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Effective range expansion in various scenarios of EFT($\not\pi$)

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Abstract – Using rigorous solutions, we compare the ERE parameters obtained in three different scenarios of EFT($\not\pi$) in nonperturbative regime. A scenario with unconventional power counting (like KSW) is shown to be disfavored by the PSA data, while the one with elaborate prescription of renormalization but keeping conventional power counting intact seems more promising.

Introduction. – Since Weinberg's seminal papers in the 1990s [1], nuclear forces can now be pretty systematically understood and calculated within the framework of effective field theories (EFT) basing on chiral symmetry; for reviews on various achievements in this growing area, we refer to refs. [2,3]. In this course, there also appear some intriguing issues. For example, according to the recent review by Machleidt and Entem [4], the satisfactory renormalization and power counting of the NN sector within the framework of EFT is still an open issue. There are roughly two main types of choices towards this issue: one insists on the cut-off independence with renormalization implemented through subtractions according to new power counting schemes [5–10] that effectively invoke certain "perturbative" expansion around some leading nonperturbative components; the other one stresses the nonperturbative tractability of the whole approach [11,12] with finite cutoff in the sense of renormalization *à la* Lepage [13]. For approaches that do not obviously fall in the above two choices, see refs. [14–16]. Recently, there appear new evidences or arguments that seem to disfavor the first-type choices and the associated power counting schemes [17,18]. Thus, this issue is closely tied with the rationality and feasibility of various practical choices of the EFT power counting of couplings and the prescription of renormalization in nonperturbative regime. In this regard, it is much helpful to explore this issue using closed-form T-matrices. As it is hard to do so in the presence of pion exchanges, we turn to simplified situations to gain useful hints.

In this report, we will explore the rationality of some typical choices for power counting and renormalization prescriptions through examining their predictions for the form factors in effective range expansion (ERE) and confronting with the PSA data [19], within the realm of EFT($\not\pi$) whose renormalization has been clearly settled. This EFT is employed due to its technical tractability and its practical relevance to NN scattering at lower momentum scales (say below 70 MeV) and due to the fact that this EFT shares the same key nonperturbative structures like power-like divergences and truncation of potentials with the EFT containing pion exchanges, thus our studies here might be useful for the more general cases with pion exchanges. It will be shown that the choice with unconventional power counting (like KSW) is strongly disfavored by the PSA data. That means, in nonperturbative regime, modifying EFT power counting would encounter more difficulties. Thus, such a scheme is problematic in its rating of strengths of interactions with or without pion exchanges, in line with other criticisms of such schemes [17,18,20]. We feel such discussions are worth doing as the EFT approaches are now widely applied, see the recent application in systems like cold atoms [21] where unnatural scales and nonperturbative renormalization are also key issues.

This report is organized as follows: in the second section, the closed-form solutions of T-matrices in EFT($\not\pi$) are presented, where some issues related to renormalization are addressed and clarified; our main results and analysis of three typical scenarios for EFT($\not\pi$) are presented in the third section. A brief discussion and summary will be given in the fourth section.

Closed-form solutions and renormalization of EFT. – According to Weinberg, the potentials for NN scattering (or similar two-body nonrelativistic scattering) are first systematically constructed according to a reasonable set of power counting rules for the EFT up to a given order Δ and then resummed through Lippmann-Schwinger equations (LSEs) to obtain the T-matrices [1]. Below pion threshold, the NN potentials become contact ones (pionless EFT or EFT($\not\pi$)). In an uncoupled channel with angular momentum L, we have

$$V_L(q,q') = \sum_{i,j=0}^{\Delta/2-L} C_{L;ij} q^{2i+L} {q'}^{2j+L} = q^L {q'}^L U^T(q^2) C_L U({q'}^2), \quad (1)$$

with q, q' being external momenta. Introducing the column vector $U(x) \equiv (1, x, x^2, \ldots)$, the potential is turned into a matrix sandwiched between vectors. Note that: $C_{L;ij}=0$, when $i+j>\Delta/2-L$ due to EFT truncation. Here the couplings $C_{L;ij}$ are taken to be energy-independent as the energy-dependence in potentials could be removed using various methods [22]. With such contact potentials (EFT($\not\pi$)), one only needs to deal with the following divergent integrals in Lippmann-Schwinger equations:

$$\mathcal{I}_{ij}(E) \equiv \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{k^{2(i+j)}}{E-k^2/M+i\epsilon}, \; i,j \geqslant 0. \quad (2)$$

