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Recovery of Laughlin correlations with cyclotron braids

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Abstract – Cyclotron braid subgroups are defined in order to identify the topological origin of Laughlin correlations in 2D Hall systems. Flux tubes and vortices for composite fermion constructions are explained in terms of cyclotron braids. The odd and even denominator fractional lowest Landau level fillings are discussed.

Introduction. – The essence of 2D Hall system physics involves Laughlin correlations (LCs), which are expressed by the famous Laughlin wave function (LF) [1]. An analysis of the Coulomb interaction in Haldane pseudopotential terms [2–4] indicates that the LF exactly describes the ground state for N charged 2D particles at the fractional Landau level (LL) filling $1/q$, q odd integer, if one neglects the long-distance part of the Coulomb interaction expressed by a projection on the relative angular momenta of particle pairs for values greater than $q-2$. An effective model of composite fermions (CFs) [5] was next formulated in terms of auxiliary flux tubes attached to particles in order to produce the required statistical phase by employing the Aharonow-Bohm phase shift. The competitive construction of CFs utilizes so-called vortices [6,7], collective fluid-like objects that are pinned to bare fermions and reproducing LCs [6]. Both types of composite particles, with vortices or with flux tubes, are phenomenological in nature, thus the question arises as to what is a more fundamental reason for LCs in 2D charged systems. It is commonly acknowledged [8–10] that the source of exotic LCs is of a 2D peculiar topology type. To match the topological properties of quantum systems, quantization by the Feynman path integral method is particularly convenient [8,9]. In the present letter we revisit it to recover LCs by employing properties of the underlying cyclotron braid picture [11,12] without a phenomenological modeling of CFs.

Too-short cyclotron trajectories at strong magnetic fields. – One-dimensional unitary representations (1DURs) of the full braid group [9,13–15] (π_1 homotopy group of undistinguishable N-particle configuration space [13]), define weights for the path integral summation over trajectories [8,9]. If the trajectories fall into separated homotopy classes that are distinguished by non-equivalent closed loops attached to an open trajectory $\lambda_{a,b}$ (linking points a and b in the configuration space), then an additional unitary factor (the weight of the particular trajectory class) should be included [8,9] in the path integral: $I_{a\rightarrow b}=\sum_{l\in\pi_1} e^{i\alpha_l}\int d\lambda_l e^{iS[\lambda_{l(a,b)}]}$, where π_1 stands for the full braid group. The factors $e^{i\alpha_l}$ form a 1DUR of the full braid group and distinct representations correspond to distinct types of quantum particles. For the permutation group S_N, which is the full braid group for N particles in R^n, $n\geqslant 3$, there exist only two 1DURs: $\sigma_i\rightarrow e^{i\pi}$ or $\sigma_i\rightarrow e^{i0}$, ($\sigma_i$ is the interchange of the i-th and $(i+1)$-th particles) corresponding to fermions and bosons, respectively. For N particles in R^2 the braid group is substantially richer than S_N and has an infinite number of 1DURs [9,15], defined for the group generators as: $\sigma_i\rightarrow e^{i\theta}$, $i=1,\dots,N-1$, $\theta\in(-\pi,\pi]$, where each θ corresponds to a different type of anyons [1,8,9] (Abelian, as the 1DUR elements commute). The closed loops from the full braid group describe exchanges of identical particles, thus, their 1DURs indicate the statistics of the particles. Because 1DURs are periodic with a period of 2π, the statistical distinguishing of CFs linked with LCs is precluded due to the fact that they require a phase shift of $p\pi$, $p=3,5\dots$. If it is impossible to associate CFs with the 1DURs of the full braid group, we propose [11] to associate them with appropriately constructed braid subgroups instead of the full braid group and in this way to distinguish CFs from fermions.

