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Threshold fluctuations in a superconducting current-carrying bridge

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Abstract

We calculate the energy of threshold fluctuation δF_{thr} which triggers the transition of a superconducting current-carrying bridge to the resistive state. We show that the dependence $\delta F_{\text{thr}}(I) \propto I_{\text{dep}} \hbar (1 - I/I_{\text{dep}})^{5/4}/e$, found by Langer and Ambegaokar for a long bridge with length $L \gg \xi$, holds far below the critical temperature in both dirty and clean limits (here I_{dep} is the depairing current of the bridge and ξ is a coherence length). We also find that even a ‘weak’ local defect (leading to a small suppression of the critical current of the bridge $I_c \lesssim I_{\text{dep}}$) provides $\delta F_{\text{thr}} \propto I_c \hbar (1 - I/I_c)^{3/2}/e$, typical for a short bridge with $L \ll \xi$ or a Josephson junction.

Keywords: fluctuations, superconducting bridge, defects

(Some figures may appear in colour only in the online journal)

1. Introduction

The superconducting state of a bridge or wire with current is stable with respect to infinitesimally small perturbations of superconducting order parameter Δ up to currents close to the critical (depairing) current. But if fluctuation-induced change of Δ is sufficiently large, instability is developed in the superconductor even at $I < I_c$, leading to the appearance of a finite resistance and dissipation. The theory of fluctuation-induced switching was first studied in the work of Langer and Ambegaokar (LA) [1]. They considered a long (length $L \gg \xi$, with ξ the coherence length) quasi-one-dimensional (transverse dimensions smaller than ξ) superconducting bridge. To calculate the threshold fluctuation LA proposed finding a saddle-point state in the system nearest in energy to the ground state. They obtained that threshold fluctuation corresponds to a partial suppression of the superconducting order parameter in a finite segment of the bridge with size of about ξ and derived the dependence of the energy of threshold fluctuation on the applied current. Their result is described well by following approximate expression [2]:

$$\begin{aligned} \delta F_{\text{LA}} &= \frac{4\sqrt{2}}{3} F_0 \left(1 - \frac{I}{I_{\text{dep}}}\right)^{5/4} \\ &= \frac{\sqrt{6}}{2} \frac{I_{\text{dep}} \hbar}{e} \left(1 - \frac{I}{I_{\text{dep}}}\right)^{5/4}, \end{aligned} \quad (1)$$

where $F_0 = \Phi_0^2 S / 32\pi^3 \lambda^2 \xi$, Φ_0 is the magnetic flux quantum, $S = wd$ is the area of the cross section of the bridge with width w and thickness d , λ is the London penetration depth of the magnetic field and $I_{\text{dep}} = 2I_0/3\sqrt{3}$ ($I_0 = c\Phi_0 S / 8\pi^2 \lambda^2 \xi$) is the depairing current in the Ginzburg–Landau (GL) model, which coincides with the expected critical current of the long ($L \gg \xi$) bridge. In [3] the LA approach was generalized for superconducting bridges with arbitrary length and it was shown that the dependence $\delta F_{\text{thr}}(I)$ tends to the expression $\delta F_{\text{thr}} = \hbar I_c (1 - I/I_c)^{3/2}/e$ for short bridges ($L \ll \xi$; $I_c \propto 1/L$ is the critical current of the bridge). This dependence is typical for the energy of threshold fluctuation of Josephson junctions with a sinusoidal current–phase relation [4].

The energy of threshold fluctuation was also calculated for a long bridge using a microscopic approach [5, 6]. In [5] the temperature and current dependences of δF_{thr} were calculated on the basis of the Eilenberger equations [7] for a clean long superconducting bridge with only one conducting channel. However, δF_{thr} has been significantly overestimated at finite current since the contribution to δF_{thr} due to the work performed by the current source was not taken into account. In the present work we recalculate the dependence $\delta F_{\text{thr}}(I)$ at different temperatures on the basis of the Eilenberger equations, and find agreement with a power-5/4 law up to $T = 0.5T_c$, which coincides with the result found in [6] using Usadel equations [8] for a long dirty bridge. We argue that the

relation $\delta F_{\text{thr}}(0) \sim I_{\text{dep}} \hbar/e$ found in the framework of the GL model (see equation (1)) approximately holds in a broad temperature range below T_c not only for long bridges (dirty or clean ones) but also for short bridges, with the replacement of $I_{\text{dep}}(T)$ by the actual critical current of the bridge $I_c(T)$.

