



ADIABATIC BLACK HOLE GROWTH IN SÉRSIC MODELS OF ELLIPTICAL GALAXIES

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ABSTRACT

We have examined the effect of slow growth of a central black hole on spherical galaxies that obey Sérsic or $R^{1/m}$ surface-brightness profiles. During such growth the actions of each stellar orbit are conserved, which allows us to compute the final distribution function (DF) if we assume that the initial DF is isotropic. We find that black hole growth leads to a central cusp or “excess light,” in which the surface brightness varies with radius as $R^{-1.3}$ (with a weak dependence on Sérsic index m), the line-of-sight velocity dispersion varies as $R^{-1/2}$, and the velocity anisotropy is $\beta \simeq -0.24$ to -0.28 depending on m . The excess stellar mass in the cusp scales approximately linearly with the black hole mass, and is typically 0.5–0.85 times the black hole mass. This process may strongly influence the structure of nuclear star clusters in spheroidal galaxies if they contain black holes.

Key words: galaxies: evolution – galaxies: kinematics and dynamics – galaxies: nuclei – Galaxy: center

1. INTRODUCTION

The surface-brightness profiles of most elliptical galaxies are remarkably well-fit by the empirical Sérsic formula given by Equation (1)—see Sérsic (1968), Caon et al. (1993), D’Onofrio et al. (1994), and the references in Kormendy et al. (2009). This is a fitting formula containing three parameters, usually taken to be the brightness I_0 , the effective radius R_e —the projected radius containing half of the total luminosity—and the shape parameter or Sérsic index m , which controls the overall curvature of the profile and quantifies the central concentration of the galaxy (Graham et al. 2001; Trujillo et al. 2001).

The robustness of the Sérsic fit has prompted many astronomers to attribute deviations from it to the presence of additional components or physical processes occurring late in the lifetime of the galaxy (Graham 2013; Kormendy & Ho 2013 and references therein), rather than to shortcomings in the empirical fitting formula. To illustrate this, we have reproduced two surface-brightness profiles from Kormendy et al. (2009) in Figure 1. The figure shows a deviation at the center. These deviations are of two kinds: either “excess light” or “missing light” near the centers compared to the inward extrapolation of the Sérsic profile. Excess-light or “power-law” ellipticals generally have lower luminosities ($M_V \gtrsim -21.5$) than missing-light or “core” ellipticals ($M_V \lesssim -21.7$). The excess-light ellipticals are also found to have disk isophotes and Sérsic indices $m < 4$, while core ellipticals have boxy isophotes and indices $m > 4$.

The division of ellipticals into these two classes, and the explanations given above for this dichotomy, date back at least to Faber et al. (1997). The formation of a core is generally attributed to the dynamical excitation of stellar orbits due to a binary black hole (hereafter BH) (Faber et al. 1997; Milosavljević & Merritt 2001; Boylan-Kolchin et al. 2004; Merritt 2006), although there is little or no direct evidence for this hypothesis. Excess light is generally attributed to late star formation from gas that has become concentrated near the galaxy center during mergers (Mihos & Hernquist 1994; Hopkins et al. 2009a).

Excess light in elliptical galaxies is usually distinguished from nuclear star clusters in spheroidal galaxies. Spheroidal galaxies occupy an elongated locus perpendicular to the main

elliptical locus in the μ_e – r_e (surface brightness versus effective radius) diagram (Wirth & Gallagher 1984; Bender et al. 1992; Kormendy et al. 2009). It is likely that different formation mechanisms are responsible for the central density profiles in ellipticals and spheroidals, a conclusion supported by the success of merger simulations in producing elliptical—but not spheroidal—scaling relations (Boylan-Kolchin et al. 2006; Robertson et al. 2006; Hopkins et al. 2009b), indicating that spheroidals are not produced by mergers.

In this work we assume that the central BHs in elliptical and spheroidal galaxies form slowly by the accretion of gas from larger radii. In this case the surrounding stellar system will contract as the central mass grows, because the stellar orbits conserve their adiabatic invariants or actions. If the galaxy began with a Sérsic surface-brightness profile, this process will naturally produce excess light relative to the Sérsic profile near the center of the galaxy, and the corresponding excess stellar mass will be simply related to the mass of the BH and the properties of the initial Sérsic profile. The main goal of the paper is to work out these relations and to compare the predictions of this simple model to the observations.

