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To cite this article: A. Sergeev *et al* 2010 *EPL* **92** 27003

A LETTERS JOURNAL EXPLORING
THE FRONTIERS OF PHYSICS

October 2010

EPL, **92** (2010) 27003
doi: 10.1209/0295-5075/92/27003

www.epljournal.org

Thermomagnetic vortex transport: Transport entropy revisited

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received 2 July 2010; accepted in final form 11 October 2010
published online 17 November 2010

PACS 74.25.Uv – Vortex phases (includes vortex lattices, vortex liquids, and vortex glasses)
PACS 74.25.F- – Transport properties
PACS 74.25.fg – Thermoelectric effects

Abstract – Transport entropy, S_d, defines both the thermal force, $\mathbf{f}_{th} = -S_d \nabla T$, pushing a vortex along $-\nabla T$ in the Nernst effect and the thermal energy, $\epsilon_{th} = S_d T$, transferred by a vortex in the Ettingshausen effect. All current theories associate the main contribution to S_d with the electromagnetic energy of superconducting currents circulating around cores, $F^{em}(T)$. Using the universal relation between F^{em} and magnetization (DORSEY A. T., *Phys. Rev. B*, **46** (1992) 8376) we extend our concept that magnetization currents do not transfer the thermal energy (SERGEEV A. *et al.*, *Phys. Rev. B*, **77** (2008) 064501) and prove that supercurrents around cores neither produce the net force proportional to ∇T, nor participate in the heat transport. Being consistent with the London concept and Onsager relation, our approach naturally explains the absence of the thermomagnetic effects in a system of Josephson vortices in SIS junctions. It elucidates the heat current definition and justifies the magnetization subtraction from the microscopically calculated energy (not heat) flux in the vortex liquid. The revised theory is in very good agreement with the measured entropy of Abrikosov's vortices and explains the nonmonotonic behavior of $S_d(T)$ with a maximum at $\sim 0.6T_c$.

Introduction. – In recent years, experiments with high-T_c cuprates reveal large thermomagnetic effects [1–3], which are often associated with the vortex liquid [4,5]. This observation has revived interest in thermomagnetic phenomena and exposed long-standing problems in the theory, in particular, its consistency with the London concept and Onsager principle etc. [6].

The central issue of the vortex liquid is Abrikosov's notion of the quantized flux line, which consists of a normal core with the size of the coherent length, ξ, and superconducting currents circulating around the core in the area of the order of the magnetic penetration length, λ. In the Nernst effect, vortices under the thermal force, $\mathbf{f}_{th}$, move in the direction parallel to the temperature gradient ∇T, transfer the magnetic flux, and in this way induce the transverse Nernst voltage. In the Ettingshausen effect, vortices under the Lorentz force, $\mathbf{f}_L$, move in the direction perpendicular to the electric field (current) and transfer the thermal energy in this direction [5,6]. Traditionally both effects are described in terms of the "transport entropy" of vortices, S_d, which defines the thermal force $\mathbf{f}_{th} = -S_d \nabla T$ in the Nernst effect and the thermal energy of a vortex $\epsilon_{th} = TS_d$ in the Ettingshausen effect [6]. The Onsager principle requires that both the thermal force and the thermal energy are described by the same transport entropy S_d [6,7].

In current theories [7–11] the transport entropy S_d is attributed to the electromagnetic energy of superconducting currents F^{em} (for F^{em} in various models see also [12,13]). Moreover, according to all theories, the contribution of supercurrents, S_d^{em}, significantly prevails over a contribution of the core, S_d^{core}. The term S_d^{em} was investigated in various models. In refs. [7–9] the supercurrent-related entropy S_d^{em} was calculated in the Ginzburg-Landau (GL) formalism. In [10] S_d^{em} was obtained in the London model, which was developed for extreme type-II superconductors ($\xi \ll \lambda$), where cores are treated as point singularities and S_d^{core} is absent at all. The London-type models with $S_d^{core} = 0$ are widely used for numerical studies of thermomagnetic effects in high-T_c cuprates [11].

Thus, according to the current theories [7–11], the superconducting currents around a core provide the main contribution to the transport entropy, *i.e.* the

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supercurrents create the net thermal force in the Nernst effect and also transfer the heat in the Ettingshausen effect. However, the last conclusion strongly contradicts to the London concept, according to which $S_d^{em}=0$, because the supercurrent is formed by macroscopic number of particles, which move coherently in a single quantum state and do not transfer thermal energy [14,15]. Direct calculations of thermal conductivity by Abrahams *et al.* confirmed that even in the fluctuations regions above T_c, "since the superfluid carriers no entropy, it does not contribute to the thermal current" [16].