Our interest in the role of defects in $\delta F_{\text{thr}}(I)$ is motivated by recent experimental works [9–11]. In experiments one usually measures the switching current I_{sw} (which has a random value due to fluctuations) many times to find the average value $\langle I_{\text{sw}} \rangle$ and the dispersion σ , which are directly related to $\delta F_{\text{thr}}(I)$ (for the explicit relation between $\langle I_{\text{sw}} \rangle$, σ and $\delta F_{\text{thr}}(I)$ see, for example, equations (2) and (3) in [11]). Note that alternatively $F_{\text{thr}}(0)$ could be found from the temperature dependence of resistivity near T_c , because $R(T) \propto \exp(-\delta F_{\text{thr}}(0)/kT)$ [12]. Although in [10, 11] experiments were done for long superconducting bridges in a wide temperature interval below T_c good agreement with a power-3/2 law was found. To explain this result, Khlebnikov [13] recently developed a model which considered the bridge as a discrete set of nodes connected by superconducting links; in his model he neglected local suppression of the superconducting order parameter. Below we show that a power-3/2 law can be obtained for a long bridge in the framework of the LA model if one takes into consideration the presence of defects in the bridge, such as constrictions or local variation of the critical temperature or mean path length. We argue that the dependence $\delta F_{\text{thr}}(I)$ can deviate from a power-5/4 law even in case of relatively ‘weak’ defects, when the critical current I_c of the bridge with defects is not far from the depairing current I_{dep} .

2. Effect of defects on $\delta F_{\text{thr}}(I)$

Here we consider a model system consisting of a superconducting bridge with cross section S and length L connecting two superconducting banks whose cross section has the area $S_{\text{pad}} \gg S$. Assuming that the maximum characteristic transverse size is $d \sim \sqrt{S} \ll \xi$, the problem can be considered as one-dimensional and only the dependence on the longitudinal coordinate x is taken into account.

To consider the effect of defects on the dependence $\delta F_{\text{thr}}(I)$, we use the GL theory. To determine the energy of threshold fluctuation it is necessary to find the saddle state of the system corresponding to the local maximum of the free energy in presence of an external current source. Since it is a stationary state (albeit unstable), it is described by the GL equation

$$\xi_{\text{GL}}^2(0) \nabla^2 \Delta + (1 - T/T_c - |\Delta|^2/\Delta_{\text{GL}}^2(0)) \Delta = 0, \quad (2)$$

where $\xi_{\text{GL}}(0)$ and $\Delta_{\text{GL}}(0)$ are, respectively, the coherence length and the superconducting order parameter in the GL model at zero temperature [12].

We seek the solution in the form $\Delta(x)/\Delta_{\text{GL}} = f(x) \exp(i\varphi(x))$. Then the dimensionless GL equation has the form

$$\frac{d^2 f}{dx^2} - \frac{j^2}{f^3} + f - f^3 = 0, \quad (3)$$

where the condition of the constant current in the system, $I = \text{const}$, is used (here $j = f^2 d\varphi/dx = I/S$ is the current density in the bridge). In (3) the magnitude of the superconducting order parameter f , length and current density are measured in units of $\Delta_{\text{GL}} = \Delta_{\text{GL}}(0) \sqrt{1-t}$, $\xi = \xi_{\text{GL}}(0)/\sqrt{1-t}$ and $j_0 = I_0/S$ ($t = T/T_c$ is the dimensionless temperature). Equation (3) should be supplemented with boundary conditions at the ends of the bridge

$$f|_{-\frac{L}{2}} = f|_{\frac{L}{2}} = 1, \quad (4)$$

which follow from the assumption about nearly zero current density at the banks, and thus the order parameter reaches its equilibrium value $f = 1$.

The energy of threshold fluctuation can be found using the expression

$$\delta F_{\text{thr}} = F_{\text{saddle}} - F_{\text{ground}} - \frac{\hbar}{2e} I \delta \varphi, \quad (5)$$

where $\delta \varphi$ is the additional phase difference between the ends of the bridge appearing in the saddle-point state and F_{saddle} and F_{ground} are the free energies of the saddle-point and ground states, respectively. In our units these energies take the form

$$F_{\text{saddle,ground}} = -\frac{F_0}{2} \int f^4 dx. \quad (6)$$

Equation (3) with boundary conditions (4) is solved numerically for a bridge with length $L = 30\xi$. In the numerical solution, we use the relaxation method: the time derivative $\partial f/\partial t$ is added to the GL equation (3) and iterations are performed until the time derivative becomes zero within a specified accuracy. To find the saddle-point state, we use the numerical method proposed in [14]: at a given current, we fix the magnitude of the order parameter $f(0)$ at the center of the bridge and allow f to change at all other points. The state with the minimum fixed $f(0)$ value for which a steady-state solution exists is a saddle-point state.