Pioneering studies of this process were carried out by Peebles (1972) and Young (1980). These studies assumed that the galaxies were spherical, as do we, but in contrast they assumed that the stellar distribution function (DF) near the center of the galaxy was Maxwellian before the formation of the BH. They found that as the BH grew the stellar DF developed a cusp in which the density varied as $\rho \sim r^{-3/2}$, corresponding to a surface brightness $I(R) \sim R^{-1/2}$. However, a Sérsic profile does not have a Maxwellian DF, and the properties of the cusp formed by the adiabatic growth of a central BH depend strongly on the DF (Quinlan et al. 1995). In general we expect (and shall find) the cusp formed in an initial Sérsic model to be steeper than the one found by Peebles and Young.

In Section 2 we review the properties of Sérsic models and derive the dynamical quantities of interest (density, potential, DF, velocity moments) assuming spherical symmetry, constant mass-to-light ratio Υ , and isotropic velocity dispersion. We then slowly add a central point mass representing the BH and evolve the system under the assumption of adiabatic invariance

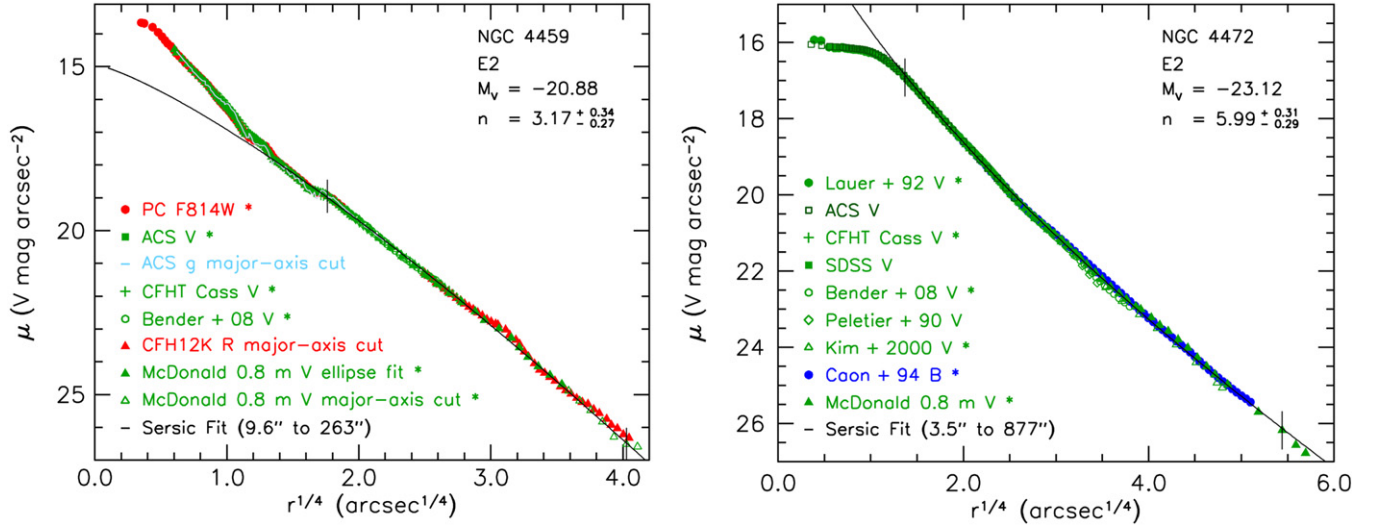


Figure 1. Surface brightness profile of the dwarf galaxy NGC 4459, an excess light elliptical with best-fitting Sérsic profile with index $m = 3.17$ (left) and the giant galaxy NGC 4472, a missing light elliptical with best-fitting Sérsic index $m = 5.99$ (right; Kormendy et al. 2009). Reproduced by permission of the AAS.

Table 1

The Dimensionless Parameters b , $|\tilde{\Phi}_m(0)|$ and $\tilde{M}_m$ for Different Values of the Sérsic Index m

m	b	$ \tilde{\Phi}_m(0) $	$\tilde{M}_m$
2	3.67	4.17×10^{-2}	2.91×10^{-2}
3	5.67	1.04×10^{-2}	5.41×10^{-3}
4	7.66	2.21×10^{-3}	8.42×10^{-4}
5	9.66	4.52×10^{-4}	1.27×10^{-4}
7	13.66	1.80×10^{-5}	2.74×10^{-5}
9	17.66	6.88×10^{-7}	5.68×10^{-8}
11	21.66	2.57×10^{-8}	1.17×10^{-9}

of the stellar orbits. The numerical methods are described in Section 3. The surface-brightness distribution, anisotropy parameter, excess mass, and line-of-sight velocity-dispersion distribution of the adiabatic models are described in Section 4 and 5, and the observations in Section 6. The conclusions are in Section 7.

