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Intermittency route to chaos for the nuclear billiard

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Abstract – We analyze on a classical 2D version of the "nuclear billiard" the onset of chaotic behaviour of the three-dimensional nucleonic trajectories in different dynamical states of the axially symmetric deformed nuclei. The coupling between the single-particle and the collective degrees of freedom in the presence of dissipation for several multipolarities is taken into account. We examine the order-to-chaos transition by studying the Shannon entropies, which quantify the time rate of information production during a certain motion. For the monopole and dipole deformations an increasing divergence of the nucleonic trajectories from the adiabatic to the resonance regime is observed. Also, a peculiar case of intermittency is reached in the vicinity of the resonance, for the monopole case. The quantity of energy transferred in a nuclear collision is shown to be the control parameter which adjusts the intermittent behaviour of the studied system.

Introduction. – Deterministic chaos is usually defined as irregular, unpredictable behaviour of the trajectories generated by nonlinear systems whose dynamical laws, involving no randomness or probabilities, predict a unique time evolution of a given system.

Over the last three decades an increasing number of papers have treated the study of the deterministic chaotic behaviour of Fermi nuclear systems (*e.g.*, [1–12]). The interest for analyzing the order-to-chaos transitions on such systems was linked to the problem of the onset of dissipation of collective systems through mainly one-body and two-body processes. Among these we mention the interaction of the nucleons with the potential well, the evaporation of individual nucleons in nuclear peripheral interactions, and the collisions between nucleons without taking into account the Pauli-blocking effect.

These kinds of analyses were performed for the first time by Burgio, Baldo *et al* [1,2] considering a system of nucleons which move within a container modelled as a Woods-Saxon–type potential and kick the container walls with a specific frequency. They discussed the damping of the movement and the relation with order-chaos transition in single-particle dynamics.

Papachristou and collaborators [3] studied the decay width of the isoscalar giant monopole resonance for various spherical nuclei, on the above classical model. Following this formalism, the beginning of the chaotic behaviour for a number of nucleons in various dynamical regions at different multipolarities was surveyed [4].

The present study of a single-particle dynamics influenced by a vibrating potential explores the interface between two complementary approaches. One of them analyzed the order-chaos interplay of the single-nucleon trajectories in average deformed potentials (for *e.g.*, [13–15]). The other, chaos in collective motions, was investigated in a number of papers based either on the interacting boson model, on the geometric collective model, or on the time-dependent relativistic mean-field model (*e.g.*, [16–21]).

For an axially symmetric deformed nucleus, the nucleonic motion can be studied in a meridian plane rotating with constant velocity around the symmetry axis. The interaction between a single-nucleon and the nuclear core containing the rest of $(A-1)$ nucleons (*i.e.*, the nuclear mean field) was modelled by Woods and Saxon [22] with a Fermi-type spherically symmetric potential. In this context we investigate the chaotic behaviour of a single nucleon in a two-dimensional (2D) deep Woods-Saxon

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potential well with a small diffuseness of the surface. Because it prevents the nucleon from escaping, such a system is considered a good approximation of a "nuclear billiard".

A detailed picture of the achievement of deterministic chaos is presented for a comparative study between the adiabatic and the resonance stage of the nuclear interaction. By using generalized information entropies (*e.g.*, [23–25]), it is emphasized that in the resonance phase of the interaction, the onset of chaotic behaviour is found to be earlier than at any other adiabatic collective frequencies of the Woods-Saxon potential well for the monopole and dipole modes.

Close to resonance we obtain for the monopole vibrations the characteristic feature of the intermittency regime, *i.e.* sudden change to a laminar behaviour (so-called intermission) of a specific signal between two turbulent phases. Albeit intermittency is a well-known phenomenon for billiards [26,27], in particular for Hamiltonian systems with divided phase space (*e.g.*, mushroom [28–30] and annular billiards [31]), and for connected Hamiltonian systems [32], we show that this property also holds for a Woods-Saxon billiard container with inelastic particle-wall interactions.

The intermittency found is described in this paper as a mere feature of the "nuclear billiard". By using phenomenological parametrizations (like in [3]), and by introducing other couplings among various nuclear degrees of freedom (*e.g.*, [4]) one can observe how the nuclear dynamics during collisions is influenced by the emergence of chaotic bursts between intermittent periods of time.