Another way to look at this problem is to associate the superconducting currents with the magnetization currents. Such approach turns out to be especially fruitful for the vortex liquid due to a universal relation between the electromagnetic energy of a vortex and the magnetization, which has been proved by Dorsey [17],

$$F^{em}=nF_\phi^{em}=n\phi_0|\mathbf{M}|, \tag{1}$$

where n is the vortex concentration, ϕ_0 is the quantum of the magnetic flux. Dorsey's formula is an exact relation based on a virial theorem in the GL formalism. So it describes strongly interacting and even overlapping vortices. Thus, in the whole GL region the electromagnetic energy F^{em} may be considered as the energy of magnetization currents. According to our recent microscopic results [18], any magnetization currents (not necessary superconducting currents) do not transfer the heat and the electromagnetic energy related to the magnetization does not contribute to thermal transport.

To resolve the above contradictions, we revise the theory of thermomagnetic vortex transport. We analyze the thermodynamics of vortices and show that the superconducting currents around cores do not transfer the thermal energy and entropy in the Ettingshausen effect. We also demonstrate that superconducting currents do not produce the net moving force in the Nernst effect, because the thermal force is universally canceled by the force due to the interaction with magnetization currents that are created by the temperature gradient. Therefore, the entropy S_d is the ordinary thermodynamic entropy transferred solely by vortex cores. We show that the last conclusion is in excellent agreement with available data and provides natural explanations to measured dependences $S_d(T)$.

Transport entropy. – First let us present the current theoretical description with more details. In the Nernst effect the electric response is induced by a transverse temperature gradient ∇T. If the entropy S_d moves from the area with the temperature T to the area with the temperature $T-\Delta T$, the ratio of the work produced by thermal force, $\mathbf{f}_{th}\cdot\Delta\mathbf{r}$, to the thermal energy, TS_d, is given by the Carnot efficiency $\Delta T/T$. Therefore, the thermal force may be expressed as $\mathbf{f}_{th}=-S_d\nabla T$ [5–8]. The thermal force $\mathbf{f}_{th}$ leads to the vortex motion with the velocity $\mathbf{v}_T=\mathbf{f}_{th}/\eta$, where η is the viscosity coefficient. Magnetic flux of vortices $n\phi_0$ generates the Nernst EMF, which is $\tilde{\mathbf{E}}_N=n\overrightarrow{\phi}_0\times\mathbf{v}_T/c$. Finally, the voltage signal in the open circuit is given by

$$\tilde{\mathbf{E}}_N=\frac{S_d}{c\eta}\ \nabla T\times\mathbf{B}, \tag{2}$$

where the magnetic field $\mathbf{B}=n\overrightarrow{\phi}_0$. In the closed circuit the Nernst EMF generates the electric current

$$\mathbf{j}^e\equiv-\alpha\nabla T\times\mathbf{e}_B=\sigma_f\tilde{\mathbf{E}}_N=-\frac{S_d}{c\phi_0}\ \nabla T\times\mathbf{e}_B, \tag{3}$$

where $\sigma_f=\eta/(\phi_0 B)$ is the flux-flow conductivity, and $\mathbf{e}_B$ is the unit vector in the direction of $\mathbf{B}$.

In the Ettingshausen effect, the heat current is induced by the transverse electric current $\mathbf{j}^e=\sigma_f\mathbf{E}$. The current gives rise to the Lorentz force, $\mathbf{f}_L=(\mathbf{j}^e\times\overrightarrow{\phi}_0)/c$, which leads to the vortex motion with the velocity $\mathbf{v}_L=\mathbf{f}_L/\eta$. The thermal energy of a vortex is expressed in terms of the transport entropy as $\epsilon_{th}=TS_d$. Then, the heat current, $\mathbf{j}^h=n\epsilon_{th}\mathbf{v}_L$, may be presented as

$$\mathbf{j}^h\equiv\widetilde{\alpha}\ \mathbf{E}\times\mathbf{e}_B=\frac{nTS_d}{c\eta}\ \mathbf{j}^e\times\phi_0=\frac{TS_d}{c\phi_0}\ \mathbf{E}\times\mathbf{e}_B. \tag{4}$$

The above description of thermomagnetic effects has been developed by Stephen [7,8]. Comparing eqs. (3) and (4), we see that the Stephen formalism is in agreement with the Onsager principle: $\tilde{\alpha}=T\alpha$. This agreement is reached by presenting both the thermal force $\mathbf{f}_{th}$ and the thermal energy ϵ_{th} via the transport entropy S_d. However, after many years of extensive theoretical and experimental research, the physical sense of S_d and its relation with ordinary entropy are still unclear [5,6].