2. INITIAL SET UP

2.1. Density

The surface-brightness distribution of a spherical galaxy is modeled well by the Sérsic law (Sérsic 1963, 1968; Ciotti 1991)

$$I(\eta) = I_0 \exp(-b \eta^{1/m}) \quad \text{where} \quad \eta = R/R_e \quad (1)$$

and m is the Sérsic index. Here R_e is the effective radius, the radius on the sky that contains half the total light. This definition of R_e requires that the parameter b is given by

$$2\gamma(2m, b) = \Gamma(2m) \quad (2)$$

where γ is the incomplete gamma function. The function $b(m)$ is well fitted by the linear interpolation $b(m) = 2m - 0.324$, with relative error less than 0.001 for $0.5 < m < 10$ (Ciotti 1991; Prugniel & Simien 1997). The exact values of $b(m)$ for several values of m are given in Table 1. As $m \rightarrow \infty$,

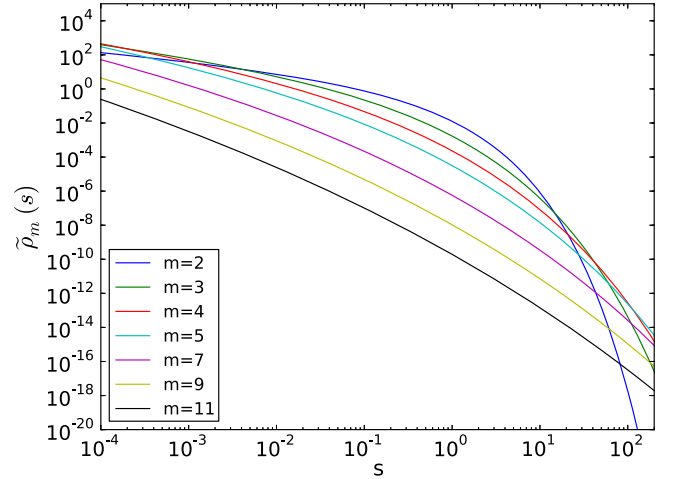


Figure 2. Dimensionless density $\tilde{\rho}_m(s)$ (Equation (4)) for $2 \leq m \leq 11$. Here $s = r/R_e$, the ratio of the radius to the effective radius (the same as Figure 1 from Ciotti 1991).

$b/m \rightarrow 2$ and $I(R) \sim R^{-2}$ (Kormendy & Djorgovski 1989) over a wide range of radii around R_e .

The luminosity density for given surface brightness $I(R)$ can be obtained by the Abel integral

$$j(r) = -\frac{1}{\pi} \int_r^\infty \frac{dR}{\sqrt{R^2 - r^2}} \frac{dI}{dR}. \quad (3)$$

If we assume that the galaxy has constant mass-to-light ratio Υ , then its mass density $\rho(r) = \Upsilon j(r)$. Substituting Equation (1) in (3) and introducing the dimensionless radius $s = r/R_e$, we get

$$\begin{aligned} \rho_m(s) &= \Upsilon \frac{I_0}{R_e} \tilde{\rho}_m(s) = \frac{\Sigma_0}{R_e} \tilde{\rho}_m(s) \quad \text{where} \quad \Sigma_0 = \Upsilon I_0, \\ \tilde{\rho}_m(s) &= \frac{b}{\pi m s^{(m-1)/m}} \int_0^1 \frac{\exp[-b(s/x)^{1/m}]}{x^{1/m}} \frac{dx}{\sqrt{1-x^2}} \end{aligned} \quad (4)$$

