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Hot and dense gas of quark quasi-particles

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Abstract – Some features of hot and dense gas of quarks which are considered as the quasi-particles of the model Hamiltonian with four-fermion interaction are studied. Being adapted to the Nambu–Jona-Lasinio model this approach allows us to accommodate a phase transition similar to the nuclear liquid-gas one at the proper scale. It allows us to argue the existence of the mixed phase of vacuum and baryonic matter (even at zero temperature) as a plausible scenario of chiral symmetry (partial) restoration.

Understanding in full and describing dependably the critical phenomena (chiral and deconfinement phase transitions) in QCD is still elusive because of the necessity to have the corresponding efficient non-perturbative methods for strongly coupled regime analyzed. For the time being such studies are pursued by invoking diverse effective models. The Nambu–Jona-Lasinio(NJL)–type models are certainly playing the most advanced role in this analysis [1]. This approach deals with the four-fermion interactions *in lieu* of a gluon field QCD dynamics and does not incorporate (albeit being supplemented by the Polyakov loops it does [2]) the property of confinement. At the same time it is quite successful in realizing the spontaneous breakdown of chiral symmetry and its restoration at nonzero temperatures or quark densities. Apparently, the non-renormalizable nature of the NJL model requires another approximation to be solved and might lead to some conclusions which are sometimes dependent on the regularization scenario. Hence, it requires a steadfast control of all inputs done to have the consistent physical results and to avoid the reasons for a skeptical attitude as it was exemplified in [3].

Instructive example was given in refs. [4] and [5] in which the ground state of the model Hamiltonian with four-fermion interactions was studied in detail. The quarks were treated as the quasi-particles of this Hamiltonian and an unexpected singularity (discontinuity) of the mean energy functional as a function of the current quark mass was found. In the particular case of the NJL model new solution branches of the equation for dynamical quark mass as a function of chemical potential have been found, and the appearance of a state filled up with quarks which is almost degenerate with the vacuum state both in quasi-particle chemical potential and in ensemble pressure has been discovered.

Here we are going to study the quark ensemble features at finite temperature and fixed baryonic chemical potential and to analyse the first-order phase transition which takes place in such a system of free quasi-particles. The analysis is performed within the framework of two approaches which are supplementary, in a sense, albeit giving identical results. One of these approaches, based on the Bogolyubov transformations, is especially informative to study the process of filling the Fermi sphere up because at this point the density of quark ensemble develops a continuous dependence on the Fermi momentum. It allows us to reveal an additional structure in the solution of the gap equation for dynamical quark mass just in the proper interval of parameters characteristic for phase transition and to trace its evolution. The result is that quark ensemble might be found in two aggregate states, gas and liquid, and the chiral condensate is partially restored in a liquid phase. In order to make these conclusions easily perceptible we deal with the simplest version of the NJL model (with one flavor and one of the standard parameter sets) and, actually, do not aim to adjust the result obtained with

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well-known nuclear liquid-gas phase transition. Besides, it seems our approach might be treated as a sort of microscopic ground of the conventional bag model and those states filled up with quarks are conceivable as a natural "construction material" for baryons.

Now as an input to start with we remind the key moments of the approach developed. The corresponding Hamiltonian includes the interaction term taken in the form of a product of two coloured currents located in the spatial points $\mathbf{x}$ and $\mathbf{y}$ which are connected by a form-factor and its density reads as

$$\mathcal{H}=-\bar{q}(i\gamma\nabla+im)q-\bar{q}t^a\gamma_\mu q\int \mathrm{d}\mathbf{y}\bar{q}'t^b\gamma_\nu q'\langle A^a_\mu A'^b_\nu\rangle, \qquad (1)$$

where $q=q(\mathbf{x})$, $\bar{q}=\bar{q}(\mathbf{x})$, $q'=q(\mathbf{y})$, $\bar{q}'=\bar{q}(\mathbf{y})$ are the quark and antiquark operators,

$$\begin{aligned} q_{\alpha i}(\mathbf{x}) \;=\; \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\frac{1}{(2|p_4|)^{1/2}}\,&\big[a(\mathbf{p},s,c)u_{\alpha i}(\mathbf{p},s,c)e^{i\mathbf{px}} \\ &+ b^+(\mathbf{p},s,c)v_{\alpha i}(\mathbf{p},s,c)e^{-i\mathbf{px}}\big], \end{aligned} \qquad (2)$$

