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Nilpotent (anti-)BRST symmetry transformations for dynamical non-Abelian 2-form gauge theory: Superfield formalism

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Abstract – We derive the off-shell nilpotent and absolutely anticommuting Becchi-Rouet-Stora-Tyutin (BRST) and anti-BRST symmetry transformations for the dynamical four (3 + 1)-dimensional (4D) non-Abelian 2-form gauge theory within the framework of geometrical superfield formalism. We obtain the (anti-)BRST-invariant coupled Lagrangian densities that respect the above nilpotent symmetry transformations. We discuss, furthermore, this (anti-)BRST invariance in the language of the superfield formalism. One of the novel features of our present investigation is the observation that, in addition to the horizontality condition, we are theoretically compelled to invoke some other physically relevant restrictions in order to deduce the precise (anti-)BRST symmetry transformations for *all* the fields of a topologically massive 4D non-Abelian gauge theory.

Introduction. – The central theme of our present investigation is to exploit the potential and power of the superfield formalism (see, *e.g.*, [1]), that has been successfully applied in the context of (non-)Abelian 1-form, Abelian 2-form and 3-form gauge theories (see, *e.g.*, [2–4]), in the description of the four (3 + 1)-dimensional (4D) topologically massive non-Abelian gauge theory where there is an explicit coupling between the non-Abelian 2-form ($B^{(2)} = \frac{1}{2!}(dx^\mu \wedge dx^\nu)B_{\mu\nu}$) gauge field $B_{\mu\nu} = B_{\mu\nu} \cdot T$ and the non-Abelian 1-form ($A^{(1)} = dx^\mu A_\mu$) gauge field $A_\mu = A_\mu \cdot T$ through the famous topological ($B^{(2)} \wedge F^{(2)}$) term where the 2-form $F^{(2)} = \frac{1}{2!}(dx^\mu \wedge dx^\nu)F_{\mu\nu}$ defines the curvature tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$ corresponding to the 1-form gauge potential A_μ. Here all the gauge fields are defined in the adjoint representation of the semi-simple non-Abelian gauge group $SU(N)$.

Since the Higgs particles (that are responsible for generating masses for the gauge particles and fermions in the domain of standard model of high-energy physics) have not yet been observed experimentally, it has become an issue of paramount importance to construct gauge-invariant theories that could provide masses to the gauge particles and fermions without taking any recourse to the Higgs mechanism. In this context, the study of 4D topologically massive gauge theories of Abelian and non-Abelian types has become quite popular because the latter do provide a theoretical basis for generating masses for the gauge bosons without exploiting any inputs from the Higgs mechanism (see, *e.g.*, [5]).

Recently, we have studied the 4D topologically massive Abelian gauge theory within the framework of BRST formalism [6]. Its straightforward generalization to the non-Abelian topologically massive theory is non-trivial because of some very strong no-go theorems [7]. There are, at least, a couple of models [8,9], however, that circumvent the severe structures laid down by the above no-go theorems. In our present endeavor, we shall focus on the dynamical non-Abelian 2-form gauge theory [9] and study its BRST and anti-BRST structures by exploiting its *usual* "scalar" gauge symmetry transformations within the framework of the geometrical superfield formalism proposed in [1–3].

One of the highlights of our findings is that the gauge-invariant restrictions are invoked, in addition to

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the horizontality condition, for the exact derivation of *all* the off-shell nilpotent and absolutely anticommuting (anti-)BRST transformations. This observation, to the best of our knowledge, is a new result within the framework of the application of superfield formalism to a gauge theory (without any interaction with matter fields).

Let us begin with the Lagrangian density for the four (3 + 1)-dimensional (4D) topologically massive non-Abelian gauge theory[1] (see, *e.g.* [9] for details)

$$\begin{aligned}\mathcal{L}_0 \;=\; -\frac{1}{4}\, F^{\mu\nu}\cdot F_{\mu\nu} + \frac{1}{12}\, H^{\mu\nu\eta}\cdot H_{\mu\nu\eta} \\ +\frac{m}{4}\,\varepsilon_{\mu\nu\eta\kappa} B^{\mu\nu}\cdot F^{\eta\kappa},\end{aligned} \tag{1}$$