We consider three types of defect. The first type corresponds to variation of critical temperature T_c along the bridge. To describe such a defect in the model, we write the GL equation at the defect region (placed in the center of the bridge) in the form

$$\frac{d^2 f}{dx^2} - \frac{j^2}{f^3} + \alpha f - f^3 = 0, \quad (7)$$

where the parameter $\alpha = (1 - t^*)/(1 - t)$ characterizes the deviation from the critical temperature of the rest of the bridge (here $t^* = T/T_c^*$). Absence of a defect corresponds to the case $\alpha = 1$, and decrease in local critical temperature $T_c^* < T_c$ corresponds to $\alpha < 1$.

We consider defects with lengths $l = 0.5\xi$, ξ and 2ξ and calculate dependences $\delta F_{\text{thr}}(I)$ at different α . The results of our calculations for length $l = 0.5\xi$ are shown in figure 1, where we also present the fitting expression $\delta F_{\text{thr}} = \delta F_{\text{thr}}(0)(1 - I/I_c)^b$. For a bridge with critical current $I_c = 0.95I_{\text{dep}}$ ($\alpha = 0.6$) we have $b \approx 1.36$, for a bridge with $I_c = 0.74I_{\text{dep}}$ ($\alpha = -0.55$) $b \approx 1.45$, and for a bridge with $I_c = 0.66I_{\text{dep}}$ ($\alpha = -1.05$) is well fitted by $b \approx 1.5 = 3/2$

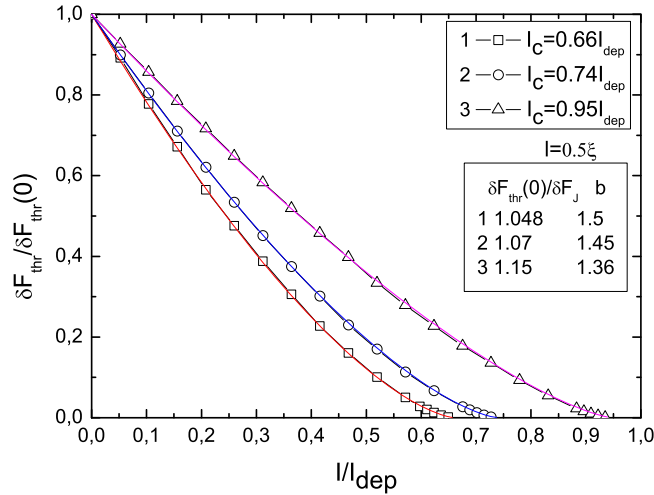


Figure 1. Dependence of the energy of threshold fluctuation on current for bridges with local variation of T_c (on length $l = 0.5\xi$ in the center of the bridge). Fitting functions $\delta F_{\text{thr}}(0)(1 - I/I_c)^b$ are shown by the solid lines, the parameters $\delta F_{\text{thr}}(0)$ and b are shown in the inset. Here $\delta F_J = \hbar I_c/e$.

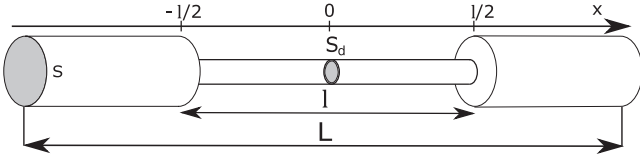


Figure 2. Superconducting bridge with cross-sectional area S and length L , containing a constriction of length l and cross section S_d .

typical for short bridge [3] and Josephson junction [4]. Besides we find that in all cases $\delta F_{\text{thr}}(0) \simeq \hbar I_c/e$ (see inset in figure 1), which is typical for a Josephson junction and resembles the result found in the framework of the GL model in limiting cases of both long $L \gg \xi$ and short $L \ll \xi$ bridges.

The second type of defect models the inhomogeneity of the cross-sectional area of the bridge. We assume that there is region with cross-sectional area $S_d < S$ and length l in the center of the bridge (see figure 2). To describe such a constriction, the boundary condition (4) is supplemented by conditions similar to the conditions from [3]

$$\left. \frac{df^L}{dx} \right|_{-l/2} = \frac{S}{S_d} \left. \frac{df^C}{dx} \right|_{-l/2}, \quad \left. \frac{S}{S_d} \frac{df^C}{dx} \right|_{l/2} = \left. \frac{df^R}{dx} \right|_{l/2}, \quad (8a)$$

$$f^L|_{-l/2} = f^C|_{-l/2} = f^C|_{l/2} = f^R|_{l/2}, \quad (8b)$$

where f^L, f^C, f^R are the magnitudes of the order parameter to the left of the defect, in the defect and to the right of the defect, respectively. The condition (8a) appears from the variation of the GL functional for a superconductor with cross section depending on x (which is responsible for the appearance of the derivative $d/dx(S(x)df/dx)$). Here S/S_d is not the actual ratio of cross-sectional areas but it is a reference parameter characterizing a change in the derivative of the function f in the x direction at the transition through the bridge-defect interface.