The intermittent dynamics, as described by a long-range temporal persistence of correlations [33], could particularly be studied when the nuclear multifragmentation process at various excitation energies is linked to the nuclear liquid-to-gas phase transitions (*e.g.*, [34–40]), being a possible way of extracting information about the nuclear equation of state.

Basic formalism. – We use for its simplicity (as in, for *e.g.*, [1,2]) a classical version of the vibrating potential model for finite nuclei [41]. The physical system contains a number of A spinless and chargeless nucleons, with no internal structure. Due to the axial symmetry of the deformed nucleus it would be sufficient to study the single-particle dynamics in a 2D deep Woods-Saxon potential well considered as a "nuclear billiard". The oscillating surface of the well is periodically hit with a certain frequency ω.

The Bohr Hamiltonian in polar coordinates is a sum of two components: kinetic ($E_{\text{kin.}}$) and potential ($E_{\text{pot.}}$), the kinetic one decoupling into radial (E_{r}), centrifugal (E_{L}), and collective terms ($E_{\text{coll.}}$):

$$E_{\text{kin.}} = E_{\text{r}} + E_{\text{L}} + E_{\text{coll.}} = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{p_\alpha^2}{2M}, \quad (1)$$

$$E_{\text{pot.}} = V\left(r, R\left(\theta, \alpha\right)\right) + \frac{mR_0^2\Omega^2\alpha^2}{2}. \quad (2)$$

As only axially symmetrical terms are considered, the phase space is thus reduced to three degrees of freedom (d.o.f.), being defined by the single-nucleon and collective coordinates and their conjugate momenta: (r, p_r), (θ, p_θ) and (α, p_α). The collective coordinate α oscillates with Ω frequency, and the nucleon mass m takes the value: 938 MeV.

The Woods-Saxon potential is constant inside the billiard and a very steeply rising function on the surface:

$$V\left(r, R\left(\theta, \alpha\right)\right) = \frac{V_0}{1 + \exp\left[\frac{r - R(\theta,\alpha)}{a}\right]}, \quad (3)$$

with $V_0 = -1500$ MeV, deep enough to prevent the escape of the nucleons regarded as classical objects for the present analysis. For the same reason, the diffusivity coefficient a has a very small value 0.01 fm. The vibrating surface can be written as in [1,2], depending on the collective variable and Legendre polynomials $P_L(\cos\theta)$, $R_0 = 6$ fm being chosen for consistency with previous papers [1,2]:

$$R = R\left(\theta, \alpha\right) = R_0\left[1 + \alpha P_L\left(\cos\theta\right)\right]. \quad (4)$$

We choose for this study the first two multipolarities L of the potential well: 0 for the monopole, and 1 for the dipole case. Because it can not be regarded as a physical oscillation of the potential surface, but rather as a global or local displacement of the protons and neutrons [42,43], an adequate computational description of the dipole mode would require two distinct nucleonic fluids, oscillating against each other (see, for *e.g.*, [4]). As the present analysis is restrained to an isospin-free single-particle dynamics in a "nuclear billiard", the $L = 1$ mode is only generically designated as a dipole deformation of the potential well (*e.g.*, [44,45]).

Once the Hamiltonian is chosen, the numerical simulations are based on the solution of the Hamilton equations:

$$\dot{r} = \frac{p_r}{m}, \qquad \dot{p_r} = \frac{p_\theta^2}{mr^3} - \frac{\partial V}{\partial r}, \quad (5)$$

$$\dot{\theta} = \frac{p_\theta}{mr^2}, \qquad \dot{p_\theta} = -\frac{\partial V}{\partial R}\cdot\frac{\partial R}{\partial \theta}, \quad (6)$$

$$\dot{\alpha} = \frac{p_\alpha}{mR_0^2}, \qquad \dot{p_\alpha} = -mR_0^2\Omega^2\alpha - \frac{\partial V}{\partial R}\cdot\frac{\partial R}{\partial \alpha}. \quad (7)$$

A Runge-Kutta–type algorithm (order 2–3) with an optimized step size was used for solving the system of differential equations, while keeping the absolute errors for the phase space variables under 10^{-6} and conserving the total energy with relative error: $\Delta E/E \approx 10^{-8}$.