These integrals, which span a matrix $\mathcal{I}(E) \equiv (\mathcal{I}_{ij}(E))$, could be generally parametrized as $(p=\sqrt{ME})$

$$\mathcal{I}_{ij}(E) \equiv \sum_{m=1}^{i+j} J_{2m+1} p^{2(i+j-m)} - \mathcal{I}_0 p^{2(i+j)}, \quad (3)$$

with $\mathcal{I}_0 \equiv J_0 + i\frac{Mp}{4\pi}$ and the arbitrary parameters $[J_0, J_{2m+1}, m>0]$ parametrizing any sensible prescription of regularization and/or renormalization.

Note that at any given order Δ, the matrix $\mathcal{I}(E)$ of finite rank characterizes the entire nonperturbative structures of the divergences or ambiguities to be fixed, so only finitely many divergences are involved, NOT infinitely many. This feature should persists even in the presence of pion exchanges. With these preparations, we find that the on-shell T-matrix for channel L takes the following form at any given order of truncation:

$$\frac{1}{T_L(p)} = \mathcal{I}_0 + \frac{\mathcal{N}_L}{\mathcal{D}_L p^{2L}}. \quad (4)$$

The coupled channels could be treated in the same fashion. The on-shell T-matrices for the channels ${}^3S_1$-${}^3D_1$ read

$$\frac{1}{T_{ss}(p)} = \mathcal{I}_0 + \frac{\mathcal{N}_0 + \mathcal{I}_0 \mathcal{N}_1 p^4}{\mathcal{D}_0 + \mathcal{I}_0 \mathcal{D}_1 p^4}, \quad \frac{1}{T_{dd}(p)} = \mathcal{I}_0 + \frac{\mathcal{N}_0 + \mathcal{I}_0 \mathcal{D}_0}{[\mathcal{N}_1 + \mathcal{I}_0 \mathcal{D}_1] p^4}. \quad (5)$$

In whatever channels, the factors $[\mathcal{N}_{\ldots}, \mathcal{D}_{\ldots}]$ are real polynomials in terms of $[C_{\ldots}]$, $[J_{2m+1}, m>0]$ and on-shell momentum p, the concrete expressions will be given in a detailed report [23], where it will also be shown that the relation $\mathcal{D}_{sd}^2 + \mathcal{D}_1 \mathcal{N}_0 = \mathcal{N}_1 \mathcal{D}_0$ in ${}^3S_1$-${}^3D_1$. The ${}^1S_0$ case has been discussed in detail in our previous work [24]. Our analysis below will be basing on the solutions given in eqs. (4), (5). To proceed, we define the following notations:

$$\mathcal{N}_{\ldots} = \sum_{l=0,1,\cdots} \mathcal{N}_{\ldots;l} p^{2l}, \quad \mathcal{D}_{\ldots} = \sum_{l=0,1,\cdots} \mathcal{D}_{\ldots;l} p^{2l}. \quad (6)$$

We also note that the factors $[\mathcal{N}_{\ldots}, \mathcal{D}_{\ldots}]$ are independent of the parameter $\mathcal{I}_0$ or J_0, a fact that is quite consequential [24–26]: Since the functional dependence of the on-shell T's upon p is physical, the nonperturbative compact form of T's implies that $\mathcal{I}_0$ is already physical or renormalization group invariant, no longer a prescription-dependent parameter in contrast to the perturbative regimes.