where the 2-form $F^{(2)} = dA^{(1)} + i\,A^{(1)}\wedge A^{(1)} \equiv \frac{1}{2!}(dx^\mu\wedge dx^\nu)F_{\mu\nu}\cdot T$ defines the curvature tensor $F_{\mu\nu}$ for the gauge potential A_μ, 3-form $H^{(3)} = \frac{1}{3!}\,(dx^\mu\wedge dx^\nu\wedge dx^\eta)\,H_{\mu\nu\eta}\cdot T$ defines the compensated curvature tensor in terms of the dynamical 2-form gauge potential $B_{\mu\nu}$ and 1-form $(K^{(1)} = dx^\mu K_\mu\cdot T)$ auxiliary field K_μ as

$$\begin{aligned}H^a_{\mu\nu\eta} \;=\; (\partial_\mu B^a_{\nu\eta} + \partial_\nu B^a_{\eta\mu} + \partial_\eta B^a_{\mu\nu}) - \big[(A_\mu\times B_{\nu\eta})^a \\ + (A_\nu\times B_{\eta\mu})^a + (A_\eta\times B_{\mu\nu})^a\big] \\ - \big[(K_\mu\times F_{\nu\eta})^a + (K_\nu\times F_{\eta\mu})^a + (K_\eta\times F_{\mu\nu})^a\big],\end{aligned} \tag{2}$$

and the last term in the above Lagrangian density (1) corresponds to the topological mass term where the curvature tensor $F_{\mu\nu}$ corresponding to the non-Abelian 1-form gauge field and the dynamical 2-form gauge field $B_{\mu\nu}$ are coupled together through $B^2\wedge F^{(2)}$.

The above Lagrangian density respects the *usual* infinitesimal "scalar" gauge transformations δ_g corresponding to the non-Abelian 1-form gauge theory as (see, *e.g.*, [9])

$$\begin{aligned}\delta_g A_\mu \;&=\; D_\mu\Omega \equiv \partial_\mu\Omega - (A_\mu\times\Omega), \quad \delta_g F_{\mu\nu} = -(F_{\mu\nu}\times\Omega),\\ \delta_g B_{\mu\nu} \;&=\; -(B_{\mu\nu}\times\Omega), \quad \delta_g H_{\mu\nu\eta} = -(H_{\mu\nu\eta}\times\Omega),\\ \delta_g K_\mu \;&=\; -(K_\mu\times\Omega), \quad \delta_g\mathcal{L}_0 = 0,\end{aligned} \tag{3}$$

where $\Omega = \Omega\cdot T$ is the infinitesimal $SU(N)$-valued "scalar" gauge parameter. In addition, there exists an independent "vector" gauge symmetry transformation in the theory [9]. We shall exploit, however, the usual "scalar" gauge symmetry transformations (3) (and corresponding properties of the gauge invariance) for our present discussion of the 4D topologically massive gauge theory within the framework of superfield approach [1–3].

[1]We adopt here the conventions and notations such that the background 4D Minkowski spacetime manifold has the flat metric with signatures $(+1,-1,-1,-1)$ and the group generators T^a of the $SU(N)$ group obey the Lie algebra $[T^a, T^b] = if^{abc}T^c$ with structure constants f^{abc} (that are chosen to be totally antisymmetric in indices a, b, and c where $a, b, c \ldots = 1, 2, \ldots N^2-1$). In the algebraic space, we also have: $(V\cdot W) = V^aW^a$ and $(V\times W)^a = f^{abc}V^bW^c$ for the sake of brevity. The 4D Levi-Civita tensor $\varepsilon_{\mu\nu\eta\kappa}$ (with $\mu,\nu,\eta\ldots = 0,1,2,3$) satisfies $\varepsilon_{\mu\nu\eta\kappa}\varepsilon^{\mu\nu\eta\kappa} = -4!, \varepsilon_{\mu\nu\eta\kappa}\varepsilon^{\mu\nu\eta\sigma} = -3!\delta^\sigma_\kappa$, etc., and $\varepsilon_{0123} = +1$.

Our present paper is organized as follows. In the second section, we recapitulate the bare essentials of the superfield approach [1–3] to derive the (anti-)BRST symmetry transformations and Curci-Ferrari (CF) restriction [10] for the non-Abelian 1-form gauge theory where the horizontality condition (HC) plays a decisive role. Our third section is devoted to the derivation of (anti-)BRST symmetry transformations for the non-Abelian 2-form gauge field and 1-form auxiliary field by exploiting a couple of physically relevant restrictions that are distinctly different from the HC. We discuss, in the fourth section, the (anti-)BRST invariance of the topologically massive non-Abelian gauge theory that is described by the coupled Lagrangian densities. Finally, in the fifth section, we summarize our results and make some concluding remarks.