$p_4^2=-\mathbf{p}^2-m^2$, i is the colour index, α is the spinor index in the coordinate space, a^+, a and b^+, b are the creation and annihilation operators of quarks and anti-quarks, $a|0\rangle=0$, $b|0\rangle=0$, $|0\rangle$ is the vacuum state of free Hamiltonian and m is a current quark mass. The summation over indices s and c is meant everywhere, the index s describes two spin polarizations of the quark and the index c plays a similar role for the colour. As usual $t^a=\lambda^a/2$ are the generators of $SU(N_c)$ colour gauge group and m is the current quark mass. The Hamiltonian density is considered in the Euclidean space and γ_μ denote the Hermitian Dirac matrices, $\mu,\nu=1,2,3,4$. $\langle A^a_\mu A'^b_\nu\rangle$ stands for the form factor of the following form:

$$\langle A^a_\mu A'^b_\nu\rangle=\delta^{ab}\frac{2\widetilde{G}}{N_c^2-1}\,[I(\mathbf{x}-\mathbf{y})\delta_{\mu\nu}-J_{\mu\nu}(\mathbf{x}-\mathbf{y})], \qquad (3)$$

where the second term is spanned by the relative distance vector and the primed gluon field denotes that in the spatial point $\mathbf{y}$. The effective Hamiltonian density (1) results from averaging the ensemble of quarks influenced by the intensive stochastic gluon field A^a_μ, see ref. [4]. For the sake of simplicity we neglect the contribution of the second term of the above formula in what follows. The ground state of the system is searched as the Bogolyubov trial function composed of the quark-antiquark pairs with opposite momenta and with vacuum quantum numbers, *i.e.*

$$\begin{aligned} &|\sigma\rangle=\mathcal{T}|0\rangle, \\ &\mathcal{T}=\Pi_{p,s}\exp\{\varphi[a^+(\mathbf{p},s)b^+(-\mathbf{p},s)+a(\mathbf{p},s)b(-\mathbf{p},s)]\}. \end{aligned} \qquad (4)$$

In this formula and below, in order to simplify the notations, we refer to only one complex index which means both the spin and colour polarizations. The parameter $\varphi(\mathbf{p})$ which describes the pairing strength is determined by the minimum of mean energy $E=\langle\sigma|H|\sigma\rangle$. By introducing the "dressing transformation" we define the creation and annihilation operators of quasi-particles as $A=\mathcal{T}a\mathcal{T}^{-1}$, $B^+=\mathcal{T}b^+\mathcal{T}^{-1}$ and for fermions $\mathcal{T}^{-1}=\mathcal{T}^\dagger$. Then the quark field operators are presented as

$$\begin{aligned} q(\mathbf{x}) \;&=\; \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\frac{1}{(2|p_4|)^{1/2}}\,\big[A(\mathbf{p},s)U(\mathbf{p},s)e^{i\mathbf{px}} \\ &\quad + B^+(\mathbf{p},s)V(\mathbf{p},s)e^{-i\mathbf{px}}\big], \\ \bar{q}(\mathbf{x}) \;&=\; \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\frac{1}{(2|p_4|)^{1/2}}\,\big[A^+(\mathbf{p},s)\overline{U}(\mathbf{p},s)e^{-i\mathbf{px}} \\ &\quad + B(\mathbf{p},s)\overline{V}(\mathbf{p},s)e^{i\mathbf{px}}\big], \end{aligned}$$

moreover, the transformed spinors U and V are given by the following forms:

$$\begin{aligned} U(\mathbf{p},s) \;&=\; \cos(\varphi)u(\mathbf{p},s)-\sin(\varphi)v(-\mathbf{p},s), \\ V(\mathbf{p},s) \;&=\; \sin(\varphi)u(-\mathbf{p},s)+\cos(\varphi)v(\mathbf{p},s), \end{aligned} \qquad (5)$$

where $\overline{U}(\mathbf{p},s)=U^+(\mathbf{p},s)\gamma_4$, $\overline{V}(\mathbf{p},s)=V^+(\mathbf{p},s)\gamma_4$ are the Dirac conjugated spinors.

