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Electron-ion collision operator in strong electromagnetic fields

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Abstract – The pair electron-ion collisions operator in strong electromagnetic fields is considered. In strong EM fields, the collision operator is derived allowing for the complex stochastic particle dynamics at the scattering. The resulting expression can be conditionally separated into the diffusion part (having a Landau-like operator form) and the fast-particle source.

Collisions play a fundamental role in plasma. They determine the form and evolution of the distribution function and, as a consequence, instabilities of any kind, the emission and the heating of plasma. The significance of collisions can be hardly overestimated. In this connection, a question arises about the form of the collision operator in different conditions. Note that one needs to know not only the collision frequency, heating rate and so on, but also the kinetic properties of electron-ion collisions in strong laser fields. This becomes very important for strong electromagnetic (EM) fields, because the heating rate does not decrease for the high field intensity [1–3]. At this, direct calculations of the laser-plasma interaction by particle-in-cell (PIC) simulations could not take into account electron-ion collisions correctly due to a too big difference between the spatio-temporal scales of collisions and the scales of the plasma density or the wave pulse [4]. In principle, the particle tree code [5] can account for the exact particle-particle Coulomb interaction at small distances, while it uses approximate multipole interactions of various orders at larger distances. But this may require extensive computer power and calculation time. However, the collision operator can be found analytically for strong fields and exactly this procedure is presented in our paper.

The collision operator in plasma in weak electrical and magnetic fields is well known [6]. It is the collision operator in the Landau form based on the idea that the main contribution to the overall scattering is made by distant collisions of particles with small-angle scattering. In this case, the particle drift motion may be regarded as an almost straight line. Thus, the collision operator can be easily found with logarithmic accuracy. The resulting logarithmic factor ("Coulomb logarithm") includes limitations imposed on the minimal and maximal ion distances (scattering on large angles or quantum effects, and collective effects or adiabatic motion in the wave field, respectively). Therefore, one can assume that the scattering is small-angled. Obviously, the approximation of straight-line drift trajectories is broken at small values of the logarithmic factor and a more complicated problem about the exact variation of particle momentum during the scattering should be considered.

Unfortunately, difficulties are also associated with electron-ion collisions in strong fields [1,7]. Seemingly, the logarithmic factor is formally great, and collisions may be regarded as small-angled. However, because of the attracting character of the electron-ion interaction with sufficiently low electron drift momentum,

$$p_T = \sqrt{mT_e/2} \ll p_{\rm osc} = eE/\omega,$$

there are trajectories of scattering particles which cannot be assumed to be a straight line. Here e, m, T_e are electron charge, mass and temperature, E and ω are the amplitude and the frequency of the EM wave. Correspondingly, the conditions of validity of the collision operator in the Landau form are broken.

Indeed, if the field is sufficiently strong, so that the electron oscillation radius $r_{\rm osc}$ is much longer than the characteristic scattering scale (the Rutherford radius estimated with respect to the oscillatory velocity)

$$b_{\rm osc} = \frac{e^2 Z}{m v_{\rm osc}^2} \ll r_{\rm osc} = \frac{eE}{m\omega^2}, \tag{1}$$

then the electron makes a lot of oscillations passing the ion. During each pass, it scatters on a small angle,

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but during the period $T=2\pi/\omega \gg t_{\mathrm{coll}} \sim b_{\mathrm{osc}}/v_{\mathrm{osc}}$ it approaches the ion noticeably. As a result, each subsequent scattering becomes stronger than the previous one, *i.e.*, the effect is cumulative. Moreover, by virtue of condition (1), a situation may occur, when the particle with impact parameter $\rho=\sqrt{2\pi}r_E$ hits the ion exactly after a period! Here, $r_E=\sqrt{r_{\mathrm{osc}}b_{\mathrm{osc}}}=\sqrt{eZ/E}$ is the characteristic non-linear scale of the scattering in strong fields.

Moreover, particles also may hit the ion after $2,3,\ldots(v_{\mathrm{osc}}/v)$ passes, which leads to even stronger intensification of all the characteristics of the scattering [1,2,7]. Thus, in strong fields $r_{\mathrm{osc}} \gg b_{\mathrm{osc}}$, the pattern of collisions changes cardinally. In particular, the electron-ion collision frequency proves to be much higher than the collision frequency estimated within the approximation of straight-line drift trajectories. Fast particles and coherent emission start to be generated. Correspondingly, the collision operator in strong fields should change too. This paper is devoted to the derivation of the pair collision operator in strong fields (1). Herein, we confine ourselves to the consideration of non-relativistic collisions. However, due to the Hamiltonian formalism used in this work, practically all computations are extended identically to the relativistic intensities of the EM fields.