Nilpotent symmetry transformations for the non-Abelian 1-form gauge theory: superfield formalism. – In the superfield approach to BRST formalism [1–3], one generalizes the 4D basic non-Abelian gauge field $(A_\mu = A_\mu\cdot T)$ and fermionic (anti-)ghost fields ($\bar C = \bar C\cdot T, C = C\cdot T$) to the superfields defined on the (4, 2)-dimensional supermanifold. These superfields are expanded along the Grassmannian directions of the supermanifold as (see, *e.g.*, [1,2])

$$\begin{aligned}\mathcal{B}_\mu(x,\theta,\bar\theta) &= A_\mu(x) + \theta\,\bar R_\mu(x) + \bar\theta\,R_\mu(x) + i\,\theta\,\bar\theta\,S_\mu(x),\\ \mathcal{F}(x,\theta,\bar\theta) &= C(x) + i\,\theta\,\bar B_1(x) + i\,\bar\theta\,B_1(x) + i\,\theta\,\bar\theta\,s(x),\\ \bar{\mathcal{F}}(x,\theta,\bar\theta) &= \bar C(x) + i\,\theta\,\bar B_2(x) + i\,\bar\theta\,B_2(x) + i\,\theta\,\bar\theta\,\bar s(x),\end{aligned} \tag{4}$$

where the secondary fields $(\bar R_\mu(x), R_\mu(x), s(x), \bar s(x))$ are fermionic and the other secondary fields $(S_\mu(x), B_1(x), \bar B_1(x), B_2(x), \bar B_2(x))$ are bosonic in nature. These secondary fields are determined in terms of the basic and auxiliary fields of the 4D non-Abelian 1-form gauge theory by exploiting the mathematical power of the HC.

Under the celebrated HC, the $SU(N)$ gauge-invariant kinetic term $(-\frac{1}{4}\,F^{\mu\nu}\cdot F_{\mu\nu})$ of the 4D non-Abelian gauge theory is required to remain invariant when we generalize the 4D local non-Abelian theory onto the (4, 2)-dimensional supermanifold in terms of the superfields. In other words, the super 2-form $\tilde{\mathcal{F}}^{(2)} = \tilde d\tilde A^{(1)} + i\,\tilde A^{(1)}\wedge\tilde A^{(1)} = \frac{1}{2!}(dZ^M\wedge dZ^N)\tilde{\mathcal{F}}_{MN}$, defined on the (4, 2)-dimensional supermanifold with the following inputs

$$\begin{aligned}\tilde d \;&=\; dZ^M\partial_M \equiv dx^\mu\,\partial_\mu + d\theta\,\partial_\theta + d\bar\theta\,\partial_{\bar\theta},\\ \partial_M \;&=\; (\partial_\mu, \partial_\theta, \partial_{\bar\theta}),\\ \tilde A^{(1)} \;&=\; dZ^M A_M \equiv dx^\mu\,\mathcal{B}_\mu(x,\theta,\bar\theta) + d\theta\,\bar{\mathcal{F}}(x,\theta,\bar\theta)\\ &\quad + d\bar\theta\,\mathcal{F}(x,\theta,\bar\theta),\end{aligned} \tag{5}$$

is equated to the ordinary 2-form $F^2 = dA^{(1)} + i\,A^{(1)}\wedge A^{(1)}$ in the HC. The latter defines the ordinary curvature tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$. In the above, the super multiplet $A_M = (\mathcal{B}_\mu, \mathcal{F}, \bar{\mathcal{F}})$ is defined on the (4, 2)-dimensional supermanifold which is characterized in terms of the superspace coordinates $Z^M = (x^\mu, \theta, \bar\theta)$

In the HC, all the Grassmannian components of the super curvature $\tilde{\mathcal{F}}_{MN}$ are set equal to zero. This requirement leads to the following relationships [1,2]:

$$
\begin{aligned}
&R_\mu = D_\mu C, \quad \bar R_\mu = D_\mu \bar C, \quad B_1 = -\tfrac{i}{2}(C\times C),\\
&s = -(\bar B_1 \times C),\\
&S_\mu = D_\mu B_2 + i(D_\mu C\times \bar C) \equiv -D_\mu \bar B_1 - i(C\times D_\mu \bar C),\\
&\bar B_2 = -\tfrac{i}{2}(\bar C\times\bar C),\ \bar B_1 + B_2 = -i(C\times\bar C),\\
&\bar s = -(B_2\times\bar C).
\end{aligned} \tag{6}
$$