The equilibrium deformation parameter $\bar{\alpha}$, which is the mean collective variable, can be calculated (*e.g.*, for $L = 0$) by equating the mechanical pressure of the wall, P_{wall} with $P_{\text{part.}}$, the pressure exerted by the particles:

$$P_{\text{wall}} = \frac{m\Omega^2}{2\pi}\cdot\frac{\bar{\alpha}}{1+\bar{\alpha}}, \qquad P_{\text{part.}} = \frac{T}{\pi R_0^2\left(1+\bar{\alpha}\right)^2}. \quad (8)$$

Thus, one gets the equation for the equilibrium value of the collective coordinate in 2D, in the monopole case:

$$\bar{\alpha}\left(1+\bar{\alpha}\right)=\frac{2T}{mR_0^2\Omega^2}\,. \tag{9}$$

Then a small perturbation of this collective variable was considered $\alpha(t=0\,\mathrm{fm}/c)=\bar{\alpha}+0.15$ [1,2] and the evolution of the physical system was thoroughly investigated.

On the resonance condition. – One can choose the wall oscillation taking place close to adiabatic conditions, imposing a wall frequency smaller than the single-particle one. Thus, the frequency of vibration $\Omega_{\mathrm{ad.}}$ was chosen less than $0.05\,c/\mathrm{fm}$, which corresponds to an oscillation period:

$$\tau_{\mathrm{wall}}=\frac{2\pi}{\Omega_{\mathrm{ad.}}}\geqslant 125.66\,\mathrm{fm}/c. \tag{10}$$

By introducing the maximum particle speed:

$$v=\sqrt{\frac{2T}{m}}\,, \tag{11}$$

and the parameters as in [1,2]: $R_0=6\,\mathrm{fm}$ and the 2D kinetic energy, $T=36\,\mathrm{MeV}$, one can obtain the value for the single-particle period:

$$\tau_{\mathrm{part.}}=\frac{2\pi}{\omega}=\frac{2R_0}{v}\approx 43.33\,\mathrm{fm}/c\,. \tag{12}$$

In addition to [1,2] we introduced a physical constraint to this elementary physical system and continued that type of analysis necessary for the study of a nonintegrable dynamical system. At the beginning we considered a physical situation and we chose instead a static vibrating "nuclear billiard", a projectile nucleus having the same properties, colliding with a target nucleus. It is well known that the nuclear interaction, at incident energies ranging from MeV to GeV, can result in a multitude of processes from the nuclear evaporation to complete fragmentation or multifragmentation, according to the impact parameter.

It was shown in [46,47] that during this kind of processes even for peripheral events an unnegligible amount of energy is transferred by nucleon-nucleon scattering to the nucleons of the projectile and not only the transverse momentum distributions, but also the longitudinal momentum distributions as measured in the projectile fragmentation rest frame can reveal the centrality status of the interaction. It can also offer a hint on the apparent temperature of a Fermi gas of nucleons which was found to be [47] near the isotopic temperatures, *i.e.* several MeV [48–50].

It was therefore supposed that the target fragmentation can be associated with a resonance process. In order to obtain such behaviour, the wall frequency was gradually increased to the resonance frequency $\Omega_{\mathrm{res.}}=0.145\,c/\mathrm{fm}$. However, nuclear evaporation or plain breakup of a projectile nucleus can take place long before this regime is achieved by redistributing energy between the nucleons themselves and also between single-particle degrees of freedom and collective ones. Individual nucleons or clusters can thus have enough kinetic energy to escalade the wall barrier.

We should also emphasize that we can either have the case that can be put in correspondence with a nuclear collision process, *i.e.* the variation of the nucleonic frequency oscillation as the apparent temperature of the nucleons in the nuclei increases (see eq. (11) and eq. (12)), maintaining the potential well vibration constant, or respectively, the inverse situation in which the period between two consecutive collisions of the nucleon with the self-consistent mean field is kept invariable, while modifying the oscillation modes of the nuclear surface. The latter regards our studied case and is the reversed physical case previously described. It was used because of the specific choice of the "toy model" parameters described in [1,2].