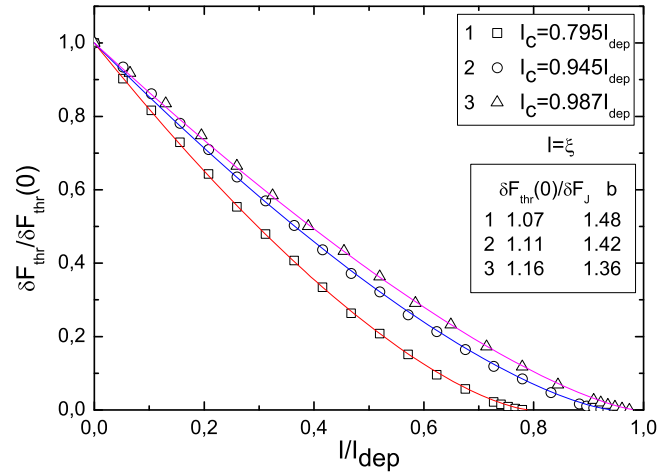


Figure 3. Dependence of the energy of threshold fluctuation on current for a bridge with different constrictions (the length of constriction is fixed: $l = \xi$). The fitting functions $\delta F_{\text{thr}}(0)(1 - I/I_c)^b$ are shown by the solid lines, the parameters $\delta F_{\text{thr}}(0)$ and b are shown in the inset. Here $\delta F_J = \hbar I_c/e$.

The calculated dependences $\delta F_{\text{thr}}(I)$ for the constriction with $l = \xi$ and different cross sections S_d are shown in figure 3 together with the fitting expressions $\delta F_{\text{thr}} = \delta F_{\text{thr}}(0)(1 - I/I_c)^b$. For a bridge with $S_d = 0.9S$ critical current $I_c = 0.987I_{\text{dep}}$ and $b \approx 1.36$, for a bridge with $S_d = 0.76S$ $I_c = 0.945I_{\text{dep}}$ ($b \approx 1.42$) and for bridge with $S_d = 0.5S$ $I_c = 0.795I_{\text{dep}}$ and $b \approx 1.48$. This result demonstrates that even a small variation in the cross-sectional area can significantly change the dependence $\delta F_{\text{thr}}(I)$ and neither a power-5/4 law nor a power-3/2 law are suitable to fit the current dependence of δF_{thr} . As in case of local variation of T_c , even a relatively ‘weak’ constriction ‘provides’ a power-3/2 law and $\delta F_{\text{thr}}(0) \simeq \hbar I_c/e$ (see inset in figure 3).

Very similar results could be obtained if there is local variation in the bridge of mean path length ℓ (a third type of defect). In principle, to calculate $\delta F_{\text{thr}}(I)$ one can use analytical results for the distribution of f and phase along the superconducting bridge from [15] but we use a numerical procedure because the dependence of f on the coordinate is expressed via special functions. We find that when ℓ is five times smaller in the region with length $l = 0.5\xi$ the dependence $\delta F_{\text{thr}}(I) \simeq 1.06\delta F_J(1 - I/I_c)^{3/2}$ with $I_c \simeq 0.73I_{\text{dep}}$.

The change of exponent from 5/4 to 3/2 for the types of defect considered can be understood in the following way. In a long defectless bridge the length of the critical nucleus (the region with suppressed Δ) diverges as $I \rightarrow I_{\text{dep}}$ [1], while in a bridge with defect its length is restricted by the length of defect plus $\sim 2\xi$ (when $I_c \lesssim 0.7I_{\text{dep}}$). This resembles the situation with a short superconducting bridge which behaves like a Josephson junction and has $\delta F_{\text{thr}} \propto (1 - I/I_c)^{3/2}$.

3. The energy of threshold fluctuation at arbitrary temperature

The results obtained in section 2 are valid near critical temperature T_c since they are based on the GL model. Below

we show example of defectless bridges for which the dependences $\delta F_{\text{thr}} = \delta F_{\text{thr}}(0)(1 - I/I_c)^b$ ($b = 5/4, 3/2$) and $\delta F_{\text{thr}}(0) \propto I_c$ are valid at $T \ll T_c$ too.