If we make the identifications: $\bar B_1 = \bar B, B_2 = B$, the above Curci-Ferrari restriction $\bar B_1 + B_2 = -i(C\times\bar C)$ changes to its well-known form $B+\bar B = -i(C\times\bar C)$. Plugging in the above relationships in the expansions (4), we obtain the following expressions for the superfields along the Grassmannian directions of the supermanifold, namely;

$$
\begin{aligned}
\mathcal{B}^{(h)}_\mu(x,\theta,\bar\theta) &= A_\mu(x) + \theta\,(D_\mu \bar C(x)) + \bar\theta\,(D_\mu C(x))\\
&\quad + \theta\,\bar\theta\,[i\,D_\mu B(x) - (D_\mu C\times C)(x)],\\
\mathcal{F}^{(h)}(x,\theta,\bar\theta) &= C(x) + \theta\,(i\bar B(x)) + \bar\theta\left[\frac{1}{2}(C\times C)(x)\right]\\
&\quad + \theta\,\bar\theta\,[-i\,(\bar B\times C)(x)],\\
\bar{\mathcal{F}}^{(h)}(x,\theta,\bar\theta) &= \bar C(x) + \theta\left[\frac{1}{2}(\bar C\times\bar C)(x)\right]\\
&\quad + \bar\theta\,(iB(x)) + \theta\,\bar\theta\,[-i\,(B\times\bar C)(x)],
\end{aligned} \tag{7}
$$

which can be expressed in terms the off-shell nilpotent ($s^2_{(a)b}=0$) (anti-)BRST symmetry transformations $s_{(a)b}$ for the non-Abelian 1-form gauge theory as follows[2]

$$
\begin{aligned}
\mathcal{B}^{(h)}_\mu(x,\theta,\bar\theta) &= A_\mu(x) + \theta\,(s_{ab}A_\mu(x)) + \bar\theta\,(s_b A_\mu(x))\\
&\quad + \theta\,\bar\theta\,(s_b s_{ab} A_\mu(x)),\\
\mathcal{F}^{(h)}(x,\theta,\bar\theta) &= C(x) + \theta\,(s_{ab}C(x)) + \bar\theta\,(s_b C(x))\\
&\quad + \theta\,\bar\theta\,(s_b s_{ab} C(x)),\\
\bar{\mathcal{F}}^{(h)}(x,\theta,\bar\theta) &= \bar C(x) + \theta\,(s_{ab}\bar C(x)) + \bar\theta\,(s_b \bar C(x))\\
&\quad + \theta\,\bar\theta\,(s_b s_{ab}\bar C(x)),
\end{aligned} \tag{8}
$$

where the superscript (h) on the superfields denotes the expansions of the superfields after the application of the horizontality condition.

The spacetime component of the super curvature tensor $\tilde{\mathcal{F}}_{MN}$ is $\tilde{\mathcal{F}}^{(h)}_{\mu\nu}(x,\theta,\bar\theta) = \partial_\mu \mathcal{B}^{(h)}_\nu - \partial_\nu \mathcal{B}^{(h)}_\mu + i[\mathcal{B}^{(h)}_\mu, \mathcal{B}^{(h)}_\nu]$. This can be written, using the expansion for $\mathcal{B}^{(h)}_\mu(x,\theta,\bar\theta)$ in (7), as

$$
\begin{aligned}
\tilde{\mathcal{F}}^{(h)}_{\mu\nu}(x,\theta,\bar\theta) &= F_{\mu\nu} - \theta\,(F_{\mu\nu}\times\bar C) - \bar\theta\,(F_{\mu\nu}\times C)\\
&\quad + \theta\bar\theta\,[(F_{\mu\nu}\times C)\times\bar C - i\,F_{\mu\nu}\times B].
\end{aligned} \tag{9}
$$

The above expression does imply clearly that the kinetic term remains invariant under the horizontality condition (*i.e.* $-\frac{1}{4}\tilde F^{\mu\nu(h)}(x,\theta,\bar\theta)\cdot\tilde F^{(h)}_{\mu\nu}(x,\theta,\bar\theta) = -\frac{1}{4}F^{\mu\nu}\cdot F_{\mu\nu}$).