The most realistic evolution of the nucleons in a chosen potential can assume a simultaneous variation of both angular frequencies. The resonance condition of the coupled classical oscillators should remain however an important condition for a rapid appearance of a deterministic chaotic behaviour of the physical system in study at different time scales. A proper analysis of a system should provide the variation of the collision radian frequency of the nucleons inside the "billiard" as the apparent temperature increases and the change in the vibrating potential period, supposing that the multipolarity increases when pumping energy in the "nuclear reservoir" during interaction. We can for example use in simulations, for nuclei with a large number of nucleons, the liquid-drop model or the collective model, which predict a frequency of vibration as a function of the multipolarity:

$$\Omega_L=\sqrt{\frac{C_L}{B_L}}\,, \tag{13}$$

with C_L being the elasticity coefficient, and B_L the mass coefficient for the oscillator of L multipolarity.

Shannon entropy analysis. – To investigate route to classical chaos, we paid attention to the time evolution of the generalized information entropy (or Shannon entropy) (*e.g.*, [23–25]), $N(t)$ being the number of gradually occupied cells until the time t:

$$S_{\mathrm{Shannon}}\,(t)=-\sum_{k=1}^{N(t)}p_k\cdot\ln p_k\,. \tag{14}$$

This type of entropy is actually a number which quantifies the time rate of information production for a chaotic trajectory. We consider in the first place the case of a particle that at every moment occupies a cell of the two-dimensional lattice phase space with a p_k probability:

$$p_k=1/N_{\mathrm{tot.}}\,,\qquad N_{\mathrm{tot.}}=N_r\cdot N_{p_r}\cdot N_\theta\,, \tag{15}$$

where $N_{\mathrm{tot.}}$ is the total cells number and N_r, N_{p_r}, and N_θ are the number of bins of the (r,p_r,θ) lattice.

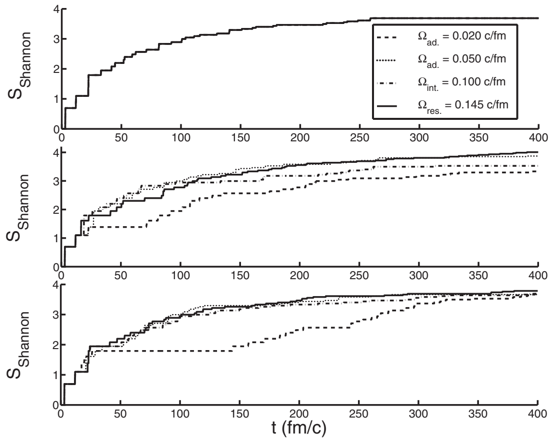

Fig. 1: The Shannon entropy of the one-particle phase space for all frequencies studied ($\Omega = 0.02$–$0.145\,c/\text{fm}$), when the single and collective degrees of freedom are uncoupled (upper panel) and coupled: $L = 0$ (central panel), $L = 1$ (lower panel).

Table 1: The computed $S_{\text{Shannon}}(t = 800\,\text{fm}/c)$ of the $(r \leftrightarrow p_r \leftrightarrow \theta)$ one-particle phase space maps.

Oscillation frequency	UCE	$L=0$	$L=1$
$\Omega_{\text{ad.}} = 0.020\,c/\text{fm}$	3.6889	3.7136	3.7842
$\Omega_{\text{ad.}} = 0.050\,c/\text{fm}$	3.6889	4.0775	4.0775
$\Omega_{\text{int.}} = 0.100\,c/\text{fm}$	3.6889	3.6889	3.8501
$\Omega_{\text{res.}} = 0.145\,c/\text{fm}$	3.6889	4.1589	4.0431

Since p_θ is a constant of motion for the monopole and the uncoupled equations (UCE) cases, we use for comparisons only these three phase space variables.