Since the collisions in strong EM fields differ from those in rather weak EM fields, we start the derivation from the collision operator in the Boltzmann form [6,8] generalized to the case of varying EM fields [9]. This operator can be written in the form of the general integral operator

$$St_{\mathrm{ei}}[f]=\int w_{\mathrm{ei}}(\boldsymbol{p},\boldsymbol{p}_0)f(\boldsymbol{p}_0)\,\mathrm{d}^3p_0, \tag{2}$$

affecting the electron distribution function $f(\boldsymbol{p})$. Here we assume that ions are heavy and the changes of their velocities are negligibly small. The core w_{ei} of the collision operator can be rewritten as an integral along the trajectories of test particles [9]. For example, its average over the period of the EM field is equal to

$$\langle w_{\mathrm{ei}}(\boldsymbol{p},\boldsymbol{p}_0)\rangle=\frac{n_i}{T}\lim_{\Xi\to-\infty}\int\limits_{\Xi}^{\Xi+\zeta}\!\!\int[\delta(\boldsymbol{p}_+-\boldsymbol{p})-\delta(\boldsymbol{p}_0-\boldsymbol{p})]\mathrm{d}\xi\,\mathrm{d}^2\rho_0. \tag{3}$$

Here, $\zeta=\frac{2\pi}{\omega}\frac{p_0}{m}$ is the distance travelled by the electron over the period, $\boldsymbol{p}_+(\boldsymbol{p}_0,\boldsymbol{r}_0)$ is the electron momentum after the collision (at $t\to+\infty$) for the electron with initial parameters $\boldsymbol{p}_0$ and $\boldsymbol{r}_0=\boldsymbol{\rho}_0+\boldsymbol{\ell}(\Xi+\xi)$, where $\boldsymbol{\ell}=\boldsymbol{p}_0/p_0$ and $\boldsymbol{\rho}_0=\boldsymbol{r}_0-\boldsymbol{p}_0\frac{(\boldsymbol{r}_0,\boldsymbol{p}_0)}{p_0^2}$ are the vectors of the impact parameters in the plane perpendicular to the initial electron momentum.

In fact, this is very similar to the derivation of the well-known formula for the collision rate $\nu=n_iv\sigma$ [6]. But for periodic EM field the collision rate should be additionally averaged over the phase of the EM wave. Such averaging can be rewritten as integration over the spatial layer with thickness ζ:

$$\nu(\boldsymbol{p}_0)\equiv\int\frac{p^2}{p_{\mathrm{osc}}^2}w_{\mathrm{ei}}\,\mathrm{d}^3p=\frac{n_i}{T}\lim_{\Xi\to-\infty}\iint\limits_{\Xi}^{\Xi+\zeta}\frac{\boldsymbol{p}_+^2-\boldsymbol{p}_0^2}{p_{\mathrm{osc}}^2}\mathrm{d}\xi\,\mathrm{d}^2\rho_0.$$

The term "test particles" means the particles which satisfy the motion equation with Hamiltonian $H=p^2/2m-e^2Z/|\boldsymbol{r}+\boldsymbol{r}_{\mathrm{osc}}(t)|$ or, in the laboratory coordinates, $H=p^2/2m-e^2Z/|\boldsymbol{R}|+e\boldsymbol{E}\boldsymbol{R}$. The first expression is written in the drift coordinates for which the particle momenta are constant during motion in free space with the EM wave. The relation of the drift and laboratory coordinates has a simple form in the non-relativistic case:

$$\boldsymbol{R}=\boldsymbol{r}+\boldsymbol{r}_{\mathrm{osc}},\quad \boldsymbol{P}=\boldsymbol{p}+\boldsymbol{p}_{\mathrm{osc}}.$$

The same relation can also be found for the relativistic case [10], but the expressions become more complicated. In quantum mechanics, the drift coordinates used are commonly called as Kramers-Henneberger frame [11].