First we consider, similar to Zharov *et al* [5], the case of a long clean one-dimensional superconducting bridge ($\ell \gg \xi_0$, with $\xi_0 = \hbar v_F / \pi \Delta_0$ the coherence length in clean limit at $T = 0$) containing only one conduction channel. To find the saddle state in that case we use the one-dimensional Eilenberger equations for the normal and anomalous Green's functions, $g(x, \omega_n, v_F)$ and $f(x, \omega_n, v_F)$ respectively

$$\begin{aligned} \hbar v_F \frac{dg}{dx} + \Delta f^+ - \Delta g &= 0, \\ -\hbar v_F \frac{df}{dx} - 2\omega_n f + 2\Delta g &= 0, \\ \hbar v_F \frac{df^+}{dx} - 2\omega_n f^+ + 2\Delta g &= 0, \end{aligned} \quad (9)$$

where v_F is the Fermi velocity, $\omega_n = 2\pi k_B T(n + 1/2)$ is the Matsubara frequency. The Green's functions obey the normalization condition $g^2 + ff^+ = 1$. These equations are completed with the self-consistency equation for the order parameter Δ

$$\frac{\Delta(x)}{\lambda} = \pi N_0 k_B T \sum_{\omega_n} \frac{1}{2} [f(x, \omega_n, v_F) + f(x, \omega_n, -v_F)], \quad (10)$$

and the expression for the supercurrent density

$$j = -2\pi i e N_0 k_B T \sum_{\omega_n} \frac{v_F}{2} [g(x, \omega_n, v_F) - g(x, \omega_n, -v_F)]. \quad (11)$$

Here λ is the coupling constant and N_0 is the density of states on the Fermi level. The summation is going over all Matsubara frequencies.

Following [5], we seek the solution in the form of plane waves $\Delta, f \propto e^{ikx}$ with complex amplitudes and solve (9). To calculate the energy of threshold fluctuation, we use the expression (5) derived by Eilenberger in [7]. Using this expression and the saddle-point solution of (9), one can calculate the energy of threshold fluctuation

$$\begin{aligned} \delta F_{\text{thr}} &= S N_0 \pi k_B T \hbar v_F \text{Re} \sum_{\omega_n} \left(\ln \frac{a_+}{a_-} - \frac{2\Delta_{R0}}{\sqrt{\omega_n'^2 + \Delta_0^2}} \right) \\ &\quad - \frac{\hbar}{e} I \arctan \frac{\Delta_{R0}}{|\Delta_I|}. \end{aligned} \quad (12)$$

Here $a_{\pm} = \Delta_0^2 - i\Delta_I \omega_n' \pm \Delta_{R0} \sqrt{\omega_n'^2 + \Delta_0^2}$, $\omega_n' = \omega_n + i\hbar k/2$, Δ_0 is the absolute value of the complex amplitude of the order parameter and Δ_{R0} and Δ_I are the real and imaginary parts of the complex amplitude, which is determined by the equations

$$\pi k_B T \sum_{\omega_n} \left(\text{Re} \frac{1}{\sqrt{\omega_n'^2 + \Delta_0^2}} - \frac{1}{|\omega_n|} \right) = \ln T, \quad (13)$$

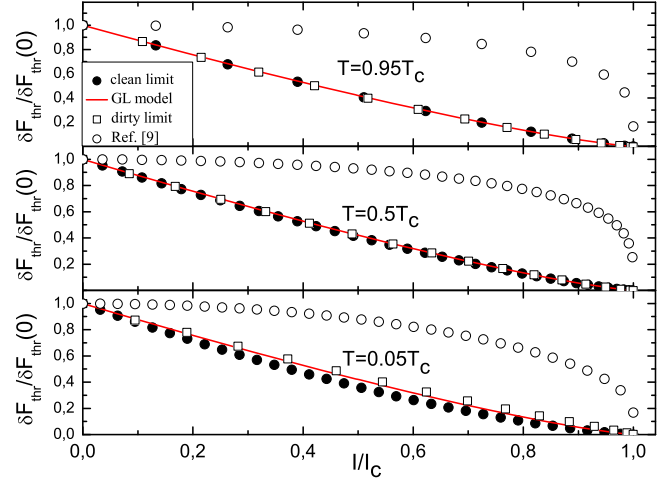


Figure 4. Dependence of the energy of threshold fluctuation on current for a long bridge ($L \gg \xi(T)$) at different temperatures in the clean and dirty limits. We compare this with the results following from the GL model (solid lines, equation (1)) and [5] (white circles, equation (12) without last term).

$$\sum_{\omega_n} \text{Im} \left[\frac{1}{\omega_n' + i\Delta_I} \frac{1}{\sqrt{\omega_n'^2 + \Delta_0^2}} \right] = 0. \quad (14)$$

In [5] expression (12) does not contain last term, which includes the work performed by the current source on the system during the transition of the system from the ground state to the saddle-point state. The comparison of our results with the results of [5] and the LA theory is shown in figure 4. It is seen that accounting for this term significantly changes the dependence $\delta F_{\text{thr}}(I)$ and brings it to a form that is similar to (1) in the wide temperature range below the critical temperature (only at $T/T_c = 0.05$ is there noticeable deviation from the power-5/4 law).