In the paper, ref. [5], the process of filling in the Fermi sphere with the quasi-particles of quarks was studied by constructing the state of the Sletter determinant type $|N\rangle=\prod_{|\mathbf{P}|<P_F;S}A^+(\mathbf{P};S)|\sigma\rangle$, which possesses the minimal mean energy over the state $|N\rangle$. The polarization indices run over all permissible values here and the quark momenta are bounded by the limiting the Fermi momentum P_F. The momenta and polarizations of states forming the quasi-particle gas are marked by capital letters similar to the above formula. In all other cases small letters are used.

As is known the ensemble state at finite temperature T is described by the equilibrium statistical operator ρ. Here we use the Bogolyubov-Hartree-Fock approximation in which the corresponding statistical operator is presented by the following form:

$$\rho=\frac{e^{-\beta\,\hat{H}_{\mathrm{app}}}}{Z_0},\qquad Z_0=\mathrm{Tr}\{e^{-\beta\hat{H}_{\mathrm{app}}}\}, \qquad (6)$$

where some approximating effective Hamiltonian H_{app} is quadratic in the creation and annihilation operators of quark and antiquark quasi-particles A^+, A, B^+, B and is defined in the corresponding Fock space with the vacuum state $|\sigma\rangle$ and $\beta=T^{-1}$. We do not need to know the exact form of this operator henceforth because all the quantities of our interest in the Bogolyubov-Hartree-Fock approximation are expressed by the corresponding averages (a density matrix):

$$\begin{aligned} &n(P)=\mathrm{Tr}\{\rho A^+(\mathbf{P};S)A(\mathbf{P};S)\}, \\ &\bar{n}(Q)=\mathrm{Tr}\{\rho B^+(\mathbf{Q};T)B(\mathbf{Q};T)\}, \end{aligned}$$

which are found by solving the following variational problem. One needs to determine the statistical operator

ρ in such a form in order to have at the fixed mean charge

$$\bar{Q}_4 = \mathrm{Tr}\{\rho Q_4\} = V 2N_c \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}[n(p)-\bar{n}(p)], \qquad (7)$$

where

$$Q_4 = \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\frac{-ip_4}{|p_4|}\left[A^+(p)A(p)+B(p)B^+(p)\right],$$

for the diagonal component (of our interest here, $Q_4 = -\int \mathrm{d}\mathbf{x}\bar{q}i\gamma_4 q$) and the fixed mean entropy

$$\begin{aligned}\bar{S} &= -\mathrm{Tr}\{\rho\ln\rho\} = -V2N_c\int\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\,[n(p)\ln n(p) \\ &+ (1-n(p))\ln(1-n(p)) + \bar{n}(p)\ln\bar{n}(p) \\ &+ (1-\bar{n}(p))\ln(1-\bar{n}(p))],\end{aligned} \qquad (8)$$

($S = -\ln\rho$), the minimal value of mean energy of quark ensemble $E = \mathrm{Tr}\{\rho H\}$. The definition of mean charge is given here up to the unessential (infinite) constant coming from permuting the operators BB^+ in the charge operator Q_4. It may not be out of place to remind that the mean charge should be treated in some statistical sense because it characterizes quark ensemble density and has no colour indices.

Calculating the corresponding matrix elements one can obtain the following result for mean energy density per one quark degree of freedom (in this brief letter we omit the details which can be found in [6]) $w = \mathcal{E}/2N_c$, $\mathcal{E} = E/V$, where E is a total energy of the ensemble,

$$\begin{aligned}w &= \int\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}|p_4| + \int\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}|p_4|\cos\theta[n(p)+\bar{n}(p)-1] \\ &- G\int\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\sin(\theta-\theta_m)\,[n(p)+\bar{n}(p)-1] \\ &\times\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\sin(\theta'-\theta'_m)\,[n(q)+\bar{n}(q)-1]\ I.\end{aligned} \qquad (9)$$

(up to the constant unessential for our consideration here). In this formula the following notations are used: $p = |\mathbf{p}|$, $q = |\mathbf{q}|$, $\theta = 2\varphi$, $\theta' = \theta(q)$, $I = I(\mathbf{p}+\mathbf{q})$, and the angle $\theta_m(p)$ is determined by the condition as follows: $\sin\theta_m = m/|p_4|$. It is interesting to notice that the existence of such an angle stipulates the discontinuity of the mean energy functional mentioned above and found out in [4]. It was quite practical to single out the colour factor in the four-fermion coupling constant as $G = 2\widetilde{G}/N_c$ also.