Usually, one simplifies expressions (2), (3) assuming that the momentum change during collisions is small [6,8,9]. In this case, the resulting integrals can be integrated analytically and the Landau form of the collision operator can be obtained. For small drift velocities ($p \ll p_{\mathrm{osc}}$, $b_{\mathrm{osc}} \ll r_{\mathrm{osc}}$), the electron dynamics is complicated [1,2,7]. But even for these conditions, the collision operator can be found using the results of [7].

Direct use of (3) is difficult due to the complicated, stochastic form of the test particle trajectories. Expression (3) should be simplified using some features of the particle dynamics in strong fields. As was shown in [1,7], the dynamics of such particles is the essential attraction to the ion practically without any change in the absolute value of the drift velocity; an abrupt "hard" ion impact; and an escape from the region of the scattering.

The momentum of particles stays almost constant after leaving the collision region. This allows one to replace the final momentum $\boldsymbol{p}(t=+\infty)$ with the value of the momentum after the hard impact. Moreover, by replacing the integration variable $\boldsymbol{r}_0$ by the coordinate before the hard impact $\boldsymbol{r}_c$, one obtains

$$\begin{aligned}\langle w_{\mathrm{ei}}(\boldsymbol{p},\boldsymbol{p}_0,t)\rangle \;=\; \frac{n_i}{T}\int J(\boldsymbol{r}_c,\boldsymbol{p}_0)\cdot(\delta(\boldsymbol{p}_0-\boldsymbol{p})\\ -\delta(\boldsymbol{p}_0+\delta\tilde{\boldsymbol{p}}+\Delta\boldsymbol{p}-\boldsymbol{p}))\,\mathrm{d}^3r_c.\end{aligned} \tag{4}$$

Here, $\delta\tilde{\boldsymbol{p}}$ is the small variation in the particle momentum at the stage of the ion attraction, $J(\boldsymbol{r}_c,\boldsymbol{p}_0)$ is the Jacobian of the transition from $\boldsymbol{r}_0$ to $\boldsymbol{r}_c$. Note that in the case of the multi-flow character of the particle dynamics (as it is in strong fields) the summation over the ambiguity regions should be performed in the Jacobian. The value $\Delta\boldsymbol{p}$ is the momentum variation at the hard impact:

$$\Delta\boldsymbol{p}=-\left(\frac{2\boldsymbol{\rho}mP/b}{1+\rho^2/b^2}+\frac{2\,m\boldsymbol{P}}{1+\rho^2/b^2}\right)\underset{\rho\gg b}{\approx}-2mP\frac{\boldsymbol{\rho}}{\rho}\frac{b}{\rho}, \tag{5}$$

where $\boldsymbol{\rho}=\boldsymbol{R}_c-\boldsymbol{P}\frac{(\boldsymbol{R}_c,\boldsymbol{P})}{|P|^2}$ and $b=e^2Z/mP^2$ are the impact parameter before the hard impact and the Rutherford radius determined by the total (laboratory) particle momentum, respectively. As before, $\boldsymbol{R}_c$, $\boldsymbol{P}$ are the total coordinate and the momentum at the impact moment. The expressions for $\boldsymbol{\rho}$, $\boldsymbol{P}$, which can be simplified in the case of small velocities $v \ll v_{\rm osc}$, are considered for the linearly polarized pumping wave:

$$\boldsymbol{P}\simeq \boldsymbol{z}_0 p_{\rm osc}(z), \quad \boldsymbol{\rho}\simeq x\boldsymbol{x}_0+y\boldsymbol{y}_0. \tag{6}$$

Here we consider transparent plasmas $\omega^2 \gg \omega_p^2 = 4\pi e^2 n/m$, which allows us [12] to use results of electron-ion collisions in rare plasma [1,7]. Indeed, the following relations are fulfilled in a transparent plasma:

$$r_D=\frac{v}{\omega_p}\gg r_a=\frac{v}{\omega}>r_E\gg b_{\rm osc}.$$

So, one can neglect the Debye shielding too (which is important at scales $r\geqslant r_D$).