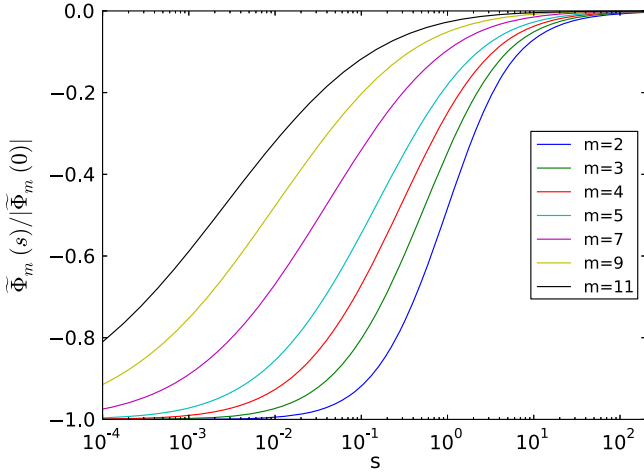


Figure 3. Normalized potential $\tilde{\Phi}_m(s)/|\tilde{\Phi}_m(0)|$ (Equations (8) and (10)) for $2 \leq m \leq 11$. Here $s = r/R_e$, the ratio of the radius to the effective radius. A similar plot is given by Ciotti (1991), Figure 2.

where $x = s/\eta$. For $m > 1$ the dimensionless density $\tilde{\rho}_m(s)$ diverges at the origin as

$$\tilde{\rho}_m(s) \sim \frac{b}{2\pi m} \frac{1}{s^{(m-1)/m}} B\left[\frac{1}{2}, \frac{1}{2}(1 - 1/m)\right] \quad (5)$$

where B is the beta function. The dimensionless density $\tilde{\rho}_m(s)$ is plotted in Figure 2 for several values of m .

2.2. Potential

The potential is obtained by solving the Poisson equation

$$\nabla^2 \Phi = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\Phi}{dr} \right) = 4\pi G \rho. \quad (6)$$

If we define $\Phi_m(r) = 4\pi G \Sigma_0 R_e \tilde{\Phi}_m(s)$, then in dimensionless variables we have

$$\frac{1}{s^2} \frac{d}{ds} \left(s^2 \frac{d\tilde{\Phi}_m}{ds} \right) = \tilde{\rho}_m. \quad (7)$$

In integral form,

$$\tilde{\Phi}_m(s) = - \int_s^\infty \frac{\tilde{M}_m(s')}{s'^2} ds' \quad \text{where} \quad \tilde{M}_m(s) = \int_0^s \tilde{\rho}_m(s') s'^2 ds' \quad (8)$$

is the mass of the system measured in units of $4\pi \Sigma_0 R_e^2$. One cannot solve the integral in general for the density given by Equation (4) but we can obtain the central potential (Ciotti 1991), which is useful in what follows:

$$\Phi(0) = -4G\Upsilon \int_0^\infty dR I(R) = -4\pi G \int_0^\infty r \rho(r) dr \quad (9)$$

from which the dimensionless central potential is found to be:

$$\tilde{\Phi}_m(0) = - \frac{\Gamma(m+1)}{\pi b^m}. \quad (10)$$

Thus the central potential is finite even though the density diverges at the center for $m > 1$. The values of the dimensionless central potential and dimensionless mass for several values of the Sérsic index m are given in Table 1.

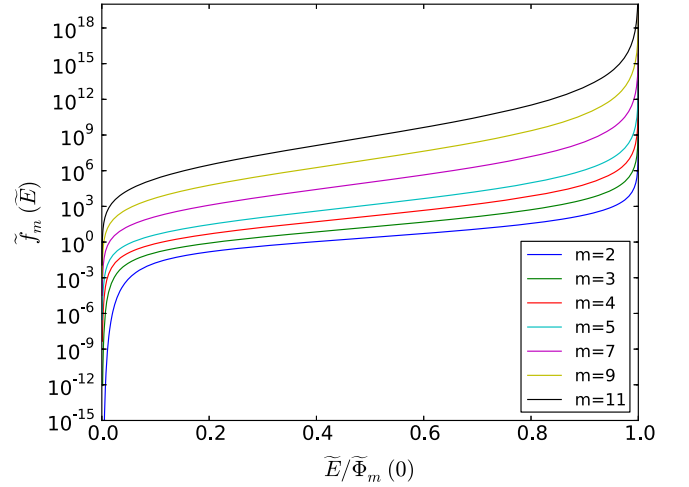


Figure 4. DF $\tilde{f}_m(\tilde{E})$ as a function of normalized energy $\tilde{E}/\tilde{\Phi}_m(0)$ for $2 \leq m \leq 11$. A similar plot is given by Ciotti (1991), Figure 3.

Poisson's equation (7) has to be solved numerically with the initial condition (10), taking $d\tilde{\Phi}_m/ds = 0$ at $s = 0$. Plots of the potential for different values of m are shown in Figure 3.