Obviously, the renormalization of the T-matrices in eqs. (4), (5) could not be simply achieved with conventional means without ruining their nonperturbative forms, see refs. [24–26] for a transparent demonstration. In ref. [26] it is noted the intrinsic mismatch between the EFT couplings and the nonperturbative divergences precludes the conventional counterterms from working: short of couplings (due to truncation) for the "unmatched" divergences. Consequently, subtractions have to be done at loop level, with some residual constants becoming independent parameters to be determined through additional physical boundaries, while in perturbative contexts each divergence could be absorbed into the EFT couplings and makes the couplings "run".

Among the two main choices mentioned above, the second one could be roughly seen as one instance of subtraction at loop level [27]: The integral cutoff is kept finite as an independent parameter in addition and determined by fitting to data rather than absorbed into the couplings. Here, our general parametrization of the subtractions instead of a single cutoff is feasible as only finitely many divergences are involved due to truncation. In our view, this is the true source of tractability of EFT in nonperturbative regime.

The origin of additional parameters could be seen in this way. Suppose we work with an underlying theory where no divergence shows up. In the EFT limit or projection, some of the operators would show up in the EFT Lagrangian at a given order, while the high-energy "details" become "regulators". Part of the "details" could be reabsorbed into the EFT couplings they "match" with, there would also be "unmatched" part due to truncation, which have to be fixed as independent parameters in EFT. The good news is that, only finitely many of such parameters would be involved due to truncation [24–26].

In fact, the EFT upper scales are physical in the sense that they correspond to the thresholds of the EFT expansion, not the ordinary regularization scales to be removed later. For example, in EFT($\not\pi$), $\Lambda \sim m_\pi$; in the presence of pion exchanges, $\Lambda \sim m_\rho$.

Evidently, within EFT($\not\pi$), physics are encoded in the functional dependence of on-shell T's upon on-shell momentum, or more specifically in the parameter $\mathcal{I}_0$ and the ratios $[\mathcal{N}_{\dots;i}/\mathcal{N}_{\dots;0}, \mathcal{D}_{\dots;j}/\mathcal{N}_{\dots;0}]$ (cf., eq. (6)), which are rational functions in terms of $[C_{\dots}]$ and $[J_{\dots}]$. Thus, a power counting scheme must conspire with appropriate prescriptions to yield desired behaviors in EFT approach. This requires that the two sets of parameters be treated on the same footing, giving rise to the concept of scenario. Different scenarios (modulo equivalence) would give rise to different physics. Therefore, for one specific system, only one scenario (and its equivalents) of EFT would be a correct or sensible choice.

In general, EFT parameters could be functions of the EFT expansion parameter $\epsilon(\equiv \frac{Q}{\Lambda})$ with Q being an ordinary EFT scale and Λ the upper scale. For later convenience, we introduce the following dimensionless parameters: $C_{ij} = \frac{4\pi}{M}\frac{\tilde{c}_{ij}(\epsilon)}{2^{i+j}\Lambda^{2i+2j+1}}$, $J_{2k+1} = \frac{M}{4\pi}\mu^{2k+1}\tilde{j}_{2k+1}(\epsilon)$, with $\mu \sim Q$ being an ordinary renormalization scale. In fact, the functional forms of $[\tilde{c}_{ij}(\epsilon)]$ and $[\tilde{j}_{2k+1}(\epsilon)]$ embody the detailed patterns of fine tuning in a scenario. For the purpose of demonstration below, it suffices to define the fine tuning in a scenario in terms of $[\tilde{c}_{ij}(\epsilon)]$ only:

$$\frac{\tilde{c}_{ij}(\epsilon)}{|\tilde{c}_{ij;0}(\epsilon)|} = \pm 1 + \mathcal{O}(\epsilon^{\sigma}), \quad \sigma \in [1, \kappa], \tag{7}$$

where $\tilde{c}_{ij;0}(\epsilon)$ denotes the leading term in the ϵ-expansion and κ denotes the lowest exponent of the higher-order contributions to the ERE parameters measured in terms of the ϵ-expansion. This is the guiding principle for determining the fine-tuning exponent σ in the following discussions.