Off-shell nilpotent transformations for non-Abelian 2-form gauge and 1-form auxiliary fields. – Exploiting (3), it can be checked that $\delta_g(B_{\mu\nu}\cdot F_{\eta\kappa}) = 0, \delta_g(K_\mu\cdot F_{\nu\eta}) = 0$. Thus, we propose the following gauge-invariant restrictions (GIRs) in terms of the (super)fields

$$
\begin{aligned}
\tilde{\mathcal{B}}_{\mu\nu}(x,\theta,\bar\theta)\cdot\tilde{\mathcal{F}}^{(h)}_{\eta\kappa}(x,\theta,\bar\theta) &= B_{\mu\nu}(x)\cdot F_{\eta\kappa}(x),\\
\tilde{\mathcal{K}}_\mu(x,\theta,\bar\theta)\cdot\tilde{\mathcal{F}}^{(h)}_{\nu\eta}(x,\theta,\bar\theta) &= K_\mu(x)\cdot F_{\nu\eta}(x),
\end{aligned} \tag{10}
$$

as analogues of the horizontality condition ($\tilde{\mathcal{F}}^{(2)} = F^{(2)}$). The expansions of the superfields $\tilde{\mathcal{B}}_{\mu\nu}(x,\theta,\bar\theta)$ and $\tilde{\mathcal{K}}_\mu(x,\theta,\bar\theta)$ on the (4, 2)-dimensional supermanifold are

$$
\begin{aligned}
\tilde{\mathcal{B}}_{\mu\nu}(x,\theta,\bar\theta) &= B_{\mu\nu}(x) + \theta\bar R_{\mu\nu}(x)) + \bar\theta R_{\mu\nu}(x) + i\theta\,\bar\theta S_{\mu\nu}(x),\\
\tilde{\mathcal{K}}_\mu(x,\theta,\bar\theta) &= K_\mu(x) + \theta\bar P_\mu(x) + \bar\theta P_\mu(x) + i\,\theta\,\bar\theta\,Q_\mu(x),
\end{aligned} \tag{11}
$$

where the secondary fields ($R_{\mu\nu}, \bar R_{\mu\nu}, P_\mu, \bar P_\mu$) are fermionic and $(S_{\mu\nu}, Q_\mu)$ are bosonic in nature. These secondary fields would be determined by exploiting the above restrictions (10) where the HC plays a decisive role, too, in a subtle manner.

It is straightforward to check that the following relationships ensue from (10):

$$
\begin{aligned}
&R_{\mu\nu} = -(B_{\mu\nu}\times C)\ \bar R_{\mu\nu} = -(B_{\mu\nu}\times\bar C),\\
&S_{\mu\nu} = -(B_{\mu\nu}\times B) - i[(B_{\mu\nu}\times C)\times\bar C],\\
&P_\mu = -(K_\mu\times C), \quad \bar P_\mu = -(K_\mu\times\bar C),\\
&Q_\mu = -(K_\mu\times B) - i\,[(K_\mu\times C)\times\bar C].
\end{aligned} \tag{12}
$$

The expansions, that emerge after the application of the gauge-invariant restrictions, are

$$
\begin{aligned}
\tilde{\mathcal{B}}^{(g)}_{\mu\nu}(x,\theta,\bar\theta) &= B_{\mu\nu}(x) - \theta\,[(B_{\mu\nu}\times\bar C)(x)] - \bar\theta[(B_{\mu\nu}\times C)(x)]\\
&\quad + \theta\,\bar\theta\,[\{(B_{\mu\nu}\times C)\times\bar C - i\,B_{\mu\nu}\times B\}(x)],\\
&\equiv B_{\mu\nu}(x) + \theta\,(s_{ab}B_{\mu\nu}(x)) + \bar\theta\,(s_b B_{\mu\nu}(x))\\
&\quad + \theta\,\bar\theta\,(s_b s_{ab} B_{\mu\nu}(x)),\\
\tilde{\mathcal{K}}^{(g)}_\mu(x,\theta,\bar\theta) &= K_\mu(x) - \theta\,[(K_\mu\times\bar C)(x)] - \bar\theta\,[(K_\mu\times C)(x)]\\
&\quad + \theta\,\bar\theta\,[\{(K_\mu\times C)\times\bar C - i\,K_\mu\times B\}(x)],\\
&\equiv K_\mu(x) + \theta\,(s_{ab}K_\mu(x)) + \bar\theta\,(s_b K_\mu(x))\\
&\quad + \theta\,\bar\theta\,(s_b s_{ab} K_\mu(x)),
\end{aligned} \tag{13}
$$

where the superscript (g) denotes the super expansions obtained after the application of GIRs. From the preceding discussions, it is clear that we have obtained all the off-shell nilpotent (anti-)BRST transformations for the basic fields $(B_{\mu\nu}, A_\mu)$, auxiliary field (K_μ) and the

[2] The full off-shell nilpotent transformations $s_{(a)b}$ (cf. (16) below) are absolutely anticommuting on a surface described by the Curci-Ferrari field equation $[B+\bar B + i(C\times\bar C) = 0]$ in the 4D spacetime manifold.