For the Jacobian evaluation, one can use the fact that the physical meaning of the Jacobian is the particle density presented in new variables, which has been found earlier [10]. In the general case, the density for the linearly polarized field can be written as

$$J(\boldsymbol{r}_c,\boldsymbol{p}_0)=\left(1+\frac{p}{p_{\rm osc}}\frac{b(\boldsymbol{p}_0)}{\rho}\right)\underset{\rho\to 0}{\approx}\alpha(\boldsymbol{p}_0)\frac{b_{\rm osc}}{\rho}, \tag{7}$$

$$\alpha=\frac{p_{\rm osc}}{p_0}\,\Theta\!\left(\tfrac{\boldsymbol{p}_{\rm osc}\boldsymbol{p}_0}{p_{\rm osc}p_0}\right)\gg 1. \tag{8}$$

Here α is the factor of the electron attraction anisotropy depending on the initial electron momentum. Moreover, by using the smallness of the variation in the particle velocity at the attraction stage, one can set $\delta\tilde{\boldsymbol{p}}\approx 0$. As a result, the expression for the collision core has the form

$$\langle w_{\rm ei}\rangle=\frac{n_i}{T}\int J(\boldsymbol{r}_c,\boldsymbol{p}_0)(\delta\left(\delta\boldsymbol{p}-\Delta\boldsymbol{p}(\boldsymbol{r}_c,\boldsymbol{v}_0)\right)-\delta(\delta\boldsymbol{p}))\mathrm{d}^3 r_c, \tag{9}$$

where $\delta\boldsymbol{p}=\boldsymbol{p}_0-\boldsymbol{p}$. Note, that formula (9) is similar to the classical one [8,13], but it takes into account the electron attraction (factor $J(\boldsymbol{r}_c,\boldsymbol{p}_0)$) during scattering in the presence of a strong EM wave. Moreover, this factor significantly changes the final expressions at strong EM fields.

Such a neglect of the particle velocity variation is insignificant for the consideration of the energy exchange with the field. So, the characteristic variation of the particle energy is comparable with or greater than the initial particle energy (for example, for the description of particles in the trailing edge of the distribution function). However, it introduces significant errors in the transport characteristics of the scattering, to which, as in weak fields, distant small-angle collisions give the major contribution.

The expression of the collision core (9) is similar to the form of eq. (3), but there is an important difference: *an explicit multi-flowness* is excluded from the integrand. Indeed, formula (9) describes the multi-flow regime. However, it is done implicitly, *i.e.*, via the particle density $n(\boldsymbol{r}_c,\boldsymbol{v}_0)$ before the last impact. In this case, the density should be calculated precisely allowing for complex (stochastic) particle dynamics. One may find a simpler way, *e.g.*, write some approximation (7) for the density before the last impact and use the fact that the value $J(\boldsymbol{r}_c)$ comes into the integrand and all approximation errors will be "smoothed" during the integration.

Formally, the Landau-like term of the collision operator,

$$St_{\rm ei}=\frac{\partial}{\partial p_i}B_{ij}\frac{\partial f(\boldsymbol{p})}{\partial p_j},$$

for small-angle scattering can be derived from formula (9) by the standard way [6,8,9] taking into account the integrand factor $J\sim 1/\rho$. This gives the coefficient B_{ij} in the form

$$\begin{aligned}B_{ij} &= n_i\int \frac{\alpha b_{\rm osc}}{\rho}\cdot\frac{\partial U}{\partial r_i}\frac{\partial U}{\partial r_j}\bigg|_{\boldsymbol{r}_c\to\boldsymbol{r}_0+\boldsymbol{v}t}\mathrm{d}t\,\mathrm{d}^3r = \\ &\frac{2\pi\alpha n_i e^4 Z^2 m}{p_{\rm osc}}\left(\delta_{ij}-\frac{P_iP_j}{|P|^2}\right)\cdot\int\frac{b_{\rm osc}\,\mathrm{d}r}{r^2}\simeq \\ &\frac{\pi^2\alpha n_i e^4 Z^2 m}{p_{\rm osc}}\left(\delta_{ij}-\frac{P_iP_j}{|P|^2}\right),\end{aligned} \tag{10}$$

where $U=e^2Z/r$ is the ion Coulomb potential. Formula (10) contains the integral $\int b_{\rm osc}\,\mathrm{d}r/r^2$ which is divergent at the lower limit. The value of the Rutherford radius estimated over the total velocity $b\approx b_{\rm osc}$ has been taken as the lower limit for its calculation, since it is the limit of the expansion (5) of the momentum variation at the scattering on small angles ($r\gg b\approx b_{\rm osc}$).

The form of tensor (10) is the only form of the simplest differential operator which yields the correct expression for the collision frequency in strong fields [1,7] in the approach in which most of electrons scatter transversely to the wave field. Note that exactly the same "diffusion" part is also obtained for the instantaneous (not averaged over time) collision operator. However, in this case the total particle momentum P also depends on time.