Various scenarios of EFT($\not\pi$). – Below, we examine three typical scenarios of EFT. At this stage, we remind that our formulation is applicable to all nonrelativistic dynamics governed by short-distance interactions, where the working energy is well below the threshold of the quanta mediating the short-distance interactions so that contact potentials could be effectively employed to describe the physical processes.

Natural and unnatural scenarios. Let us define three typical scenarios:

$$\text{A:} \quad \tilde{c}_{ij} \sim \mathcal{O}(1); \quad \tilde{j}_{2k+1} \sim \mathcal{O}(1); \quad J_0 \sim \frac{M}{4\pi}Q; \tag{8}$$

$$\text{B:} \quad \tilde{c}_{ij} \sim \frac{\mathcal{O}(1)}{\epsilon^{i+j+1}}; \quad \tilde{j}_{2k+1} \sim \mathcal{O}(1); \quad J_0 \sim \frac{M}{4\pi}Q; \tag{9}$$

$$\text{C:} \quad \tilde{c}_{ij} \sim \mathcal{O}(1); \quad \tilde{j}_{2k+1} \sim \mathcal{O}(1); \quad J_0 \sim \frac{M}{4\pi}\Lambda. \tag{10}$$

Obviously, scenario A is defined with conventional EFT power counting for couplings and usual renormalization prescription, it will lead to natural ERE parameters, hence a natural scenario. Scenario B comprises of an unconventional power counting of couplings and a usual prescription, the couplings are unnaturally large. It will indeed lead to unnatural scattering. In scenario C, conventional power counting of couplings is preserved, and the renormalization prescription is as usual, but the scale of $J_0(=\mathrm{Re}(\mathcal{I}_0))$ is chosen to scale differently from the preceding two scenarios. This is because as a physical (RG invariant) parameter [24–26] (see the discussions below eq. (6)), J_0 could take values as large as $M\Lambda/(4\pi)$ since the upper scale Λ itself is a physical parameter of normal size, NOT a regularization cutoff as mentioned above. Thus, this scenario is "natural" in the sense that all the scales involved are naturally sized. But it is indeed able to lead to unnatural scattering upon fine tuning, see below.

The exponent κ could be read off from the concrete expressions of $1/T$. For example, in $^1S_0$, we have (at $\Delta = 4$)

$$\begin{aligned}
&\mathrm{Re}\left(4\pi/(M\Lambda T_0(p))\right) \sim \\
\text{A:}\quad & \epsilon + \frac{1 + o(\epsilon^3) + \frac{p^2}{\Lambda^2}\epsilon^3\mathcal{O}(1 + o(\epsilon^3)) + \cdots}{\tilde{c}_0 + o(\epsilon^5) + \frac{p^2}{\Lambda^2}\mathcal{O}(1 + o(\epsilon^3)) + \cdots}; \\
\text{B:}\quad & \epsilon + \frac{1 + o(\epsilon) + \frac{p^2}{\Lambda^2}\mathcal{O}(1 + o(\epsilon)) + \cdots}{\tilde{c}_0 + o(\epsilon) + \frac{p^2}{\Lambda^2}\epsilon^{-2}\mathcal{O}(1 + o(\epsilon)) + \cdots}; \\
\text{C:}\quad & 1 + \frac{1 + o(\epsilon^3) + \frac{p^2}{\Lambda^2}\epsilon^3\mathcal{O}(1 + o(\epsilon^3)) + \cdots}{\tilde{c}_0 + o(\epsilon^5) + \frac{p^2}{\Lambda^2}\mathcal{O}(1 + o(\epsilon^3)) + \cdots}. \\
& \kappa_{\mathrm{A}} = 3; \quad \kappa_{\mathrm{B}} = 1; \quad \kappa_{\mathrm{C}} = 3.
\end{aligned} \tag{11}$$

The related details would be given in ref. [23].