When shifting towards many-body quantum chaos, the quantum counterpart of the information entropy is used to measure the degree of complexity of individual wave functions, expanded as superposition of ψ_k states with C_k coefficients [51–53]:

$$S_{\text{quantum}} = \sum_{k=1}^{N} |C_k|^2 \cdot \ln |C_k|^2 \ . \qquad (16)$$

As an alternative measure for the classicaly defined entropy (eq. (14)) we also used the cumulative filling percentage of the one-nucleon phase space:

$$\eta\,(t) = \frac{N\,(t)}{N_{\text{tot.}}} \cdot 100\,(\%)\,. \qquad (17)$$

For a given 2D phase space lattice formed of $N_{\text{tot.}} = 4^3$ bins we present in fig. 1 a comparison between the information entropies of the physical system in study, starting from the adiabatic stage of interaction and gradually increasing the frequency towards the resonance value, $\Omega_{\text{res.}} = 0.145\,c/\text{fm}$. The slopes for the resonance frequency case were found to be significantly higher than for the adiabatic one ($\Omega_{\text{ad.}} = 0.02\,c/\text{fm}$) for the multipolarities involved.

Table 2: The filling percentage η of the $(r \leftrightarrow p_r \leftrightarrow \theta)$ one-particle phase space maps.

Oscillation frequency	UCE	$L=0$	$L=1$
$\Omega_{\text{ad.}} = 0.020\,c/\text{fm}$	62.50	64.06	68.75
$\Omega_{\text{ad.}} = 0.050\,c/\text{fm}$	62.50	92.19	92.19
$\Omega_{\text{int.}} = 0.100\,c/\text{fm}$	62.50	62.50	73.44
$\Omega_{\text{res.}} = 0.145\,c/\text{fm}$	62.50	100.00	89.06

Table 3: The time (in fm/c) at which the information entropies of the $(r \leftrightarrow p_r \leftrightarrow \theta)$ one-particle phase space maps have the maximum value (*i.e.* $\eta = 100\%$).

Oscillation frequency	UCE	$L=0$	$L=1$
$\Omega_{\text{ad.}} = 0.020\,c/\text{fm}$	$> 10^5$	6023	6359
$\Omega_{\text{ad.}} = 0.050\,c/\text{fm}$	$> 10^5$	1618	4223
$\Omega_{\text{int.}} = 0.100\,c/\text{fm}$	$> 10^5$	11442	3241
$\Omega_{\text{res.}} = 0.145\,c/\text{fm}$	$> 10^5$	729	1887

Table 4: The time (in fm/c) at which the one-particle Shannon entropies of a pack of $w = 5$ close orbits begin having the maximum value.

Oscillation frequency	UCE	$L=0$	$L=1$
$\Omega_{\text{ad.}} = 0.020\,c/\text{fm}$	$> 10^4$	1095	555
$\Omega_{\text{ad.}} = 0.050\,c/\text{fm}$	$> 10^4$	855	476
$\Omega_{\text{int.}} = 0.100\,c/\text{fm}$	$> 10^4$	4133	333
$\Omega_{\text{res.}} = 0.145\,c/\text{fm}$	$> 10^4$	279	327

Another comparison revealed significant differences between the onset times of the quasi-constant Shannon entropy values, for all cases taken into consideration. Thus, for four vibrational radian frequencies and for three coupling modes of the Hamilton equations we show the information entropy values after $800\,\text{fm}/c$ (table 1) and the associated phase space filling degrees (table 2). Also, in table 3, are presented the periods of time after which the filling percentages η equal unity.

We continue the analysis by further defining the Shannon entropy for a group of w nearby orbits:

$$S_{\text{pack}}\,(t) = \ln N_w(t), \qquad (18)$$

so that the number of occupied cells is: $1 \leqslant N_w(t) \leqslant w$, thus describing the spread of the trajectories at each moment of time t. When reaching the maximum divergence, the entropy for five distinct phase space paths gets its highest value (*i.e.* $S_{\text{pack}} = 1.6094$) (see table 4).

We begin the analysis with the UCE case. The single and collective uncoupled d.o.f. give birth to a quasi-laminar behaviour with a weak development of chaotic states. The one-particle information entropy shows an identical evolution, no matter the frequency chosen. The orbit covers, after $800\,\mathrm{fm}/c$, only 62.50% of the entire lattice (table 2) and does not reach 100%, even after $\Delta t = 100000\,\mathrm{fm}/c$ (table 3). Also, the phase space is not covered up by all five trajectories for the whole range of $10000\,\mathrm{fm}/c$ considered, when analyzing S_{pack} (table 4).