Now we consider the case of dirty superconducting bridge ($\ell \ll \xi_0$). To calculate the energy of the saddle-point state we use the Usadel equation [8] for the normal $g(\omega_n, x)$ and anomalous $f(\omega_n, x)$ Green's functions in standard parametrization [16]

$$g(\omega_n, x) = \cos \theta(\omega_n, x), f(\omega_n, x) = \sin \theta(\omega_n, x) e^{i\chi(x)}, \quad (15)$$

where θ and χ are real functions. With that parametrization the Usadel equation reads as

$$\frac{\hbar D}{2} \frac{d^2 \theta}{dx^2} - \left(\omega_n + \frac{D}{2\hbar} q_s^2 \cos \theta \right) \sin \theta + \Delta \cos \theta = 0, \quad (16)$$

while the self-consistency equation and the expression for the supercurrent density take the forms

$$\Delta \ln \frac{T}{T_c} = 2\pi k_B T \sum_{\omega_n > 0} \left(\sin \theta - \frac{\Delta}{\omega_n} \right), \quad (17)$$

$$j = 4eN_0 D \pi T \frac{q_s}{\hbar} \sum_{\omega_n > 0} \sin^2 \theta. \quad (18)$$

Here D is the diffusion coefficient and $q_s = \hbar (d\chi/dx)$ is the superfluid momentum. The free energy in (5) can be written

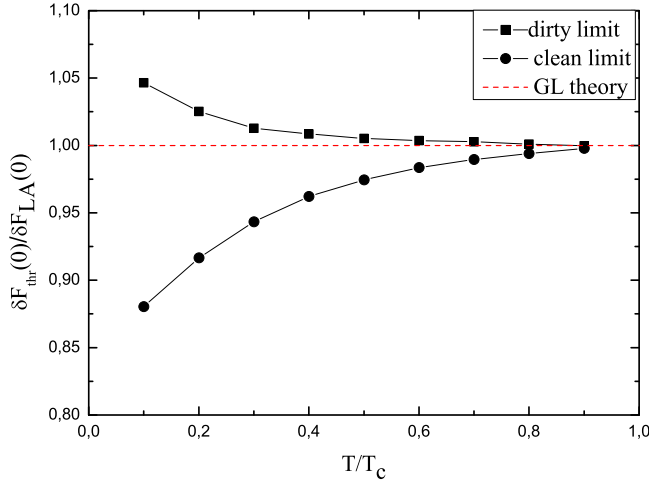


Figure 5. Temperature dependence of the energy of threshold fluctuation at zero current for a long bridge ($L \gg \xi(T)$) in the clean and dirty limits. The energy is measured in units of $\delta F_{\text{LA}}(0) = \sqrt{6} I_{\text{dep}} \hbar / 2e$.

as

$$F = 2\pi N_0 k_B T S \sum_{\omega_n > 0} \int dx \left\{ \frac{\hbar D}{2} \left[\left(\frac{d\theta}{dx} \right)^2 + \left(\frac{q_s \sin \theta}{\hbar} \right)^2 \right] - 2\omega_n (\cos \theta - 1) - 2\Delta \sin \theta + \frac{\Delta^2}{\omega_n} \right\} + N_0 S \int dx \Delta^2 \ln \frac{T}{T_c}. \quad (19)$$

Equations (16)–(18) are numerically solved for a long bridge using Newton's method with the boundary conditions $\theta = \theta_\infty$ at $x = \pm 15\xi_{T_c}$ ($\xi_{T_c} = \sqrt{\hbar D / k_B T_c}$), where θ_∞ is the solution of the uniform Usadel equation

$$-\left(\omega_n + \frac{D}{2\hbar} q_s^2 \cos \theta_\infty \right) \sin \theta_\infty + \Delta_\infty \cos \theta_\infty = 0. \quad (20)$$

The search for the saddle state is performed in a similar way as that based on the GL theory, with the only difference that we fix the ratio $\sin \theta(0) / \sin \theta_\infty$ at $x = 0$ instead of the magnitude of the order parameter. The dependence $\delta F_{\text{thr}}(I)$ is shown in figure 4. It can be seen that for a dirty long bridge the current dependence of δF_{thr} remains close to the dependence described by equation (1). $\delta F_{\text{thr}}(0) \simeq \delta F_{\text{LA}}(0)$ (see figure 5) for a broad range of temperatures below T_c and if one uses for $I_{\text{dep}}(T)$ the result from microscopic calculations and not the GL depairing current. In the clean limit the deviation is stronger, reaching about 15% for $\delta F_{\text{thr}}(0)$ as $T \rightarrow 0$ [5].