We are interested in minimizing the following functional $\Omega = E - \mu\bar{Q}_4 - T\bar{S}$, where μ and T are the Lagrange factors for the chemical potential and temperature, respectively. The approximating Hamiltonian which we discussed above $\hat{H}_{\mathrm{app}}$, is constructed simply by using the information on $E-\mu\bar{Q}_4$ of the presented functional (see, also below). For the specific contribution per one quark degree of freedom $f = F/2N_c$, $F = \Omega/V$, we obtain

$$\begin{aligned}f &= \int\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\,[|p_4|\cos\theta(n+\bar{n}-1)-\mu(n-\bar{n})] \\ &+\int\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}|p_4| - G\int\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\sin(\theta-\theta_m)\,(n+\bar{n}-1) \\ &\times\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\sin(\theta'-\theta'_m)\,(n'+\bar{n}'-1)I \\ &+T\int\frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\,[n\ln n+(1-n)\ln(1-n) \\ &+\bar{n}\ln\bar{n}+(1-\bar{n})\ln(1-\bar{n})].\end{aligned} \qquad (10)$$

Here the primed variables correspond to the momentum q. The optimal values of parameters are determined by solving the following system of equations ($\mathrm{d}f/\mathrm{d}\theta = 0$, $\mathrm{d}f/\mathrm{d}n = 0$, $\mathrm{d}f/\mathrm{d}\bar{n} = 0$):

$$\begin{aligned}&|p_4|\sin\theta - M\cos(\theta-\theta_m) = 0,\\ &|p_4|\cos\theta - \mu + M\sin(\theta-\theta_m) - T\ln\left(n^{-1}-1\right) = 0,\\ &|p_4|\cos\theta + \mu + M\sin(\theta-\theta_m) - T\ln\left(\bar{n}^{-1}-1\right) = 0,\end{aligned} \qquad (11)$$

where we denoted the induced quark mass as

$$M(\mathbf{p}) = 2G\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}(1-n'-\bar{n}')\sin(\theta'-\theta'_m)\,I(\mathbf{p}+\mathbf{q}). \qquad (12)$$

Turning to the presentation of the obtained results in the form customary for the mean-field approximation, we introduce a dynamical quark mass M_q parameterized as

$$\sin(\theta-\theta_m) = \frac{M_q}{|P_4|}, \qquad |P_4| = (\mathbf{p}^2 + M_q(\mathbf{p}))^{1/2},$$

and ascertain the interrelation between induced and dynamical quark masses. From the first equation of system (11) we fix the pairing angle $\sin\theta = pM/(|p_4||P_4|)$ and, making use of the identity

$$(|p_4|^2 - Mm)^2 + M^2p^2 = [p^2 + (M-m)^2]\ |p_4|^2,$$

find out that

$$\cos\theta = \pm\frac{|p_4|^2 - mM}{|p_4||P_4|}.$$

For clarity we choose the upper sign "plus". Then, as the analysis of the NJL model teaches us, the branch of equation solution for negative dynamical quark mass is the most stable one. Let us recall here that we are dealing with the Euclidean metrics (though it is not a principal point) and the quark mass appears in the corresponding expressions as an imaginary quantity. Now substituting the calculated expressions for the pairing angle into the trigonometrical factor expression

$$\sin(\theta-\theta_m) = \sin\theta\frac{p}{|p_4|} - \cos\theta\frac{m}{|p_4|},$$

and performing some algebraic transformations of both parts of the equation, we come to determine

$$M_q(\mathbf{p}) = M(\mathbf{p}) - m.$$

And, in particular, the equation for the dynamical quark mass, eq. (12), takes the form characteristic for the mean-field approximation:

$$M = 2G \int \frac{\mathrm{d}\mathbf{q}}{(2\pi)^3} (1 - n' - \bar{n}') \, \frac{M'_q}{|P'_4|} \, I(\mathbf{p}+\mathbf{q}).$$