For the dipole mode at $\Omega_{\mathrm{ad.}} = 0.05\,c/\mathrm{fm}$, it appears that, after only $800\,\mathrm{fm}/c$, the entropy closes in upon its maximum value: $S_{\mathrm{Max.}} = \ln N_{\mathrm{tot.}} = 4.1589$ (table 1). However, on long periods of time, the real tendency is towards filling up the nucleonic phase space as rapid as the vibrational frequency is increased (table 3). The pattern is repeated when studying the Shannon entropy for closeby nucleonic trajectories (table 4).

We found quite the same feature for the monopole case, except for the intermittent "window" at $\Omega_{\mathrm{int.}} = 0.1\,c/\mathrm{fm}$ (tables 1 and 2). The occupying rate is so small in the monopole intermittent zone, that just at $11442\,\mathrm{fm}/c$, the particle would have covered the whole phase space (see table 3). A similar conclusion can be drawn from table 4. The trajectory pack information entropy reaches its highest value after the longest one-particle evolution time of all monopole cases considered: $4133\,\mathrm{fm}/c$. The Shannon entropy analysis thus revealed that the found intermittency can not be here regarded as a classical alternation between laminar and chaotic behaviours, but as an alternation between consecutive turbulent states, although with very different onset times of chaoticity.

However, one has to bear in mind that the validity of the aforementioned results is restricted to classical chaos only. For quantum models there is a departure from similar results obtained on a classical premise [51], the so-called "quantum suppression of classical chaos" [52]. Even in the case of a good quantitative concordance in the chaotic regime of the semi-classical predictions with the corresponding quantum ones, a few eigenstates were found to present a diminished delocalization [53,54].

Thus, the above classical results, obtained in the approximation of the microcanonical ensemble, should only be qualitatively regarded as appropriately close to similar results from quantum many-body approaches. This only happens whenever considering that the physical system reaches a statistical thermal equilibrium and the phase coherence of the eigenfunctions can be regarded to have a low statistical weight (*e.g.*, the case of an initially coherent state of a heavy-ion collision after a sufficient time of relaxation (*i.e.*, phase decoherence)) [51].

Conclusions. – A comparative study was done between specific physical regimes of nuclear interaction: adiabatic and resonance, giving a detailed picture of a possible scenario towards a pure deterministic behaviour of chaotic type of the studied nucleonic system in a two-dimensional deep Woods-Saxon potential well.

By comparing the order-to-chaos transition for these cases of interest, it was shown that the couplings between the one-particle and collective dynamics significantly decrease the onset of the chaotic nucleonic motion towards realistic nuclear interaction time scale.

The study is based on an analysis of the Shannon entropy type, pointing out that the paths to chaos for the "nuclear billiard" are quite similar for the first two multipole degrees, as we noticed a more rapid emergence of chaotic states as moving on towards higher radian frequencies of oscillation.

The main result obtained consists in locating a phase alternation of distinct degrees of chaotic dynamics in the monopole case of nuclear wall oscillation at $\Omega_{\mathrm{int.}} = 0.1\,c/\mathrm{fm}$, revealing a quasi-laminar behaviour prior to the resonance stage of interaction. The intermittency was thus shown to be a specific property for "nuclear billiards" defined by inelastic particle-wall collisions.

Further studies are currently in progress, investigating the intermittent behaviour for several vibrational modes on an extended frequency domain, associated to the dynamics of a nuclear collision. The amount of energy transferred in such a collision, reflected by the nucleonic frequency oscillation ω, or equivalently, by the collective vibrational frequency Ω, was shown to be the control parameter which tunes up the intermittent behaviour of the studied nuclear system. Establishing the type of intermittent dynamics found, by calculating its associated laminar length distribution [55], is regarded as a step forward.

The used formalism can be phenomenologically improved by adding spin and charge to the nucleons. Also, by choosing realistic initial conditions and potentials parameterized, and by considering other types of couplings beside collective ones (like the spin-orbit interaction).

A semi-quantal treatment of this problem, including the Pauli-blocking effect, is hoped to shed more light on the discussed issue in the near future. While the chaotic dynamics was proved to prevail in giant resonances [56,57], the detection of the interplay between laminar and turbulent behaviours could provide a hint on the role played by the quantum chaos in the fragmentation and damping of giant resonance excitations [58].

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