Alongside the case of long bridges we also study short bridges ($L \ll \xi(T)$) in the dirty limit. In this case we can neglect non-gradient terms in (16) inside the bridge, as done by Kulik and Omelyanchuk [17], and we obtain the following equation:

$$\frac{\hbar D}{2} \frac{d^2 \theta^C}{dx^2} - \frac{D}{2\hbar} q_s^2 \cos \theta^C \sin \theta = 0, \quad (21)$$

where θ^C defines θ inside the bridge. In [17] the solution of this equation was found together with the current–phase relation $I(\phi)$

$$I(\phi) = \frac{4\pi k_B T}{e R_N} \sum_{\omega_n > 0} \frac{\Delta_\infty \cos \frac{\phi}{2}}{\delta} \arctan \frac{\Delta_\infty \sin \frac{\phi}{2}}{\delta}, \quad (22)$$

where $\delta = \sqrt{(\Delta_\infty \cos \phi / 2)^2 + \omega_n^2}$ and ϕ is the phase difference across the bridge. In (22) for each current there are two values of ϕ corresponding to two different states—the smaller ϕ corresponds to the ground state and the larger ϕ corresponds to the saddle state. The strategy for finding δF_{thr} is as follows: for fixed current we find two values of ϕ , then with these ϕ we use the analytical solution from [17] for θ^C , while for θ outside the bridge we numerically solve equations (16) and (17), neglecting the pair breaking effect of the current/supercurrent in the banks (which is applicable when the cross section of the banks $S_{\text{pad}} \gg S$). Solutions in the bridge and in the banks are matched using the boundary conditions

$$\begin{aligned} \frac{d\theta^{L,R}}{dx} \Big|_{\pm \frac{L}{2}} &= \frac{S}{S_{\text{pad}}} \frac{d\theta^C}{dx} \Big|_{\pm \frac{L}{2}} \\ &= \frac{S}{S_{\text{pad}}} \frac{2}{L} \frac{\omega_n \sin \frac{\phi}{2} \arctan \frac{\Delta_\infty \sin \frac{\phi}{2}}{\delta}}{\delta}, \end{aligned} \quad (23)$$

$$\theta^L|_{-\frac{L}{2}} = \theta^C|_{-\frac{L}{2}} = \theta^C|_{\frac{L}{2}} = \theta^R|_{\frac{L}{2}}, \quad (24)$$

$$\theta^L|_{-\frac{L_{\text{sys}}}{2}} = \theta^R|_{\frac{L_{\text{sys}}}{2}} = \theta_\infty, \quad (25)$$

where θ^L, θ^R are the functions θ in the left bank and right bank, respectively. Here $L_{\text{sys}} = 40\xi_{T_c} + L$ is the length of the modeled system, including the bridge (with length L) and the banks with cross section S_{pad} and length $(L_{\text{sys}} - L)/2$ which are in contact with much wider banks where θ is equal to its value at the given temperature and zero current. The above conditions appear from the conservation law for spectral currents [18] and is similar to the boundary conditions (8).

The calculated $\delta F_{\text{thr}}(I)$ are shown in figure 6. For $L \ll \xi(T)$ ($L = 0.2\xi_{T_c}$) the power-3/2 law is approximately valid at all temperatures (note the noticeable difference at $I \gtrsim 0.8I_c$ for $T = 0.5T_c$ and $T = 0.05T_c$) while for a bridge with $L = 0.6\xi_{T_c}$ the condition $L \ll \xi(T)$ is not applicable at low temperatures, which leads to stronger deviation from the power-3/2 law in a wide range of currents near I_c . Note that $\delta F_{\text{thr}}(0) \simeq \hbar I_c / e$ (see inset in 6) has its largest deviation at low temperatures.

Finally, in the dirty limit we find how $\delta F_{\text{thr}}(0)$ depends on the length of the bridge. Earlier, in [3] we claimed that the dependence $\delta F_{\text{thr}}(0, L)$ may have a minimum at $L \simeq 2 - 3\xi(T)$ for a proper choice of bank and bridge widths. We performed calculations (to determine the saddle state, the condition $\theta(x = 0, y) = 0$ is added) using the two-dimensional Usadel equation in the the same geometry as in [3] (see figure 4 there) and the same geometrical parameters but we did not find a minimum (see figure 7). Instead $\delta F_{\text{thr}}(0)$ monotonically increases as L decreases following increase of