The full braid group contains all accessible closed multi-particle classical trajectories —braids (with initial and final orderings of particles that may differ by permutation). We base our analysis on the observation that inclusion of a magnetic field substantially changes

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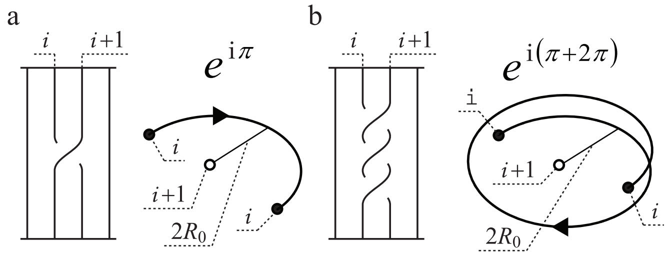

Fig. 1: The generator σ_i of the full braid group and the corresponding relative trajectory of the i-th and $(i+1)$-th particles exchange (a); the generator of the cyclotron braid subgroup, $b_i^{(p)} = \sigma_i^p$ (in the figure, $p=3$), corresponds to additional $\frac{p-1}{2}$ loops when the i-th particle interchanges with the $(i+1)$-th one ($2R_0$ is the inter-particle separation) (b).

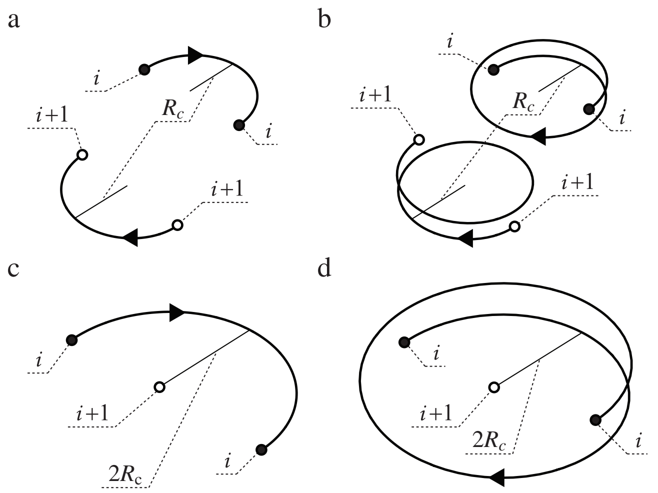

Fig. 2: Half of the individual particle cyclotron trajectories of the i-th and $(i+1)$-th particles ((a), (b)) and the corresponding relative trajectories ((c), (d)) for interchanges of the i-th and $(i+1)$-th 2D particles under a strong magnetic field, for $\nu = 1$ (left) and for $\nu = \frac{1}{3}$ (right), respectively (3D for better visualization).

these trajectories —a classical cyclotron motion confines a variety of accessible braids. When the separation of particles is greater than twice the cyclotron radius, which occurs at fractional lowest LL fillings, the exchanges of particles along single-loop cyclotron trajectories are precluded, because the cyclotron orbits are *too short* for interchanges. Particle interchanges, however, are necessary for defining the statistics and in order to allow exchanges again, the cyclotron radius must somehow be *enhanced*. An enhancement could be achieved by either lowering the effective magnetic field or lowering the effective particle charge. These two possibilities lead to the two phenomenological concepts of CFs —with the lowered field in Jain's construction [5] and with the screened charge in Read's construction of vortices [6]. Both of these constructions seem to have nothing in common with braid groups, but actually both of these phenomenological tricks correspond to the same, more basic and natural concept, of restricting the braids by excluding inaccessible trajectories [11,12]. We argue that at sufficiently high magnetic fields in 2D charged N-particle systems, multi-loop braids allow for the enlargement of cyclotron orbits, thus restoring particle exchanges in a natural way [12]. These multi-loop braids form a subgroup of the full braid group and, in the presence of strong magnetic field, the summation in the Feynman propagator will be thus confined to the elements of this subgroup (its semigroup, for fixed magnetic field orientation, however, with the same 1DURs as of the subgroup).

Cyclotron braid subgroups —the response to too short trajectories. – More precisely, we associate composite particles and LCs with the 1DURs of *cyclotron braid subgroups* that are generated by the following generators: $b_i^{(p)} = \sigma_i^p$, $(p = 3, 5\ldots)$, $i = 1, \ldots, N-1$, where each p corresponds to a different type of cyclotron braid subgroup (and to a different type of corresponding composite particles), and σ_i are the generators of the full braid group. The group element $b_i^{(p)}$ represents the interchanges of the i-th and $(i+1)$-th particles with $\frac{p-1}{2}$ loops, which is clear by virtue of the definition of the single interchange σ_i (cf. fig. 1). The 1DURs of the full group confined to the cyclotron subgroup do not depend on i and yield the cyclotron subgroup 1DURs: $b_i^{(p)} \to e^{ip\alpha}$, $i = 1, \ldots, N-1$, where p is an odd integer and $\alpha \in (-\pi, \pi]$. These 1DURs, enumerated by the *pairs* (p, α), describe composite anyons (CFs, for $\alpha = \pi$). Thus in order to distinguish various types of composite particles one has to consider (p, α) 1DURs of cyclotron braid subgroups.