Primary and qualitative results. Now, we perform the magnitude analysis of the ERE parameters in S-waves in the three scenarios defined above using the closed-form T-matrices obtained in the second section with the contact potentials truncated at order $\Delta = 4$. To proceed, the following primary fine tunings of the lowest coupling $\tilde{c}_{00}$ are considered:

$$\begin{aligned}
\text{Tuning I:}\quad & \tilde{c}_{00} \sim -1 + \mathcal{O}(\epsilon) \quad (\text{scenario A, C}), \\
& \epsilon\tilde{c}_{00} \sim -1 + \mathcal{O}(\epsilon) \quad (\text{scenario B}); \\
\text{Tuning II:}\quad & \tilde{c}_{00} \sim -1 + \mathcal{O}(\epsilon^2) \quad (\text{scenario A, C}).
\end{aligned}$$

The results for $^3S_1$ are listed in table 1 with tuning I for scenario A, B and C. In order to yield a scattering length of size $(\epsilon\Lambda^{-1}\mathcal{O}(1 + o(\epsilon))$ in scenario B, one should use $\epsilon\tilde{c}_{00} \sim +1 + \mathcal{O}(\epsilon)$ instead, with the rest of ERE parameters being the same as in table 1. Evidently, scenario A corresponds to systems with natural size of ERE parameters while B and C to those with unnatural ones. Moreover, scenario B seems to correspond to systems with more unnatural scales than scenario C. The results of the $^1S_0$ case are presented in table 2, where the tuning II is used in scenario C to yield a much larger (ϵ^{-2}) scattering length in $^1S_0$, while tuning I is used in scenarios A and B. Due to $\kappa_{\mathrm{B}} = 1$, it does not make sense to consider tuning II in scenario B at all, and tuning II would lead essentially the same scaling in scenario A as it is a natural scenario.

Interestingly, the leading term for r_e is very simple and involves the coupling $\tilde{c}_{01}(=\tilde{c}_{10})$ only, so it is listed out explicitly. Moreover, it is in fact of the same size in the

Table 1: Scaling of ERE parameters in $^3S_1$.

ERE	Scenario A	Scenario B	Scenario C
$\Lambda \cdot a$	$\mathcal{O}(1+o(\epsilon))$	$\epsilon^{-2}\mathcal{O}(1+o(\epsilon))$	$\epsilon^{-1}\mathcal{O}(1+o(\epsilon))$
$\Lambda \cdot r_e$	$(2\tilde{c}_{01}+o(\epsilon))$	$(2\epsilon^2\tilde{c}_{01}+o(\epsilon))$	$(2\tilde{c}_{01}+o(\epsilon))$
$\Lambda^3 \cdot v_2$	$\mathcal{O}(1+o(\epsilon))$	$\epsilon^{-1}\mathcal{O}(1+o(\epsilon))$	$\mathcal{O}(1+o(\epsilon))$
$\Lambda^5 \cdot v_3$	$\mathcal{O}(1+o(\epsilon))$	$\epsilon^{-2}\mathcal{O}(1+o(\epsilon))$	$\mathcal{O}(1+o(\epsilon))$
$\Lambda^7 \cdot v_4$	$\mathcal{O}(1+o(\epsilon))$	$\epsilon^{-3}\mathcal{O}(1+o(\epsilon))$	$\mathcal{O}(1+o(\epsilon))$

Table 2: Scaling of ERE parameters in $^1S_0$.

ERE	Scenario A	Scenario B	Scenario C
$\Lambda \cdot a$	$\mathcal{O}(1+o(\epsilon))$	$\epsilon^{-2}\mathcal{O}(1+o(\epsilon))$	$\epsilon^{-2}\mathcal{O}(1+o(\epsilon^2))$
$\Lambda \cdot r_e$	$(2\tilde{c}_{01}+o(\epsilon^2))$	$(2\epsilon^2\tilde{c}_{01}+o(\epsilon))$	$(2\tilde{c}_{01}+o(\epsilon^2))$
$\Lambda^3 \cdot v_2$	$\mathcal{O}(1+o(\epsilon^2))$	$\epsilon^{-1}\mathcal{O}(1+o(\epsilon))$	$\mathcal{O}(1+o(\epsilon^2))$
$\Lambda^5 \cdot v_3$	$\mathcal{O}(1+o(\epsilon^2))$	$\epsilon^{-2}\mathcal{O}(1+o(\epsilon))$	$\mathcal{O}(1+o(\epsilon^2))$
$\Lambda^7 \cdot v_4$	$\mathcal{O}(1+o(\epsilon^2))$	$\epsilon^{-3}\mathcal{O}(1+o(\epsilon))$	$\mathcal{O}(1+o(\epsilon^2))$