However, the gain of the large-angle scattering is of the same order as the small-angle one (this also was shown in [1,7]). Indeed, let us apply the momentum method to formula (9) using (5). The first-order momentum

$$\frac{\mathrm{d}\langle p_i\rangle}{\mathrm{d}t}=\frac{\pi^2 n_i e^4 Z^2\alpha m}{p_{\rm osc}^3}\cdot p_{{\rm osc},i}(t)=\frac{\partial B_{ij}}{\partial p_j},$$

is the same as the one from (10). However, the second momentum

$$\frac{\mathrm{d}\langle p_ip_j\rangle}{\mathrm{d}t}=\frac{\pi^2 n_i e^4 Z^2\alpha m}{p_{\rm osc}}\cdot(\delta_{ij}+\delta_{iz}\delta_{jz})\neq B_{ij}$$

already gives a relation being closer to the isotropic scattering, whereas eq. (10) yields the scattering transverse to the total (oscillatory) velocity.

Therefore, for an accurate (not evaluative) description of the collisions, one should use operator (9). Let us

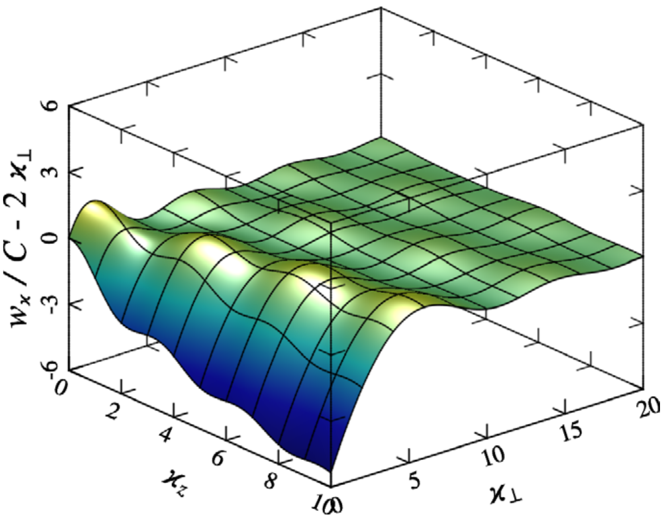

Fig. 1: (Color online) Fourier representation (12) of the collision operator core.

rewrite it in explicit form taking into account the phase bunching [7] of test electrons at the phase of the oscillatory velocity maximum:

$$\langle w_{\rm ei}\rangle = \frac{v_{\rm osc}n_i}{2\pi}\int J(\boldsymbol{r}_c,\boldsymbol{p}_0)\left[\delta\left(\delta\boldsymbol{p}_\perp - \frac{2p_{\rm osc}b_{\rm osc}\boldsymbol{\rho}}{\rho^2+b_{\rm osc}^2}\right)\cdot\delta\left(\delta p_z \pm \frac{p_{\rm osc}b_{\rm osc}^2}{\rho^2+b_{\rm osc}^2}\right) - \delta(\delta\boldsymbol{p})\right]{\rm d}x\,{\rm d}y. \qquad (11)$$

Here, the $\pm$ signs denote electron bunching near the phases with the maximal values of the oscillatory momentum. Of course such assumption is rather strong but it allows us to derive the expression for the collision operator explicitly and to give proper dependencies for heating rate, hot electron distribution and others, which are in good agreement with the results of numerical simulations.

Operator (11) can be simplified by its Fourier representation

$$w_\varkappa(\boldsymbol{p}_0) = \int \langle w(\delta\boldsymbol{p},\boldsymbol{p}_0)\rangle e^{i\varkappa\delta\boldsymbol{p}/p_{\rm osc}}\,{\rm d}^3\delta p.$$

By replacing the variable $\rho = b_{\rm osc}\tan(\varphi/2)$, the double integral (11) is made a single integral:

$$w_\varkappa = \alpha\nu\int_0^\pi \frac{J_0(\varkappa_\perp\sin\varphi)\cos(\varkappa_z(1+\cos\varphi)) - 1}{1+\cos\varphi}{\rm d}\varphi, \qquad (12)$$

where $\nu = v_{\rm osc}n_i b_{\rm osc}^2/2\pi$. Unfortunately, we could not perform integration (12) in the explicit form. The result of the numerical integration of (12) is presented in fig. 1. However, two important cases have analytical solutions.