The second and third equations of system (11) allow us to find for the equilibrium densities of quarks and antiquarks as $n = [e^{\beta(|P_4|-\mu)} + 1]^{-1}$, $\bar{n} = [e^{\beta\ (|P_4|+\mu)} + 1]^{-1}$, and, hence, the thermodynamic properties of our system as well, in particular, the pressure of the quark ensemble, $P = -\mathrm{d}E/\mathrm{d}V$. By definition we should calculate this derivative at constant mean entropy, $\mathrm{d}\bar{S}/\mathrm{d}V = 0$. This condition allows us, for example, to calculate the derivative $\mathrm{d}\mu/\mathrm{d}V$. However, this way is not reliable because then the mean charge $\bar{Q}_4$ might change, and it is more practical to introduce two independent chemical potentials —for quarks μ and for antiquarks $\bar{\mu}$ (following formula for $\bar{n}$ with an opposite sign). In fact, it is the only possibility to obey both conditions simultaneously. It leads to the following definitions of corresponding densities: $n = [e^{\beta(|P_4|-\mu)} + 1]^{-1}$, $\bar{n} = [e^{\beta\ (|P_4|+\bar{\mu})} + 1]^{-1}$. In fact, this kind of description makes it possible to even treat some non-equilibrium states of quark ensemble (but with losing a covariance similar to the situation which takes place in electrodynamics while one deals with electron-positron gas). But here we are interested in the particular case of $\bar{\mu} = \mu$. The corresponding derivative of specific energy $\mathrm{d}w/\mathrm{d}V$ might be presented as

$$\begin{aligned}\frac{\mathrm{d}w}{\mathrm{d}V} &= \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \left(\frac{\mathrm{d}n}{\mathrm{d}\mu}\frac{\mathrm{d}\mu}{\mathrm{d}V} + \frac{\mathrm{d}\bar{n}}{\mathrm{d}\bar{\mu}}\frac{\mathrm{d}\bar{\mu}}{\mathrm{d}V} \right) \\ &\quad \times \Bigg[|p_4| \cos\theta - 2G \sin(\theta - \theta_m) \\ &\quad \times \int \frac{\mathrm{d}\mathbf{q}}{(2\pi)^3} \sin(\theta' - \theta'_m)\,(n' + \bar{n}' - 1) I \Bigg]. \qquad (13)\end{aligned}$$

Now expressing the trigonometric factors via dynamical quark mass and exploiting eq. (12) we have for the ensemble pressure

$$P = -\frac{E}{V} - V 2N_c \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \left(\frac{\mathrm{d}n}{\mathrm{d}\mu}\frac{\mathrm{d}\mu}{\mathrm{d}V} + \frac{\mathrm{d}\bar{n}}{\mathrm{d}\bar{\mu}}\frac{\mathrm{d}\bar{\mu}}{\mathrm{d}V} \right) |P_4|.$$

The requirement of mean charge conservation,

$$\frac{\mathrm{d}\bar{Q}_4}{\mathrm{d}V} = \frac{\bar{Q}_4}{V} + V 2N_c \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \left(\frac{\mathrm{d}n}{\mathrm{d}\mu}\frac{\mathrm{d}u}{\mathrm{d}V} - \frac{\mathrm{d}\bar{n}}{\mathrm{d}\bar{\mu}}\frac{\mathrm{d}\bar{\mu}}{\mathrm{d}V} \right) = 0,$$

provides us with an equation which interrelates the derivatives $\mathrm{d}\mu/\mathrm{d}V$, $\mathrm{d}\bar{\mu}/\mathrm{d}V$. Apparently, here the regularized expression for mean charge of quarks and antiquarks is meant. Acting in a similar way with the requirement of mean-entropy conservation, $\mathrm{d}\bar{S}/\mathrm{d}V = 0$, we obtain another equation as

$$\begin{aligned}\frac{\bar{S}}{2N_c V^2} &= -\int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \frac{\mathrm{d}n}{\mathrm{d}\mu} \ln(n^{-1} - 1) \frac{\mathrm{d}\mu}{\mathrm{d}V} \\ &\quad + \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \frac{\mathrm{d}\bar{n}}{\mathrm{d}\bar{\mu}} \ln(\bar{n}^{-1} - 1) \frac{\mathrm{d}\bar{\mu}}{\mathrm{d}V}.\end{aligned}$$