Calculating S_d, the theoretical works [7–11] neglected the core contribution and restricted themselves with F^{em} only. In his pioneering paper, [7], Stephen considered thermomagnetic vortex transport near H_{c1}, where an interaction between vortices can be neglected. In this case, F^{em} per vortex can be obtained in the GL formalism [6–8,13],

$$F_\phi^{em}=\frac{\phi_0 H_{c1}}{4\pi}=\phi_0|M(H_{c1})|=\left(\frac{\phi_0}{4\pi\lambda}\right)^2\ln\frac{\lambda}{\xi}, \tag{5}$$

where $M=4\pi H_{c1}$ is the magnetization. Stephen introduced the transport entropy per vortex as [7,8],

$$\begin{aligned}S_d^{em}&=-\frac{\partial F_\phi^{em}}{\partial T}=-\phi_0\frac{\partial|M|}{\partial T}\\&=-\frac{\phi_0}{4\pi}\frac{\partial H_{c1}}{\partial T}=-\frac{\partial}{\partial T}\left(\frac{\phi_0^2}{16\pi^2\lambda^2}\ln\frac{\lambda}{\xi}\right).\end{aligned} \tag{6}$$

With the Dorsey relation (eq. (1)), the Stephen entropy (eq. (6)) could be easily calculated in the whole GL region.

Contrary to Stephen, Troy and Dorsey [9] considered the whole electromagnetic energy of vortices, $F_\phi^{em}=\phi_0|M|$, as the thermal energy, $\epsilon_{th}=TS_d$, and obtained

$S_d^{em} = \phi_0|M|/T$. Coffey [10] calculated F^{em} in the London model and extended Stephen's theory to the intermediate magnetic fields. His result for S_d^{em} differs from eq. (5) by a factor $\ln(H_{c2}/B)/\ln\kappa$. Using the London-type model, Podolsky *et al.* [11] found that S_d^{em} is proportional to the magnetization. Thus, some theories obtained S_d^{em} to be proportional to $|\mathbf{M}|$ [9,11], while the others concluded that S_d^{em} is proportional to the temperature derivative of $\mathbf{M}$ [7,8,10]. But, it is even more important that all the above theories associated S_d with the electromagnetic energy of supercurrents, F^{em}, and this approach contradicts the London concept.

Transport entropy revisited. – Now we will show that the entropy of superconducting currents in fact is zero and, in agreement with the Onsager principle, the superconducting currents do not contribute to the Nernst effect as well. First, we would like to note that in the Stephen theory [7,8] the nonzero entropy of supercurrents has been obtained due to misinterpretation of thermodynamic relations. If $S=0$, the free energy, $F^{em}=U^{em}-TS$, is equal to the internal energy, *i.e.* $U^{em}=F^{em}$ and $U_\phi^{em}=F_\phi^{em}=\phi_0|\mathbf{M}|$. Then, the entropy can be again expressed via thermodynamic relation as the temperature derivative of F^{em} (without the energy of magnetic field $H^2/8\pi$) *at the constant magnetization* [19], *i.e.*

$$S^{em} = -\Big(\frac{\partial F_\phi^{em}}{\partial T}\Big)_M = -\phi_0\Big(\frac{\partial M}{\partial T}\Big)_M = 0. \qquad (7)$$

According to eq. (1), the above proof is valid not only for single vortices, where the Stephen approach is applicable, but in the whole GL region. Of course, this consideration does not add anything new beyond London's concept. It just shows that, in fact, the temperature derivative in eq. (5) should be calculated at constant $\mathbf{M}$, which leads to zero entropy.

Let us now consider the Nernst effect. First, we will ignore vortex core and calculate the net force acting on superconducting currents in the presence of ∇T. Taking into account that $S^{em}=0$ and using eq. (1) derived by Dorsey, we can calculate the thermal force as

$$\mathbf{f}_{th} = -\frac{\partial F_\phi^{em}}{\partial \mathbf{r}} = -\phi_0\frac{\partial|M|}{\partial T}\,\nabla T. \qquad (8)$$

As is shown in fig. 1(a), the thermal force is directed from the cold to the hot area, because the vortex energy $F_\phi(T)$ decreases when T increases. Note, that our thermal force (eq. (8)) and the thermal force introduced by Stephen [5–8] have the same value, but the opposite direction (see discussion in [20]).