The N-particle wave function acquires an appropriate phase shift due to particle interchanges, because in agreement with the general rules of quantization [10,15], the wave function must transform according to the 1DUR of an appropriate element of the braid group when the particles traverse, in classical terms, a closed loop in the configuration space corresponding to this particular braid element. In this manner, the Aharonov-Bohm phase of Jain's fictitious fluxes is replaced by additional loops (each loop adds 2π to the total phase shift, if one considers 1DUR with $\alpha = \pi$ related to CFs, cf. fig. 1 (right-hand side)). Let us emphasize that the real particles do not traverse the braid trajectories, as quantum particles do not have any trajectories. But exchanges of coordinates of the N-particle wave function can be represented by braid group elements, in 2D —not permutation only [9,10,15]. Hence, for the braid cyclotron subgroup generated by $b_i^{(p)}$, $i = 1, \ldots, N-1$, we obtain the statistical phase shifts $p\pi$ for the CFs (*i.e.*, for $\alpha = \pi$), as required by the LCs, without the need to model them with flux tubes or vortices.

Each additional loop of a relative trajectory for the particle pair interchange (as defined by the generators $b_i^{(p)}$) reproduces an additional loop in the individual cyclotron trajectories for both interchanging particles —cf. fig. 2. The cyclotron trajectories are repeated in the relative trajectory (c, d) with twice the radius of the individual particle trajectories (a, b). In quantum language, with regards to classical multi-loop cyclotron trajectories, one can conclude only on the number, $\frac{BS}{N}/\frac{hc}{e}$, of flux quanta per single particle in the system, which for the filling $\frac{1}{p}$ is p, *i.e.*, the same as the number of cyclotron loops. Thus, a simple rule could be formulated: an additional loop of a cyclotron braid corresponding to particle interchange, results in *two* additional flux quanta piercing the individual particle cyclotron trajectories. This rule follows immediately from the definition of the cyclotron trajectory, which must be a *closed* individual particle trajectory related to a *double* interchange of the particle pair. In this way, the cyclotron trajectories of both interchanging particles are closed, just like the closed relative trajectory for the *double* interchange. If the interchange is simple, *i.e.*, without any additional loops, the corresponding individual particle cyclotron trajectories are also simple, *i.e.*, single-looped. However, when the interchange of particles is multi-looped, as associated with the p-type cyclotron subgroup ($p>1$), the double interchange relative trajectory has $2\frac{p-1}{2}+1=p$ closed loops, and the individual cyclotron trajectories are also multi-looped, with p loops [12].

It is important to emphasize the difference between the turns of a 3D winding (*e.g.*, of a wire) and multi-loop 2D cyclotron trajectories. In the latter case, 2D multi-loop trajectories cannot enhance the total magnetic field flux BS piercing the system, so all loops must share the same total flux. In the former case, each turn of a winding adds a new portion of the flux, just as a new turn adds a new surface, which is, however, not the case in 2D.

The additional loops in 2D take away the flux quanta simultaneously diminishing the field; this gives an explanation for Jain's auxiliary fluxes screening the external field B. CFs are actually not compositions of particles with flux tubes, though the original name can be still used. Moreover, one can use a similar name, "composite anyons", for particles associated with fractional 1DURs (*i.e.*, with fractional α) of the cyclotron subgroup instead of the full braid group.