(anti-)ghost fields $(\bar{C})C$ by exploiting the geometrical superfield formalism.

From the gauge-invariant restrictions (10) and super expansions in (9) and (13), it is clear that the topological term in (1) remains invariant when we generalize the 4D theory onto the (4, 2)-dimensional supermanifold. As a consequence, we have the equality

$$\frac{m}{4}\,\varepsilon^{\mu\nu\eta\kappa}\tilde{\mathcal{B}}^{(g)}_{\mu\nu}(x,\theta,\bar{\theta})\cdot\tilde{\mathcal{F}}^{(h)}_{\eta\kappa}(x,\theta,\bar{\theta}) = \frac{m}{4}\varepsilon^{\mu\nu\eta\kappa}B_{\mu\nu}(x)\cdot F_{\eta\kappa}(x). \tag{14}$$

It is worth pointing out that the above equality shows that, ultimately, the l.h.s. of (14) is independent of the Grassmannian varibales. In an exactly similar fashion, it can be checked that the following expression for the super curvature tensor[3] $(\tilde{\mathcal{H}}^{(g,h)}_{\mu\nu\eta}\,(x,\theta,\bar{\theta}))$

$$\begin{aligned}\tilde{\mathcal{H}}^{(g,h)}_{\mu\nu\eta}\,(x,\theta,\bar{\theta}) &= H_{\mu\nu\eta}(x) - \theta[(H_{\mu\nu\eta}\times\bar{C})(x)] \\ &\quad - \bar{\theta}\,[(H_{\mu\nu\eta}\times C)(x)] \\ &\quad + \theta\,\bar{\theta}[\{(H_{\mu\nu\eta}\times C)\times\bar{C} - iH_{\mu\nu\eta}\times B\}(x)] \\ &\equiv H_{\mu\nu\eta}(x) + \theta\,(s_{ab}H_{\mu\nu\eta}(x)) \\ &\quad + \bar{\theta}\,(s_b H_{\mu\nu\eta}(x)) \\ &\quad + \theta\,\bar{\theta}\,(s_b s_{ab} H_{\mu\nu\eta}(x)),\end{aligned} \tag{15}$$

implies that $\frac{1}{12}\tilde{\mathcal{H}}^{\mu\nu\eta(g,h)}(x,\theta,\bar{\theta})\cdot\tilde{\mathcal{H}}^{(g,h)}_{\mu\nu\eta}(x,\theta,\bar{\theta}) = \frac{1}{12}H^{\mu\nu\eta}(x)\cdot H_{\mu\nu\eta}(x)$. In other words, the l.h.s. of this equality is independent of the Grassmannian variables of the superspace coordinate $Z^M = (x^\mu,\theta,\bar{\theta})$.

Coupled Lagrangian densities and their invariance. – It can be checked from the action (corresponding to the starting Lagrangian density (1)) and the following off-shell nilpotent (anti-)BRST symmetry transformations

$$\begin{aligned} s_b A_\mu &= D_\mu C, \quad s_b C = \tfrac{1}{2}(C\times C), \quad s_b\bar{C} = iB, \\ s_b B &= 0, \quad s_b\bar{B} = -(\bar{B}\times C), \quad s_b F_{\mu\nu} = -(F_{\mu\nu}\times C), \\ s_b H_{\mu\nu\eta} &= -(H_{\mu\nu\eta}\times C), \quad s_b B_{\mu\nu} = -(B_{\mu\nu}\times C), \\ s_b K_\mu &= -(K_\mu\times C),\ s_{ab}A_\mu = D_\mu\bar{C},\ s_{ab}\bar{C} = \tfrac{1}{2}(\bar{C}\times\bar{C}), \\ s_{ab}C &= i\bar{B}, \quad s_{ab}\bar{B} = 0, \quad s_{ab}B = -(B\times\bar{C}), \\ s_{ab}F_{\mu\nu} &= -(F_{\mu\nu}\times\bar{C}), \quad s_{ab}H_{\mu\nu\eta} = -(H_{\mu\nu\eta}\times\bar{C}), \\ s_{ab}B_{\mu\nu} &= -(B_{\mu\nu}\times\bar{C}), \quad s_{ab}K_\mu = -(K_\mu\times\bar{C}), \end{aligned} \tag{16}$$

that the mass dimensions of the fields of the theory, in natural units $\hbar = c = 1$, are: $[A_\mu] = [B_{\mu\nu}] = [C] = [\bar{C}] = [M]$, $[K_\mu] = [0]$, $[F_{\mu\nu}] = [H_{\mu\nu\eta}] = [B] = [\bar{B}] = [M]^2$.