To satisfy the Onsager principle, the thermal force should be balanced by another force. The additional force overlooked in all previous works originates from the magnetization currents in the presence of ∇T,

$$\mathbf{j}^e_{\nabla T} = c\nabla\times\mathbf{M}(T) = c\nabla T\times\frac{\partial\mathbf{M}}{\partial T}. \qquad (9)$$

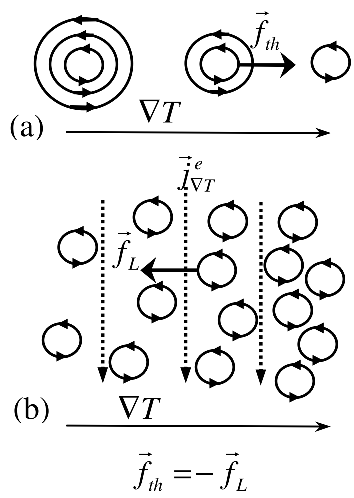

Fig. 1: Balance of two forces acting on the superconducting currents: (a) $\mathbf{f}_{th}$ is the thermal force (eq. (8)), (b) $\mathbf{f}_L$ is the Lorentz force due to the magnetization currents (eq. (10)).

As is shown in fig. 1(b), the current $\mathbf{j}^e_{\nabla T}$ leads to the Lorentz force, which acts on a single vortex in the direction perpendicular to ∇T,

$$\begin{aligned}\mathbf{f}_L &= \frac{1}{c}\,\mathbf{j}^e_{\nabla T}\times\vec{\phi}_0 = -\left(\vec{\phi}_0\cdot\frac{\partial\mathbf{M}}{\partial T}\right)\nabla T\\ &= \phi_0\frac{\partial|M|}{\partial T}\,\nabla T.\end{aligned} \qquad (10)$$

The Lorentz force $\mathbf{f}_L$ is directed from the hot to the cold area (fig. 1(b)). Thus, eqs. (8) and (10) show that the total moving force acting on vortex supercurrents is zero.

Let us highlight that our proof is completely based on the Dorsey relation (eq. (1)), which expresses the electromagnetic energy of a vortex in terms of magnetization $\mathbf{M}$. This is an exact relation, which is valid in the whole GL region [17]. Therefore, our conclusion is also valid for the entire mixed state, including interacting vortices ($1/\lambda^2 < n < 1/\xi^2$) and even vortices with overlapping cores ($n\sim 1/\xi^2$). Moreover, the Dorsey result (eq. (1)) and our proof of the cancellation of the forces (eqs. (8) and (10)) are applicable to any type of superconducting pairing. Thus, in any London-type model with point-like cores thermomagnetic effects are completely absent.

Returning to the Ettingshausen effect, we may illustrate it in the following way. During the nucleation of a vortex at the one edge of the sample the magnetic energy transforms into the kinetic energy of the coherent circulating supercurrents and condensation energy in the core. During the vortex annihilation at the opposite edge of the superconductor the energy of supercurrents is delivered back to the field. These transformations *exclude any dissipation*, so supercurrents transfer pure mechanical (not thermal) energy, which can always be used in full.

From the experimental point of view, a system of Josephson vortices in superconductor-insulator-superconductor (SIS) junctions is an ideal test ground for our conclusion, because these vortices have no normal cores and their whole energy has an electromagnetic origin. Experimental data unambiguously show the absence of thermomagnetic effects in SIS structures [21]. It is important that experiments with superconductor-normal metal-superconductor (SNS) junctions demonstrate large thermomagnetic effects analogous to that for Abrikosov's vortices [22]. These observations fully confirm our conclusions about zero transport entropy related to supercurrents.

We have shown that thermomagnetic effects are absent as long as we limit our consideration by superconducting currents around core. To get nonzero effects, we should take into account contributions of normal electrons, *i.e.* F^{core}. The transport entropy is an ordinary thermodynamic entropy counted from a background. If vortex cores do not overlap each other, *i.e.* $n\xi^2 \ll 1$, the background is homogeneous and, therefore, the transport entropy is determined by the condensation energy, $H_c^2/8\pi$, in the core area, which is $\sim \pi\xi^2$. Thus, the transport entropy per a vortex may be evaluated as

$$S_d^{core}(T) \simeq -\pi\xi^2 \ \frac{\partial}{\partial T}\frac{H_c^2(T)}{8\pi}. \qquad (11)$$

Note that close to H_{c2}, *i.e.* in the magnetic field $H \simeq B \simeq \phi_0/\xi^2$, the background is formed by cores of other vortices and eq. (11) is inapplicable. Here, the transport entropy S_d decreases due to overlapping of vortex cores and goes to zero at H_{c2}. Self-consistent description of the narrow region near H_{c2} requires microscopic consideration (see next section).