Substituting here $T \ln(n^{-1} - 1) = -\mu + |P_4|$ and $T \ln(\bar{n}^{-1} - 1) = \bar{\mu} + |P_4|$ after simple calculations (keeping in mind that $\bar{\mu} = \mu$) taking into account charge conservation we have that

$$\int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \left(\frac{\mathrm{d}n}{\mathrm{d}\mu}\frac{\mathrm{d}\mu}{\mathrm{d}V} + \frac{\mathrm{d}\bar{n}}{\mathrm{d}\bar{\mu}}\frac{\mathrm{d}\bar{\mu}}{\mathrm{d}V} \right) |P_4| = -\frac{\bar{S}T}{2N_c V^2} - \frac{\bar{Q}_4 \mu}{2N_c V^2}.$$

Finally we get for the pressure the following expression:

$$P = -\frac{E}{V} + \frac{\bar{S}T}{V} + \frac{\bar{Q}_4 \mu}{V},$$

(of course, the thermodynamic potential is $\Omega = -PV$). At small temperatures the antiquark contribution is negligible, and the thermodynamic description can be grounded on utilizing one chemical potential μ only. If the antiquark contribution is getting intrinsic, the thermodynamic picture becomes complicated due to the presence of the chemical potential $\bar{\mu}$ with the condition $\bar{\mu} = \mu$ imposed. In particular, at zero temperature the antiquark contribution is absent and we might obtain $P = -\mathcal{E} + \mu\rho_q$, where $\mu = [P_F^2 + M_q^2(P_F)]^{1/2}$, P_F is the Fermi momentum and $\rho_q = N/V$ is the quark ensemble density.

For clarity, we consider mainly the NJL model [1] in this paper, *i.e.* the correlation function behaves as the δ-function in the coordinate space. It is a well-known fact that in order to have an intelligent result in this model one needs to use a regularization cutting the momentum integration in eq. (10). We adjust the standard set of parameters [7] here with $|\mathbf{p}| < \Lambda$, $\Lambda = 631\,\mathrm{MeV}$, $m = 5.5\,\mathrm{MeV}$ and $G\Lambda^2/(2\pi^2) = 1.3$. This set of parameters at $n = 0$, $\bar{n} = 0$, $T = 0$ gives for the dynamical quark mass $M_q = 335\,\mathrm{MeV}$. In particular, it may be shown that the following representation of the ensemble energy is valid at the extremals of functional (10):

$$\begin{aligned}E &= E_{vac} + 2N_c V \int^{\Lambda} \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} |P_4| (n + \bar{n}), \\ E_{vac} &= 2N_c V \int^{\Lambda} \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} (|p_4| - |P_4|) + 2N_c V \frac{M^2}{4G}.\end{aligned}$$

It is easy to understand that this expression with the vacuum contribution subtracted looks like the energy of a gas of relativistic particles and antiparticles with mass M_q, and coincides identically with that calculated in the mean-field approximation.

Let us summarize the results of this exercise. So, we determine the density of the quark n and the antiquark

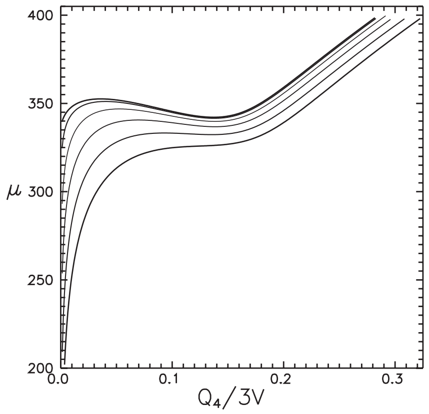

Fig. 1: The chemical potential μ (MeV) as a function of charge density $\mathcal{Q}_4 = Q_4/(3V)$ (in units of charge/fm^3). The factor 3 relates the densities of quark and baryon matter. The top curve corresponds to the situation of zero temperature. The curves following downwards correspond to the temperature values $T = 10\,\mathrm{MeV}$, ..., $T = 50\,\mathrm{MeV}$ in steps of 10 MeV.