As a consequence of the above observations, the expressions for the (anti-)BRST-invariant coupled Lagrangian densities can be written as follows:

$$\begin{aligned}\mathcal{L}_B &= -\frac{1}{4}F^{\mu\nu}\cdot F_{\mu\nu} + \frac{1}{12}H^{\mu\nu\eta}\cdot H_{\mu\nu\eta} + \frac{m}{4}\,\varepsilon_{\mu\nu\eta\kappa}B^{\mu\nu}\cdot F^{\eta\kappa} \\ &\quad + s_b s_{ab}\Big(\frac{1}{4}B^{\mu\nu}\cdot B_{\mu\nu} + \frac{i}{2}A^\mu\cdot A_\mu + C\cdot\bar{C}\Big), \\ \mathcal{L}_{\bar{B}} &= -\frac{1}{4}F^{\mu\nu}\cdot F_{\mu\nu} + \frac{1}{12}\,H^{\mu\nu\eta}\cdot H_{\mu\nu\eta} + \frac{m}{4}\varepsilon_{\mu\nu\eta\kappa}B^{\mu\nu}\cdot F^{\eta\kappa} \\ &\quad - s_{ab}s_b\Big(\frac{1}{4}B^{\mu\nu}\cdot B_{\mu\nu} + \frac{i}{2}A^\mu\cdot A_\mu + C\cdot\bar{C}\Big).\end{aligned} \tag{17}$$

It should be noted that, in the above parenthesis, we have chosen the combinations of fields that have, in totality, mass dimension equal to *two* and ghost number equal to *zero*. As a result, we have the following coupled Lagrangian densities:

$$\begin{aligned}\mathcal{L}_B &= -\frac{1}{4}\,F^{\mu\nu}\cdot F_{\mu\nu} + \frac{1}{12}\,H^{\mu\nu\eta}\cdot H_{\mu\nu\eta} + \frac{m}{4}\varepsilon_{\mu\nu\eta\kappa}B^{\mu\nu}\cdot F^{\eta\kappa} \\ &\quad + B\cdot(\partial_\mu A^\mu) + \frac{1}{2}\Big(B\cdot B + \bar{B}\cdot\bar{B}\Big) - i\,\partial_\mu\bar{C}\cdot D^\mu C, \\ \mathcal{L}_{\bar{B}} &= -\frac{1}{4}F^{\mu\nu}\cdot F_{\mu\nu} + \frac{1}{12}H^{\mu\nu\eta}\cdot H_{\mu\nu\eta} + \frac{m}{4}\varepsilon_{\mu\nu\eta\kappa}B^{\mu\nu}\cdot F^{\eta\kappa} \\ &\quad - \bar{B}\cdot(\partial_\mu A^\mu) + \frac{1}{2}\Big(B\cdot B + \bar{B}\cdot\bar{B}\Big) - iD_\mu\bar{C}\cdot\partial^\mu C.\end{aligned} \tag{18}$$

It can be checked that the Lagrangian densities $\mathcal{L}_B$ and $\mathcal{L}_{\bar{B}}$ transform under the off-shell nilpotent BRST and anti-BRST symmetry transformations (cf. (16)) as

$$\begin{aligned} s_b\mathcal{L}_B &= \partial_\mu\big[B\cdot D^\mu C\big], \quad s_{ab}\mathcal{L}_{\bar{B}} = -\partial_\mu\big[\bar{B}\cdot D^\mu\bar{C}\big] \\ s_b\mathcal{L}_{\bar{B}} &= -\partial_\mu\big[\bar{B}\cdot\partial^\mu C\big] + D_\mu\big[B + \bar{B} + i(C\times\bar{C})\big]\cdot\partial^\mu C, \\ s_{ab}\mathcal{L}_B &= \partial_\mu\big[B\cdot\partial^\mu\bar{C}\big] - D_\mu\big[B + \bar{B} + i(C\times\bar{C})\big]\cdot\partial^\mu\bar{C}.\end{aligned} \tag{19}$$

Thus, the action corresponding to the above Lagrangian densities remains invariant.