The exact results for S_d can be found from the GL formalism in the limit of large κ. In this case the condensation energy and the transport entropy are [13]

$$F_\phi^{core} = a\left(\frac{\phi_0}{4\pi\lambda}\right)^2, \qquad S_d^{core}(T) = \frac{\partial F_{core}}{\partial T}. \qquad (12)$$

In the original paper by Abrikosov, the constant a was found to be $\sim$0.08 [13], then Hu [23] corrected its value to 0.497. Comparing with eq. (5), we see that in this limiting case the correct value of S_d is approximately $2\ln(\lambda/\xi)$ times smaller than that predicted by Stephen [7,8].

In qualitative agreement with this conclusion, S_d determined from experimental data turns out to be several times smaller than that predicted by Stephen's theory. For example, for single vortices in IBaCuO not far from T_c, Palstra *et al.* [24] found that "the thermal energy transported by a vortex is about 25% of the vortex energy", where the vortex energy was defined as the electromagnetic energy, given by eq. (5). This directly shows that the thermal energy cannot be associated with the electromagnetic energy. Our evaluation with $\kappa \simeq 50$ shows that vortices (normal electrons in cores) transfer 13%, *i.e.* approximately one-half of the thermal energy determined from measurements. In agreement with other measurements [5], this evidences that in the limit of low vortex concentration the quasiparticle contribution, *i.e.* the contribution of the normal electrons outside the core, may be also substantial.

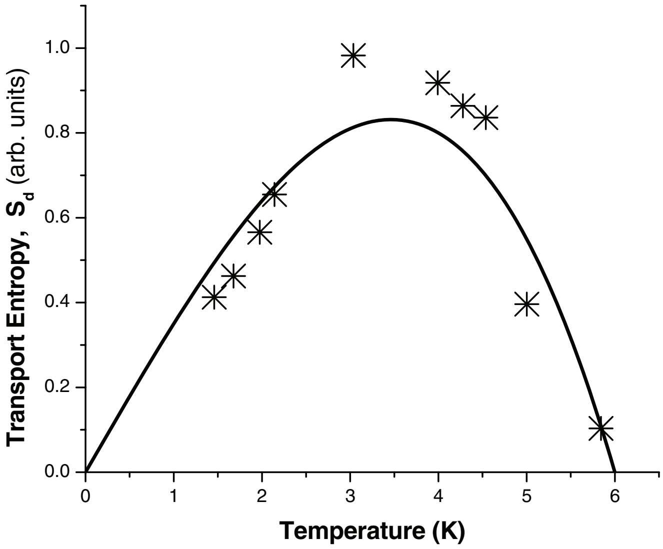

Fig. 2: The temperature dependence of the transport entropy: theory (solid line) and data from ref. [26].

Now let us analyze the temperature dependence of S_d. Comprehensive numerical analysis [25] shows that at moderate temperatures $0.2 \leqslant T/T_c \leqslant 0.9$ the radius of the vortex core ξ_1, defined by fitting the pair potential $\Delta(r)$ by the expression $\Delta(r) = \Delta_0 \tanh(r/\xi_1)$, just weakly depends on temperature. Therefore, at $0.2 \leqslant T/T_c \leqslant 0.9$, according to eq. (11) the temperature dependence of S_d is mainly determined by the dependence $H_c(T) \propto 1-(T/T_c)^2$, so S_d is proportional to $(T/T_c)[1-(T/T_c)^2]$ and has a smooth maximum at $T \simeq 0.6T_c$.

Let us now compare our conclusions with experimental data. Figure 2 shows the well-known dependence $S_d(T)$, obtained by Solomon and Otter [26] from the Ettingshausen effect in InPb films. While this dependence is presented in practically all books on vortex transport [6,27], until now its nonmonotonic character has not got any theoretical explanation. As seen from fig. 2, we get a very good agreement with these data. The linear increase of S_d at low temperatures is determined by the entropy of electrons in the core, then S_d reaches a maximum and vanishes due to the increase of the background entropy. Using the parameters of the InPb alloy [26], we evaluate that the maximum of $S_d(T)$ is $1.6 \cdot 10^{-7}$ erg/cm K, while the experiment gives $2 \cdot 10^{-7}$ erg/cm K. Thus, the proposed model provides a simple explanation of the nonmonotonic temperature dependence of S_d in ordinary superconductors.