The first case is the expansion at small $\varkappa$:

$$w_\varkappa \approx -\frac{\pi\alpha\nu}{4}(2\varkappa_z^2 + \varkappa_\perp^2) + \dots . \qquad (13)$$

This expansion corresponds to the diffusion in the momentum space

$$St_{\rm ei,dif} = \frac{\partial^2 \tilde{B}_{ij} f(\boldsymbol{p})}{\partial p_i \partial p_j} \qquad (14)$$

and gives the collision operator in the diffusion form with the coefficient (compare with (10))

$$\tilde{B}_{ij} = \frac{\pi^2\alpha(\boldsymbol{p})n_i e^4 Z^2\, m}{4p_{\rm osc}}\left(\delta_{ij} + \frac{p_{{\rm osc},i}p_{{\rm osc},j}}{|p_{\rm osc}|^2}\right). \qquad (15)$$

The second case is the expansion at large $\varkappa_\perp \to \infty$. Numerical integration shows a linear dependence here (fig. 1). Actually, the second derivative of (12) is

$$\frac{{\rm d}^2 w_\varkappa}{{\rm d}\varkappa_\perp^2} = \pi\left(J_1^2(\varkappa_\perp/2) - J_0^2(\varkappa_\perp/2)\right) \underset{\varkappa_\perp\to\infty}{\to} 0.$$

This means that $w_\varkappa$ is the linear function of $\varkappa_\perp$ at $\varkappa_\perp \to \infty$. Moreover, one can show that (fig. 1)

$$w_\varkappa \underset{\varkappa_\perp\to\infty}{\approx} 2\varkappa_\perp.$$

Assuming this dependence and using the relation

$$\int_0^\infty r^2 J_0(kr)\,{\rm d}r = -\frac{1}{k^3},$$

one can find the approximate form of the collision operator in strong EM fields:

$$\langle w_{\rm ei}\rangle \approx \frac{2\alpha\nu\delta(\Delta P_\parallel/p_{\rm osc})}{\Delta P_\perp^3}. \qquad (16)$$

Assuming that the momentum of scattered electrons is much larger than its thermal one, the collision operator can be essentially simplified. This is connected with the possibility to replace the distribution function $f(p_0)$ with the δ-function:

$$\langle St[f]_{\rm hot}\rangle = F(\boldsymbol{p})\int \alpha f(\boldsymbol{p}_0)\,{\rm d}^3 p_0, \qquad (17)$$

$$F(\boldsymbol{p}) = \frac{2\nu p_{\rm osc}\delta(p_\parallel)}{p_\perp^3}. \qquad (18)$$

This is the power-law spectrum of hot electrons, which was predicted before [14], and seems to be observed in the experiment.

Since the momentum of such a particle after the collision is large and, correspondingly, the collision frequency for them is negligibly small, then it makes sense to regard them as "escape" electrons. By analogy with the appearance of electrons escaping in the case of collisions in a static field [6], these particles almost never collide with ions in an alternating EM field henceforth and in some way may be called "lost" for the energy exchange processes. The loss frequency μ, *i.e.*, the frequency of the appearance of electrons in the trailing edge of the distribution function, may be estimated according to the relation

$$\mu \equiv \alpha(p_T)\int_{p>p_T} F(\boldsymbol{p})\,{\rm d}^2 p = \frac{n_i p_{\rm osc}}{\pi} b_{\rm osc} b_v, \qquad (19)$$

where $b_v = 2e^2 Z/T$. The distribution of escape particles over a time unit in the momentum space is given by formula (17).

So, the collision operator can be written qualitatively as a sum of 2 terms, specifically, the "diffusion" term and the hot-particle "generator":

$$St_{\mathrm{ei}}[f] = \frac{\partial^2 \tilde{B}_{ij} f(\boldsymbol{p})}{\partial p_i \partial p_j} + F(\boldsymbol{p}) \int \frac{f(\boldsymbol{p}_0)}{p_0} \mathrm{d}^3 p_0 - \mu f(\boldsymbol{p}). \quad (20)$$

Such representation gives the correct value for the plasma heating rate and for the distribution of hot electrons appearing due to the collisions. However, it is not correct for the kinetic peculiarities of the collisions. One should use collision operator (2) with core (11) or (12), if plasma kinetics were to be described accurately.

* * *

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