The role of the short-range part of the Coulomb interaction. – The Coulomb interaction is crucial for LCs [2–4] but cannot be accounted for in a manner of standard dressing particles with interactions as is typical for quasiparticles in solids, because the interaction does not have a continuous spectrum with respect to particle separation expressed in relative angular-momentum terms [2,3]. The interaction can be operationally included within the Chern-Simons (Ch-S) field theory [16,17], an effective description of the local gauge field attached to particles, which, in the area of Hall systems, suits to particles with vortices, such as anyons and CFs [18]. It has been demonstrated [3,19] that the short-range part of the Coulomb interaction stabilizes CFs against the action of the Ch-S field (its anti-Hermitian term [19,20]), which mixes states with distinct angular momenta within LL [19], in disagreement with the CF model in the Ch-S field approach [18,19]. The Coulomb interaction removes the degeneracy of these states and results in energy gaps which stabilize the CF picture, especially effectively for the lowest LL. For higher LLs, the CFs are not as useful due to possible mixing between the LLs induced by the interaction [21]. The short-range part of the Coulomb interaction also stabilizes the CFs in cyclotron braid terms [11], similarly to how it removes the instability caused by the Ch-S field for angular-momentum orbits in LL [19]. Indeed, if the short-range part of the Coulomb repulsion was reduced, the separation of particles would not be rigidly kept (adjusted to a density only in average) and then other cyclotron trajectories, in addition to those for a fixed particle separation (multi-loop at $\nu=\frac{1}{p}$), would be admitted, which would violate the subgroup construction.

Vortices —links with cyclotron braids. – For Read's CFs [6,7], LCs are modeled by collective vortices that are attached to the particles. A vortex with its center at z is defined as [6] $V(z)=\prod_{j=1}^{N}(z_j-z)^q$, where q is the vorticity. For odd q, it is linked to the Jastrow factor of the LF (resulting by the replacement of z by z_i and the addition of i ($i>j$) to the product domain, *i.e.*, binding to electrons). In particular, for $q=1$ one arrives at the Vandermonde determinant, associated with the ordinary single-loop cyclotron motion of N fermions on the plane at $\nu=1$. Because the vortices are fragments of the Laughlin function, they contain more information than just the statistical winding phase shift (the latter expressed by the factor, $\prod_{i,j}(z_i-z_j)^q/|z_i-z_j|^q$). 1DURs of the cyclotron braid subgroups define the statistical phase winding, but not the shape of the wave function, which is determined via the energy competition between various wave functions with the same statistical symmetry. Thus, vortices contain information beyond just the statistical phase shift, they also include the specific radial dependence of multi-fold zeros pinned to particles through the Jastrow polynomial. The vortex is a collective fluid-like concept that does not meet the single-particle picture. The vorticity q is selected in accordance with the *known in advance* Laughlin function, thus, similarly as CF flux tubes, it requires a motivation within the cyclotron structure.

The properties of vortices can be listed as follows [6]: 1) when traversing with an arbitrary particle z_j a closed loop around the vortex center, then the gain in phase is equal to $2\pi q$; 2) the vortex induces a depletion of the local charge density, which results in a locally positive charge (due to background jellium) that screens the charge of the electron associated to the vortex center; this positive charge is $-q\nu e$ (for $\nu=1/q$ it gives $-e$, which would completely screen the electron charge); 3) exchange of vortices results in a phase shift of $q^2\nu\pi$, (due to the charge deficit of the vortex), which for $\nu=\frac{1}{q}$ gives $q\pi$; the q-fold vortex, together with the bound electron (which contributes a charge e to the complex and produces a statistics phase shift of π), forms a complex that behaves like a composite boson with zero effective charge for odd q and like a composite fermion for even q. The bosons can condense to exactly reproduce the LF for odd q [20], while, for even q, one deals with the Fermi sea in a zero net field, as in both cases the effective charge of the complexes is zero; the latter case reproduces the Hall metal state [22–24].

The second property explains why the model with vortices works. The reduced effective charge of the electron–vortex complex, results in an increase of the cyclotron radius, which is necessary for particle exchanges at fractional fillings.