three scenarios as $\tilde{c}_{01} \sim \epsilon^{-2}$ in scenario B. From v_2 on, the leading terms are found to involve more and more couplings, leaving plenty of room for further reduction in size upon cancelation amongst the couplings, which will be discussed in detail in the future [23]. Here, they are simply assumed to be of order one to focus on our main concerns.

In order to see the rationality in choosing appropriate prescriptions rather than modifying the canonical EFT power counting, the empirical ERE parameters in S-waves are listed and analyzed with respect to scaling using the PSA data that are given in table 1 of ref. [19], and in table 8 and table 9 of ref. [28]. The results are given in table 3 with choice $\Lambda \approx m_{\pi^\pm}$ and $\epsilon \approx \frac{1}{4}$. For example, the scattering lengths scale as below: $\Lambda \cdot a_{^3S_1} \sim \epsilon^{-1}, \Lambda \cdot a_{^1S_0} \sim \epsilon^{-2}$. From this table, one can see that the PSA data lead to small v_2 in comparison to all the three schemes above. In particular, the PSA data give an extremely small v_2 in $^3S_1$ channel.

Scenarios and unnaturalness in ERE. Let us elaborate on the scenarios B and C that lead to large scattering lengths in S-waves. The magnitudes for v_2, v_3 and v_4 obtained in scenario B seem to be quite large, contrary to the PSA data that give much smaller numbers. In comparison, the numbers given by scenario C are smaller and hence seem closer to the PSA data. In the $^1S_0$ channel, the agreement between scenario C and PSA data is almost complete. Thus, the more complicated scenario B seems to be disfavored in this regard.

If one requires that the PSA data be reproduced in the two scenarios, then further fine tunings are necessary. Comparing tables 1 and 2 with table 3, it is evident that there are huge "gaps" in the size of ERE parameters between PSA data and scenario B:

$$^3S_1: \quad \frac{v_{2;B}}{v_{2:P}} \sim \epsilon^{-4}, \quad \frac{v_{3;B}}{v_{3;P}} \sim \epsilon^{-\frac{7}{2}}, \quad \frac{v_{4;B}}{v_{4;P}} \sim \epsilon^{-\frac{15}{4}}, \tag{12}$$

Table 3: PSA data and their scaling in S-waves.

ERE	$^3S_1$(data)	$^3S_1$(scaling)	$^1S_0$(data)	$^1S_0$(scaling)
$\Lambda \cdot a$	$(0.26)^{-1}$	$\epsilon^{-1}\mathcal{O}(1)$	$-(0.06)^{-1}$	$\epsilon^{-2}\mathcal{O}(1)$
$\Lambda \cdot r_e$	$(0.81)^{-1}$	$\mathcal{O}(1)$	$(0.53)^{-1}$	$2\mathcal{O}(1)$
$\Lambda^3 \cdot v_2$	$(4.13)^{-3}$	$\epsilon^3\mathcal{O}(1)$	$-(1.81)^{-3}$	$\epsilon^{\frac{5}{4}}\mathcal{O}(1)$
$\Lambda^5 \cdot v_3$	$(1.53)^{-5}$	$\epsilon^{\frac{3}{2}}\mathcal{O}(1)$	$(1.07)^{-5}$	$\mathcal{O}(1)$
$\Lambda^7 \cdot v_4$	$-(1.16)^{-7}$	$\epsilon^{\frac{3}{4}}\mathcal{O}(1)$	$-(0.92)^{-7}$	$\mathcal{O}(1)$

$$^1S_0: \quad \frac{v_{2;B}}{v_{2:P}} \sim \epsilon^{-\frac{9}{4}}, \quad \frac{v_{3;B}}{v_{3;P}} \sim \epsilon^{-2}, \quad \frac{v_{4;B}}{v_{4;P}} \sim \epsilon^{-3}. \tag{13}$$