2.3. Distribution Function

We shall assume for simplicity that the initial phase-space DF of the stars in the galaxy is ergodic³, that is, the DF depends only on the energy E and therefore the velocity distribution is isotropic at every point in space. In such a case, the DF can be calculated from the known density and potential of the galaxy as follows:

$$f(E) = \frac{1}{\sqrt{8} \pi^2} \int_E^0 \frac{d\Phi}{\sqrt{\Phi - E}} \frac{d^2 \rho}{d\Phi^2}. \quad (11)$$

Using the dimensionless potential and density, we can write the DF in dimensionless form as

$$f_m(E) = \frac{1}{(4\pi G)^{3/2} (\Sigma_0 R_e^5)^{1/2}} \tilde{f}_m(\tilde{E}) \quad \text{where} \quad \tilde{f}_m(\tilde{E}) = \frac{1}{\sqrt{8} \pi^2} \int_{\tilde{E}}^0 \frac{d\tilde{\Phi}}{\sqrt{\tilde{\Phi} - \tilde{E}}} \frac{d^2 \tilde{\rho}_m}{d\tilde{\Phi}^2}. \quad (12)$$

The DF is plotted for various values of m in Figure 4. One can see that for all values of m the DF is positive, and that it diverges as $E \rightarrow \Phi(0)$; it is straightforward to show that

$$\tilde{f}_m(\tilde{E}) \sim [\tilde{E} - \tilde{\Phi}_m(0)]^{-\alpha}, \quad \alpha = \frac{5m+1}{2(m+1)}. \quad (13)$$

2.4. Velocity Moments

For the remainder of this paper we will drop the tildes on E , Φ , ρ , etc., and it should be understood that these quantities are

³ Ergodic systems in statistical mechanics are those that uniformly explore the constant energy surface in phase space, that is, the time spent by the system in a region of phase space with fixed energy is proportional to the volume of that region. In stellar dynamics, the motion of individual stars in a spherical potential is not ergodic but the DF is said to be ergodic if it is uniform on an energy surface. The usage follows Binney & Tremaine (2008).

dimensionless. According to Jeans's theorem, a stationary spherical DF can only depend on the energy $E = \frac{1}{2}v_r^2 + \frac{1}{2}v_t^2 + \Phi(s)$ and the angular momentum $L = s v_t$, where v_r and v_t are the radial and tangential velocities. We can obtain the density, and the radial and tangential velocity dispersions for this DF as

$$\rho(s) = 4\pi \int_{\Phi(s)}^0 dE [2(E - \Phi)]^{1/2} \int_0^1 \frac{dy y f(E, L_{\max} y)}{\sqrt{1 - y^2}}, \quad (14)$$

$$\overline{v_r^2}(s) = \frac{4\pi}{\rho(s)} \int_{\Phi(s)}^0 dE [2(E - \Phi)]^{3/2} \times \int_0^1 dy y \sqrt{1 - y^2} f(E, L_{\max} y), \quad (15)$$

$$\overline{v_t^2}(s) = \frac{4\pi}{\rho(s)} \int_{\Phi(s)}^0 dE [2(E - \Phi)]^{3/2} \int_0^1 \frac{dy y^3 f(E, L_{\max} y)}{\sqrt{1 - y^2}}, \quad (16)$$

where

$$L_{\max}(E, s) = \{2 s^2 [E - \Phi(s)]\}^{1/2}, \quad y = L/L_{\max}; \quad (17)$$

$L_{\max}$ is the maximum angular momentum for an orbit of energy E that passes through radius s . Note that the velocity dispersions in the θ and ϕ directions are $\overline{v_\phi^2} = \overline{v_\theta^2} = \frac{1}{2}\overline{v_t^2}$. For numerical work, the inner integral over y can be evaluated using Gauss–Chebyshev quadrature and the outer integral over E by Gauss–Legendre quadrature.

We then project the density and velocity moments on the plane of sky to obtain the surface density

$$\Sigma(\eta) = 2 \int_\eta^\infty \frac{s \rho(s) ds}{\sqrt{s^2 - \eta^2}} \quad (18)$$

and the line-of-sight velocity dispersion

$$\sigma_\parallel^2(\eta) = \frac{2}{\Sigma(\eta)} \int_\eta^\infty \frac{s \rho(s) ds}{\sqrt{s^2 - \eta^2}} \left[\left(1 - \frac{\eta^2}{s^2}\right) \overline{v_r^2} + \frac{\eta^2}{2s^2} \overline{v_t^2} \right]. \quad (19)$$

The anisotropy parameter is

$$\beta(s) = 1 - \frac{\overline{v_r^2}(s)}{2 \overline{v_t^2}(s)} \quad (20)$$

which is 0 for an isotropic velocity distribution, $-\infty$ for circular orbits and $+1$ for radial orbits.