Finally, we note arrears of applicability of our results. Our direct proof that the total force acting on the vortex supercurrents in the Nernst effect is zero is completely based on GL formalism, which is valid close to the superconducting transition line in the H-T phase diagram. At the same time, as we stress above, this statement is a consequence of the London concept and Onsager principle, so it is general and applicable to any vortex liquid in superconductors. Calculating the transport entropy associated with a vortex core we use the two-liquid formula for $H_c(T)$ and neglect the quantization of quasiparticle spectra in the core. In weak magnetic fields these approximations are valid above 0.2–0.3 T_c. Due to quantization of spectra, the heat capacity and entropy of quasiparticles in the core are exponentially small at low temperatures [13]. This explains, why thermomagnetic effects in a weak field H are not observed below 0.2 T_c [26].

Microscopic theory: magnetization subtraction. – Finally, we discuss our conclusions in the light of the microscopic theory. As we mentioned, current microscopic theory covers only a very narrow region near H_{c2} [28]. Using TDGL and the Kubo method, Caroli and Maki [28] calculated S_d and found that it diverges at $T \to 0$. To get rid of the contradiction with the third law of thermodynamics, Maki [29] suggested to subtract from the microscopically calculated heat current "the thermodynamic thermal flux due to magnetization currents": $j^h_{mag} = -\mathbf{E} \times \mathbf{M} = n\mathbf{v}\phi_0|M|$. In a number of works it was highlighted that this modified Maki's theory is inconsistent with thermodynamics [30] and/or Onsager relations [31]: "Maki has taken account of the energy flow due to the change of the magnetization. But then Onsager's reciprocal theorem is violated, unless we add some additional electric current, the nature of which is quite unknown at present". This criticism is fully applicable to any artificial correction of the heat current. However, superconducting magnetization currents do not transfer heat [14–16]. Moreover, any magnetization currents do not transfer thermal energy and entropy [18]. Thus, microscopically calculated thermal current does not allow any artificial magnetization corrections [32].

So, how one can justify the Maki subtraction and preserve the Onsager relation? The thermal energy is defined as the energy related to chaotic motion, *i.e.* as the energy counted from the level of the electrochemical potential minus the energy of various coherent motions (coherent electrons, phonons etc.) [18]. Such definition is natural and consistent with the Onsager principle [18]. As we discussed above the electromagnetic energy of the vortex F^{em}_{ϕ} is the energy of superconducting electrons coherently rotating around a vortex core. Naturally, this energy was counted in the TDGL formalism, but should be removed from the total energy, when the thermal energy is extracted. Again, the exact relation between the electromagnetic energy and magnetization derived by Dorsey (eq. (1)) allows us to present the energy of coherent rotating motion of superconducting electrons via magnetization in the universal form, which coincides with "magnetization subtraction" proposed by Maki [29]. Thus, while the interpretation of the Maki subtraction, $n\mathbf{v}\phi_0|M|$, as the magnetization heat current is wrong and contradicts thermodynamics and Onsager relation [30,31], this correction is necessary and corresponds to the energy of the coherent circulating motion, which should be removed from the energy current to get a thermal flux.

Conclusion. – Associating the superconducting currents circulating around cores with the magnetization, we have shown that supercurrents do not transfer the thermal energy in the Ettingshausen effect and do not produce the net force proportional to ∇T in the Nernst effect. Only this approach is consistent with the thermodynamics of irreversible processes (*i.e.* the Onsager relation and the third law of thermodynamics) and the London concept. Our theory provides a natural explanation of the absence of thermomagnetic effects related to Josephson vortices in SIS structures. Our concept that magnetization currents transfer the energy, but not the heat justifies the magnetization subtraction, proposed by Maki. We have shown that the transport entropy of vortices is just the ordinary thermodynamic entropy of cores counted from the background entropy. Our results are in a very good agreement with $S_d(T)$ in ordinary superconductors. Our approach can be easily generalized for various models, which were recently suggested for the vortex liquid in cuprates.

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We are grateful to I. Aleiner, A. Gurevich, and Y. Kopelevich for valuable discussions. Research was supported by NSF under Grant No. DMR 0907126.

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