeV scale. For this the value of the effective scalar self-interaction on the brane as well as that of the bulk scalars with brane fermions must be determined.

#### Effective coupling for scalar self-interaction. –

To examine the self-interactions of a bulk scalar, we first determine the normalization constants  $N_p$  and  $N_n$ . Using the normalization conditions from eq. (8) and eq. (9), we obtain,

$$N_n \simeq \frac{e^{c\pi} A_n}{\sqrt{k' R_y}} \quad (18)$$

and

$$N_p \simeq \frac{e^{k\pi} B_p}{\sqrt{k' r_z}}. \quad (19)$$

Here,

$$A_n^2 = \int_0^1 r [J_{\nu_p}(x_{np}r)]^2 dr \quad (20)$$

and

$$B_p^2 = \int_0^1 s [J_{\nu_\phi}(x_{p\nu_\phi}s)]^2 ds, \quad (21)$$

where  $r = e^{c(y-\pi)}$  and  $s = e^{-kz}$ .

Now, we focus our attention to the self-couplings of low-lying modes. Consider a generic term in the action which is of the form

$$S_{int} = \int d^4x \int_{-\pi}^{\pi} dy \int_{-\pi}^{\pi} dz \sqrt{G} \frac{\lambda}{M^{4m-6}} \Phi^{2m}(x^\mu, y, z), \quad (22)$$

where  $\lambda$  is of order unity. Expanding in modes, the self-interactions of the light KK states are given by

$$S_{int} = \int d^4x \int_{-\pi}^{\pi} dy \int_{-\pi}^{\pi} dz a^4 b^5 R_y r_z \frac{\lambda}{M^{4m-6}} \phi_{np}^{2m}(x^\mu) \times \left( \frac{\xi_n}{\sqrt{R_y}} \right)^{2m} \left( \frac{\chi_p}{\sqrt{r_z}} \right)^{2m}. \quad (23)$$

Thus the effective four-dimensional coupling constants for the  $\phi_{np}^{2m}$  interactions are

$$\lambda_{eff} \simeq \lambda \left( \frac{k'}{M} \right)^{(2m-2)} (M e^{-c\pi})^{4-2m} e^{(3m-5)k\pi} \times \int_0^1 r^{4m-5} \left[ \frac{J_{\nu_p}(x_{np}r)}{A_n} \right]^{2m} dr \times \int_0^1 s^{5m-6} \left[ \frac{J_{\nu_\phi}(x_{p\nu_\phi}s)}{B_p} \right]^{2m} ds \quad (24)$$

The scale relevant to four-dimensional physics is therefore not  $M$  but  $(M e^{-c\pi})^{4-2m} e^{(3m-5)k\pi}$ . Note that for  $m = 2$  (*i.e.*  $\Phi^4$  theory) the effective four-dimensional coupling has an enhancement factor  $e^{k\pi}$  associated with it, which is a new feature of the multiply warped spacetime compared to its five-dimensional counterpart.



Moreover, all the higher-order couplings for  $m > 2$ , which acquires an enhancement factor  $e^{-c\pi}$  (just as in the case of couplings in the five-dimensional model), are further enhanced by the factor  $e^{(3m-5)k\pi}$ . This enhancement of the effective scalar self-couplings at all orders including the quartic coupling, due to the presence of an additional warped dimension makes the multiply warped model with a large number of KK modes phenomenologically more interesting.

**Conclusion.** – In this work we have calculated various Kaluza-Klein mode masses of a bulk scalar field in a braneworld model with two warped extra dimensions. Such bulk scalars are useful candidates for moduli stabilization of warped braneworld models [11]. We have shown:

- An increase in the number of scalar KK modes within a energy range of few TeV due to the presence of an additional extra dimension over the usual five-dimensional RS model.
- Such a clustering of modes near the TeV range develops in such a multiply warped model due to a large warping in one direction and a small warping in the other. This feature originates from the requirement that there should not be any significant hierarchy between the two moduli of the model [9].
- The effective scalar self-couplings on the standard model brane for all the KK modes are shown to receive enhancement at all orders due to the presence of the extra warped dimension.

The enhancement of the number of KK modes will take place for the other standard model fields also if they are allowed to propagate in the bulk. Extending this work for even larger number of warped dimensions [9], where two clusters of 3-branes with an energy scale close to the TeV scale and Planck scale are obtained, it is easy to show a very large proliferation of the number of KK modes for each bulk field, with masses close to the TeV scale. Such an increase in the number of KK modes is expected to modify the decay as well as the scattering amplitudes of different processes in a TeV-scale collider. Moreover, with increase in the number of warped dimensions, the effective scalar self-coupling on the visible brane acquires an additional enhancement factor which depends on the respective modulus of the extra dimension. The signature of these scalar KK modes and their enhanced self-coupling on the brane therefore may play a crucial role in determining the number of warped directions in our search for extra dimensions in the collider experiments.

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