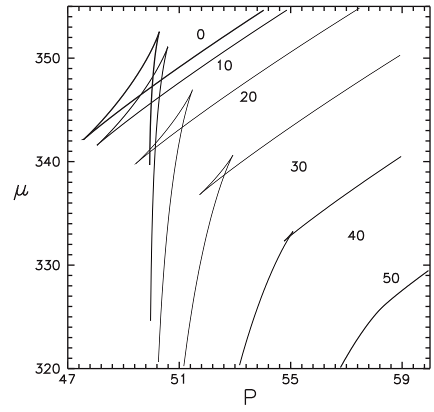

Fig. 3: The fragments of isotherms figs. 1, 2, see text. Chemical potential μ (MeV) as a function of pressure P (MeV/fm^3). The top curve corresponds to the zero isotherm and following downwards in steps of 10 MeV till the isotherm 50 MeV (the lowest curve).

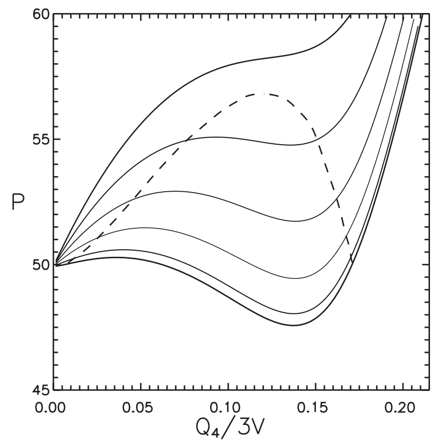

Fig. 2: The ensemble pressure P (MeV/fm^3) as a function of charge density $\mathcal{Q}_4$ at temperatures $T = 0\,\mathrm{MeV}$, ..., $T = 50\,\mathrm{MeV}$ in steps of 10 MeV. The lowest curve corresponds to zero temperature. The dashed curve shows the boundary of liquid-gas phase transition, see the text.

$\bar{n}$ quasi-particles at given parameters μ and T from the second and third equations of system (11). From the first equation we get the angle of quark and antiquark pairing, θ, as a function of dynamical quark mass M_q which is handled as a parameter. The evolution of the chemical potential as a function of charge density $\mathcal{Q}_4 = Q_4/(3V)$ (in units of charge/fm^3) with increasing temperature is depicted in fig. 1 (factor 3 relates to the quark and baryon matter densities). The top curve corresponds to zero temperature. The other curves following downwards have been calculated for the temperatures $T = 10\,\mathrm{MeV}$, ..., $T = 50\,\mathrm{MeV}$ in steps of 10 MeV. As was found in ref. [5] the chemical potential at zero temperature increases first with increasing charge density, reaches its maximal value, then decreases and at densities of the order of normal nuclear matter density[1], $\rho_q \sim 0.16/\mathrm{fm}^3$, becomes almost equal to its vacuum value. Such a behaviour of the chemical potential results from the fast decrease of dynamical quark mass with increasing Fermi momentum. It is clear from this figure that the charge density is still a multivalued function of the chemical potential at a temperature slightly below 50 MeV. Figure 2 shows the ensemble pressure P (MeV/fm^3) as a function of charge density $\mathcal{Q}_4$ at several temperatures. The lowest curve corresponds to zero temperature. The other curves following upwards correspond to temperatures $T = 10\,\mathrm{MeV}$, ..., $T = 50\,\mathrm{MeV}$ (the top curve) in steps of 10 MeV. It is interesting to recall now that in ref. [5] the vacuum pressure estimate for the NJL model was obtained as 40–50 MeV/fm^3 which is entirely compatible with the results of the conventional bag model. Besides, some hints of instability presence (rooted in the anomalous behavior of pressure $\mathrm{d}P/\mathrm{d}n < 0$) in an interval of the Fermi momenta has been found. Figure 3 shows the fragments of isotherms of figs. 1, 2 but in different coordinates (chemical potential–ensemble pressure). The top curve is calculated at zero temperature, the other isotherms following downwards correspond to temperatures increasing in steps of 10 MeV. The lowest curve is calculated at a temperature of 50 MeV. This figure obviously demonstrates the presence of states on the isotherm which are thermodynamically equilibrated and have equal pressure and chemical potential (see the characteristic Van der Waals triangle with the crossing curves). The calculated equilibrium points are shown in fig. 2 by the dashed curve. The intersection points of a dashed curve with an isotherm are fixing the boundary of the gas-liquid phase transition. The corresponding straight line $P = \mathrm{const}$, which obeys the Maxwell rule, separates the non-equilibrium and unstable fragments

[1] At Fermi momenta of dynamical quark mass order.