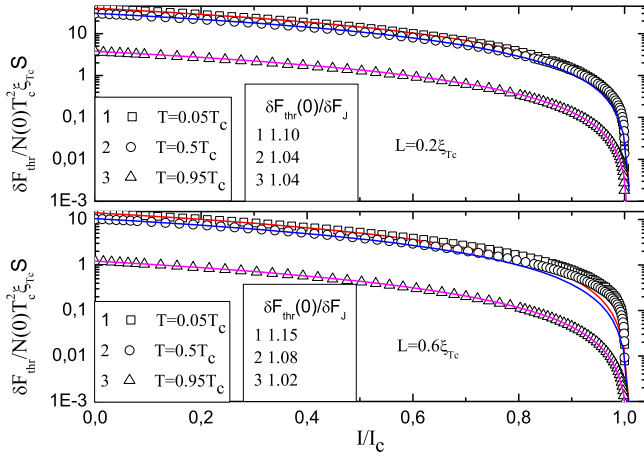


Figure 6. Current dependence of the energy of threshold fluctuation for short bridges ($L = 0.2\xi_{T_c}$ and $L = 0.6\xi_{T_c}$) at different temperatures. The functions $\delta F_{\text{thr}}(I=0)(1 - I/I_c)^{3/2}$ are shown by the solid lines. Here $\delta F_J = \hbar I_c/e$.

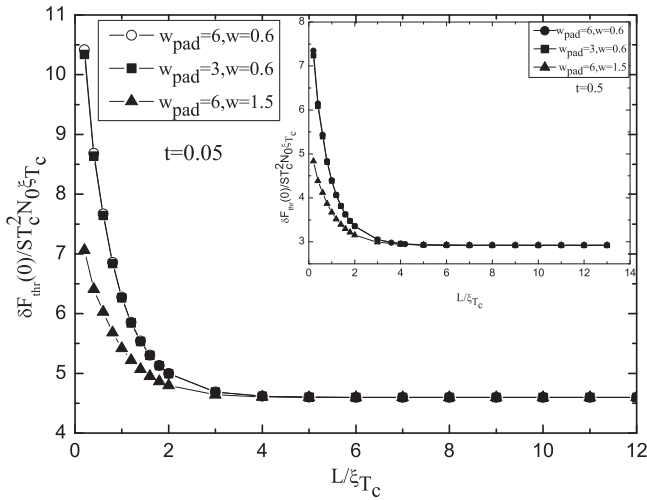


Figure 7. Energy of threshold fluctuation versus the length of the bridge at zero current for different bridge and bank widths. In the main graph results for the temperature $T = 0.05T_c$ are shown, while the inset shows results for $T = 0.5T_c$.

I_c . This result forced us to check our calculations made in the framework of the GL model [3] and we found that this result is an artifact of the grid approximation used. With a proper grid we can confirm the absence of a minimum in the dependence $\delta F_{\text{thr}}(L)$ in the GL model too.

4. Discussion

We demonstrate that functional dependence of the energy of threshold fluctuation (perturbation) on current following from the GL model remains valid at temperatures well below T_c in both dirty and clean limits if one uses the actual critical (depairing) current but not the GL depairing current. This result gives us hope that the strong effect of even a relatively

‘weak’ defect (which does not strongly suppress the critical current of the bridge and provide $I_c \simeq I_{\text{dep}}$) on the dependence $\delta F_{\text{thr}}(I)$ that was found at $T \sim T_c$ is temperature independent and could also be applicable at low temperatures.

Our results could be used for qualitative explanation of the dependence $\delta F_{\text{thr}}(I) \sim (1 - I/I_c)^{3/2}$ found in [10, 11] for long bridges/wires as due to the presence of intrinsic defects in their samples. Unfortunately we are not able to make a quantitative comparison due to the lack of important parameters (resistivity and diffusion coefficient of the bridges/wires, their width and thickness) which are needed to see how far the actual critical current of the bridge is from the depairing current. An alternative explanation of the experiments is based on a model of the bridge/wire as chain of weakly connected, via Josephson coupling, granules [13] which naturally leads to a power-3/2 law, but it is not clear how this model could be applicable to the results in [10, 11].

5. Conclusion

We calculate the energy of threshold fluctuation which switches the current-carrying superconducting bridge to the resistive state. We make calculations at arbitrary temperature, for different bridge lengths and in the presence of defects connected with local variation of T_c , mean path length ℓ or cross section of the superconductor. It is found that the presence of defects has strong influence on the form of current dependence of the energy of threshold fluctuation, changing it from $\delta F(I) \simeq (1 - I/I_{\text{dep}})^{5/4}$ valid for long defectless bridge to $\delta F(I) \simeq (1 - I/I_c)^{3/2}$ which is typical for short bridges and Josephson junctions. Additionally, using microscopic theory we show that the results, obtained on the basis of GL theory, stay valid at temperatures significantly below T_c if one uses a proper temperature-dependent critical (depairing) current.

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