It seems extremely difficult to do the fine tuning to remove these "gaps" in scenario B. In contrast, the "gaps" between PSA and scenario C are much smaller in each ERE parameter:

$$^3S_1: \quad \frac{v_{2;C}}{v_{2:P}} \sim \epsilon^{-3}, \quad \frac{v_{3;C}}{v_{3:P}} \sim \epsilon^{-\frac{3}{2}}, \quad \frac{v_{4;C}}{v_{4:P}} \sim \epsilon^{-\frac{3}{4}}, \tag{14}$$

$$^1S_0: \quad \frac{v_{2;C}}{v_{2:P}} \sim \epsilon^{-\frac{5}{4}}, \quad \frac{v_{3;C}}{v_{3:P}} \sim \epsilon^{0}, \quad \frac{v_{4;C}}{v_{4:P}} \sim \epsilon^{0}. \tag{15}$$

Thus, scenario B is also disfavored in the perspective of fine tuning.

The problem with scenario B or unnatural couplings could also be seen as follows: Suppose one insists on using the unconventional couplings of scenario B to describe the NN low-energy scattering in terms of pionless EFT. Then to reproduce the scaling in scenario C, it turns out that one has to choose the following scaling of the renormalization parameters:

$$\frac{4\pi J_{2k+1}}{M\mu^{2k+1}} \sim \epsilon^{k+1+a}, \quad a \geqslant 0, \quad \forall k > 0. \tag{16}$$

which means that NN scattering with unnatural couplings would involve subtractions at scales much smaller than normally expected, *i.e.*, a very "unnatural" prescription of renormalization. However, such unconventional choice of renormalization could not be simply excluded, though its rationales remain to be seen. Actually, in the literature using modified power counting for couplings these parameters were set to be even smaller: zero [29].

Here, we should remind again that our analysis are performed in a pionless situation. Since truncation is still necessary in cases with pion exchanges, the basic "characters" of the scenario issue depicted here might remain, though it is more difficult to work out closed-form solutions there. It would also be interesting to see how the "pictures" evolve after other contents of potential are included [30].

Although scenario C looks better than scenario B, it remains to see how the further reduction of the "gaps" in eq. (14) could happen. This issue will be addressed in the detailed report [23]. In fact, closer studies may lead to more constraints on the contact couplings in similar fashion using ERE and/or other phenomenological data, the so-determined EFT couplings may in turn provide useful targets for lattice studies basing on more fundamental theories like QCD. Such kind of analysis will also be given in our detailed report [23].

Discussions and summary. – Tables 1, 2 and 3 are our main results of this report. As is clear from the numbers listed above, the choice of employing unconventional EFT power counting and conventional subtraction for NN scattering is disfavored by the PSA data. In other words, modification of conventional power counting is disfavored in comparison with choosing appropriate prescriptions in nonperturbative regimes while keeping the conventional rating of interactions intact. Of course, a conventional power counting of couplings AND a conventional prescription could not be compatible with unnatural scattering lengths. Thus the choice like scenario C seems more promising, as our analysis done here is only a crude one, there is still much room for further tuning to remove the "gaps" as explained above. We will demonstrate instances of such tuning in the detailed report [23].

In the course of our presentation, it also occurs to us that EFT truncation turns out to be a virtue rather than a burden in the issue of renormalization of EFT in the nonperturbative regime: It is the truncation that keeps the number of nonperturbative divergences involved, here the rank of the matrix $\mathcal{I}(E)$, finite. Without truncation, the rank of $\mathcal{I}(E)$ would be infinite, an intractable situation that renders the EFT approach totally useless. This virtue might remain somehow in the presence of pion exchanges and somewhat underlies the observation that only finite nonperturbative subtractions are needed at a given order [8,14,15].

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