The 4D coupled Lagrangian densities (17) can be generalized onto (4, 2)-dimensional supermanifold and can be expressed in terms of the superfields obtained after the applications of HC and GIRs. These super Lagrangian densities, in full blaze of glory, are

$$\begin{aligned}\tilde{\mathcal{L}}_B &= -\frac{1}{4}\,\tilde{\mathcal{F}}^{\mu\nu(h)}\cdot\tilde{\mathcal{F}}^{(h)}_{\mu\nu} + \frac{1}{12}\,\tilde{\mathcal{H}}^{\mu\nu\eta(g,h)}\cdot\tilde{\mathcal{H}}^{(g,h)}_{\mu\nu\eta} \\ &\quad + \frac{m}{4}\,\varepsilon_{\mu\nu\eta\kappa}\tilde{\mathcal{B}}^{\mu\nu(g)}\cdot\tilde{\mathcal{F}}^{\eta\kappa(h)} + \frac{\partial}{\partial\bar{\theta}}\,\frac{\partial}{\partial\theta} \\ &\quad \times\Big(\frac{1}{4}\tilde{\mathcal{B}}^{\mu\nu(g)}\cdot\tilde{\mathcal{B}}^{(g)}_{\mu\nu} + \frac{i}{2}\,\mathcal{B}^{\mu(h)}\cdot\mathcal{B}^{(h)}_\mu + \tilde{\mathcal{F}}^{(h)}\cdot\bar{\mathcal{F}}^{(h)}\Big), \\ \tilde{\mathcal{L}}_{\bar{B}} &= -\frac{1}{4}\,\tilde{\mathcal{F}}^{\mu\nu(h)}\cdot\tilde{\mathcal{F}}^{(h)}_{\mu\nu} + \frac{1}{12}\,\tilde{\mathcal{H}}^{\mu\nu\eta(g,h)}\cdot\tilde{\mathcal{H}}^{(g,h)}_{\mu\nu\eta} \\ &\quad + \frac{m}{4}\,\varepsilon_{\mu\nu\eta\kappa}\tilde{\mathcal{B}}^{\mu\nu(g)}\cdot\tilde{\mathcal{F}}^{\eta\kappa(h)} - \frac{\partial}{\partial\theta}\,\frac{\partial}{\partial\bar{\theta}} \\ &\quad \times\Big(\frac{1}{4}\tilde{\mathcal{B}}^{\mu\nu(g)}\cdot\tilde{\mathcal{B}}^{(g)}_{\mu\nu} + \frac{i}{2}\,\mathcal{B}^{\mu(h)}\cdot\mathcal{B}^{(h)}_\mu + \tilde{\mathcal{F}}^{(h)}\cdot\bar{\mathcal{F}}^{(h)}\Big).\end{aligned} \tag{20}$$

[3] The superscripts (g,h), on the compensated super curvature tensor $\tilde{\mathcal{H}}^{(g,h)}_{\mu\nu\eta}\,(x,\theta,\bar{\theta})$, denote the incorporation of the constituent superfields (*i.e.* $\tilde{\mathcal{B}}^{(g)}_{\mu\nu}, \tilde{\mathcal{K}}^{(g)}_\mu, \tilde{\mathcal{F}}^{(h)}_{\mu\nu}$), that have been obtained after the application of the HC and GIRs. The latter are found to be complementary and consistent with each other.

The BRST and anti-BRST invariance of equation (19) can be translated into the language of the super Lagrangian densities (20) and the operation on them by the Grassmannian partial derivatives as: $(\partial/\partial\bar{\theta})\tilde{\mathcal{L}}_B = 0$, $(\partial/\partial\bar{\theta})\tilde{\mathcal{L}}_{\bar{B}} = 0$, $(\partial/\partial\theta)\tilde{\mathcal{L}}_B = 0, (\partial/\partial\theta)\tilde{\mathcal{L}}_{\bar{B}} = 0$.

Conclusions. – One of the key observations of our present investigation is to obtain the compelling theoretical reasons to go beyond the application of the HC in the context of superfield formulation of purely free p-form ($p = 1, 2, 3, \ldots$) gauge theories (where there is no interaction with matter fields). As it turns out, the GIRs on the superfields complement the application of the HC in the sense that we derive *all* the off-shell nilpotent (anti-)BRST symmetry transformations for the present 4D topologically massive non-Abelian gauge theory. We have exploited the GIRs in the context of (non-)Abelain 1-form gauge theory as well [2,3]. The distinct difference, however, is that, in all such theories [2,3], there is presence of matter fields. It is worth pointing out that we have tapped only the *usual* "scalar" gauge symmetry transformations for our BRST analysis and have ignored the "vector" gauge symmetry transformations (cf. the first section, for some remarks on it). It would be very interesting to exploit both these gauge symmetries *together* for the BRST analysis within the framework of superfield approach to our 4D topologically massive non-Abelian model.

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