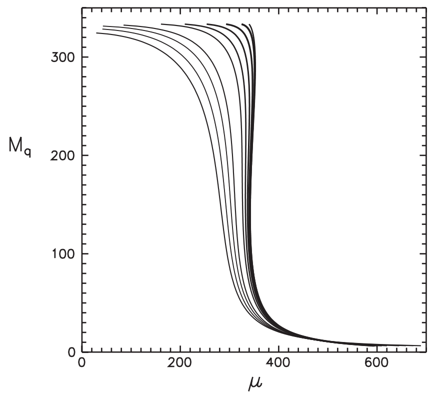

Fig. 4: The dynamical quark mass $|M_q|$ (MeV) as a function of chemical potential μ (MeV) at the temperatures $T = 0\,\text{MeV}, \ldots, T = 100\,\text{MeV}$ in steps of 10 MeV. The rightmost curve corresponds to zero temperature.

of the isotherm and describes a mixed phase. The corresponding critical temperature for the parameter we are using in this paper turns out to be $T_c \sim 45.7\,\text{MeV}$ with the critical charge density as $\bar{Q}_4 \sim 0.12\,\text{charge/fm}^3$. Usually the thermodynamic description is grounded on the mean energy functional which is the homogeneous function of particle number like $E = Nf(S/N, V/N)$ (without vacuum contribution). It is clear that such a description requires the corresponding subtractions to be introduced, however, this operation does not change the final results considerably. It was argued in ref. [5] that the states filled up with quarks and separated from the instability region look like "natural construction material" to form the baryons and to understand the existing fact of equilibrium between vacuum and octet of stable (in strong interaction) baryons[2]. The dynamical quark mass $|M_q|$ (MeV) as a function of chemical potential μ (MeV) is presented for the temperatures $T = 0\,\text{MeV}, \ldots, T = 100\,\text{MeV}$ in steps of 10 MeV in fig. 4. The rightmost curve corresponds to zero temperature. At low temperatures, below 50 MeV, the dynamical quark mass is the multivalued function of the chemical potential.

Apparently, our study of the quark ensemble thermodynamics produces quite reasonable arguments to propound the hypothesis that the phase transition of chiral symmetry (partial) restoration has already been realized as the mixed phase of physical vacuum and baryonic matter[3]. However, it is clear our quantitative estimates should not be taken as ones to be compared with, for example, the critical temperature of nuclear matter which has been experimentally measured and is equal to 15–20 MeV. Besides, the gas component (at $T = 0$) has nonzero density (as 0.01 of the normal nuclear density) but in reality this branch should correspond to the physical vacuum, *i.e.* zero baryonic density[4]. In principle, an idea of global equilibrium of gas and liquid phases makes it possible to formulate the adequate boundary conditions at describing the transitional layer arising between the vacuum and filled state and to calculate the surface tension effects. We are planning to consider these aspects of phase transition in a separate paper.

As a conclusion we would like to emphasize that in the present paper we demonstrated how a phase transition of liquid-gas kind (with reasonable values of the parameters) emerges in the NJL-type models in which the quarks are considered as the quasi-particles of the model Hamiltonian with four-fermion interaction. The constructed quark ensemble displays some interesting features for the nuclear ground state (for example, the existence of a state degenerate with the vacuum one) but needs further study of its role in the context of the existing research [9] activity to explore the complicated (or, may be, more realistic) versions of the NJL model and knowledge of the QCD thermodynamics as obtained in the lattice simulations.

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[2]The chiral quark condensate for the filled-up state discussed develops a quantity of about $(100\,\text{MeV})^3$ (at $T = 0$), see [5], that demonstrates the obvious tendency of restoring a chiral symmetry.

[3]An indirect confirmation of this hypothesis one could see, for example, in the existing degeneracy of excited baryon states, ref. [8].

[4]Similar uncertainty is present in the other predictions of chiral symmetry restoration scenarios, for example, it stretches from 2 to 6 units of normal nuclear density.