All properties of vortices or flux tubes can be grasped together by a formal local gauge transformation [20] of the original fermion particles (defined by the field operator $\Psi(\mathbf{x})$) to composite particles: $\Phi(\mathbf{x}) = e^{-J(\mathbf{x})}\Psi(\mathbf{x})$, $\Theta(\mathbf{x}) = \Psi^{+}(\mathbf{x})e^{J(\mathbf{x})}$, where: $J(\mathbf{x}) = q\int \mathrm{d}^2x'\rho(\mathbf{x}')\log(z-z') - \frac{|z|^2}{4l^2}$, and e^{-J} corresponds to a nonunitary, in general, transformation that describes the attachment of Read's vortices (or Jain's flux tubes) to the bare fermions, $\Psi(\mathbf{x})$ and $\Psi^{+}(\mathbf{x})$ (for the annihilation and creation fields, respectively). When restricting $J(\mathbf{x})$ to only its imaginary part (*i.e.*, to the imaginary part of log), one arrives at the Hermitian Ch-S field corresponding to the dressing of fermions with local flux tubes [25]. The field operators $\Phi(\mathbf{x})$ and $\Theta(\mathbf{x})$, $\Phi^{+}(\mathbf{x}) = \Theta(\mathbf{x})e^{J(\mathbf{x})+J^{+}(\mathbf{x})}$, though are not mutually conjugated (they are perfectly conjugated for the Hermitian Ch-S field), describe composite bosons (for odd q) and composite fermions (for even q) within the mean-field approach [20] (remarkably, the real part of J vanishes in the mean field, as the real part of log is canceled by the Gaussian, while the Hermitian Ch-S field is canceled by the external magnetic field). From the relation $e^{q\sum_j \log(z-z_j)} = \prod_j^N (z-z_j)^q$ (for the density operator $\rho(\mathbf{x}) = \Psi^{+}(\mathbf{x})\Psi(\mathbf{x}) \Longrightarrow \sum_{j=1}^N \delta(z-z_j)$), which coincides with the definition of Read's vortex, one can expect that the above local gauge transformation reproduces all properties of the vortices. This gauge transformation allows for the interpretation of the Laughing state as a Bose-Einstein condensate of composite bosons, at $\nu = \frac{1}{q}$, q odd, [6,20], and as a compressible fermion sea, at q even, [23,24] (the latter is unstable against BCS-like pairing) [7,26]. Assuming that the CFs are defined by the 1DURs of the cyclotron subgroup, the Hermitian term of this gauge transformation should be omitted, because it defines CFs when starting from ordinary fermions, which are already taken into account in terms of cyclotron braids.

Compressible Hall-metal state in cyclotron braid terms. – Let us finally comment on the $\nu = \frac{1}{2}$ state (Hall metal) from the point of view of the braid approach. Within Jain's model, two flux tubes attached to composite fermions completely cancel an external magnetic field in the mean-field approximation (in other words, the Hermitian Ch-S field associated with Jain's model cancels, in mean field, the external magnetic field), and this results in a Fermi sea, called the Hall metal state [22]. Within Read's approach to composite particles at $\nu = \frac{1}{2}$, the complete cancellation of charge takes place due to the charge density depletion of the vortex with $q = 2$. Mutual interchange of 2-fold vortices produces $q^2\nu\pi = 2\pi$ phase shift and including additional π due to electrons, the complexes of 2-fold vortices with electrons behave like fermions (without charge) —thus form a Fermi sea (Hall metal). The instability of the Fermi system, results next in a paired state expressed by the Pfaffian factor, restoring incompressibility due to the pairing-gap (BCS-like paired state at $\nu = 5/2$ [26,27], also considered for $\nu = 1/2$ and $1/4$ [28,29]). As Pfaffian [7] contributes with $-\pi$ to the phase shift due to particle interchanges, the total phase shift of the wave function with the Jastrow polynomial $\prod_{i>j}(z_i - z_j)^2$ [7,26] is π. This phase is given by the 1DUR of the cyclotron braid group (with $p = 3$, as such a cyclotron braid subgroup corresponds to the range $\nu \in [1/3, 1))$ assigned by $p\alpha = 3\frac{1}{3}\pi = \pi$, *i.e.*, $\alpha = \frac{1}{3}\pi$. The representation ($p = 3$, $\alpha = \frac{1}{3}\pi$) induces the fermion statistics phase shift of the many-particle wave function for $\nu = 1/2$, and in terms of braid-composite fermions, it corresponds to a net composite electron Fermi sea (since two loops take away the total external flux), in consistence with the local gauge transformation with $q = 2$, thus reproducing fermions (starting from ordinary fermions) [6,20].

In summary, we argue that, at fractional LL fillings, braid trajectories must be multi-looped, while those with lower number of loops (including single-looped) are excluded due to too short cyclotron radius. This unavoidable property of braids recovers LCs in a natural way for 2D charged systems upon strong magnetic field and explains the structure of CFs both with flux tubes or vortices. Unitary representations of cyclotron braids allow also for a self-consistent explanation of compressible states at fillings with even denominators. For example, the $\nu = 1/2$ metal Hall state corresponds to composite anyons with $p\alpha = 3\frac{1}{3}\pi = \pi$ number of 1DUR of the $p = 3$ cyclotron braid subgroup.

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