The initial Sérsic models are ergodic, so the velocity distribution is isotropic, $\beta = 0$, and the DF depends only on energy, not angular momentum. The velocity dispersion $\overline{v_r^2}(s) = \frac{1}{2}\overline{v_t^2}(s)$ is shown in Figure 5.

2.5. Actions

The radial and azimuthal actions are

$$I_r = 2 \int_{s_-}^{s_+} \sqrt{2[E - \Phi(s)] - L^2/s^2} ds \quad I_t = 2\pi L, \quad (21)$$

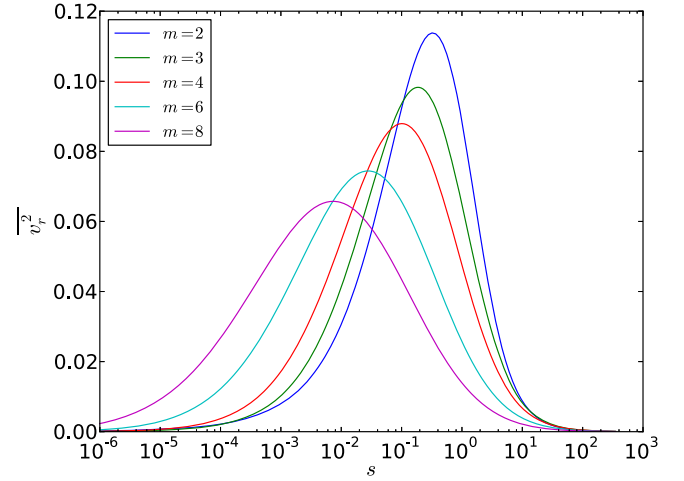


Figure 5. Initial isotropic velocity dispersion of the galaxy for varying Sérsic index m . A similar plot is given by Ciotti (1991), Figure 2.

where s_- and s_+ are the pericenter and apocenter distances of the orbit, given implicitly by

$$2 s_\pm^2 [E - \Phi(s_\pm)] - L^2 = 0. \quad (22)$$

Since actions are conserved during the adiabatic growth of the central BH, the DF is also conserved at each point in action space. In terms of energy and angular momentum, a star that is initially at (E, L) in phase space when the potential is $\Phi(s)$ will move to a new phase point (E^*, L) when the potential changes to $\Phi^*(s)$. By equating the initial and final actions, we find the relation $E = E(E^*, L)$. Then the new DF $f^*(E^*, L)$ is related to the original DF $f(E, L)$ by $f^*(E^*, L) = f[E(E^*, L), L]$.

3. NUMERICAL METHODS

The density, DF, potential and other galaxy properties are evaluated on a grid: $\rho_i = \rho(s_i)$, $\Phi_i = \Phi(s_i)$, $f_{jk} = f(E_j, L_k)$, and so on. The radial grid is equally spaced in $\log(s)$ with roughly 200 points between $s_{\min} = 10^{-6}$ and $s_{\max} = 2 \times 10^2$. The energy grid points E_j are chosen to match the potential Φ_i at the radial grid points; since $\Phi(r)$ varies as the BH grows, the locations of the energy grid points E_j also vary. The grid points for angular momentum are spaced uniformly between $x = 0$ and $x = 1$ where $x = L/L_c(E)$ and $L_c(E)$ is the angular momentum of a circular orbit at the given energy. Typically we use 200 grid points in energy and 20 in angular momentum.

Following Young (1980), the program executes the following steps for a given Sérsic index m :

1. The density $\rho_i = \rho(s_i)$ is obtained from Equation (4) at all points on the radial grid (recall that we have dropped the tildes on all dimensionless quantities).
2. The gravitational potential $\Phi_i = \Phi(s_i)$ is obtained from Poisson's Equation (8) at all points on the grid.
3. The initial DF $f_{jk} = f(E_j)$ is obtained from ρ_i and Φ_i using Equation (12). The initial values f_{jk} are independent of the angular momentum $x_k L_c(E_j)$ since the initial DF is ergodic.
4. The radial action $I_{jk} = I_r(E_j, x_k)$ is calculated for the potential $\Phi(s)$ at each point on the energy-angular momentum grid $[E_j, x_k L_c(E_j)]$, using Equation (21).

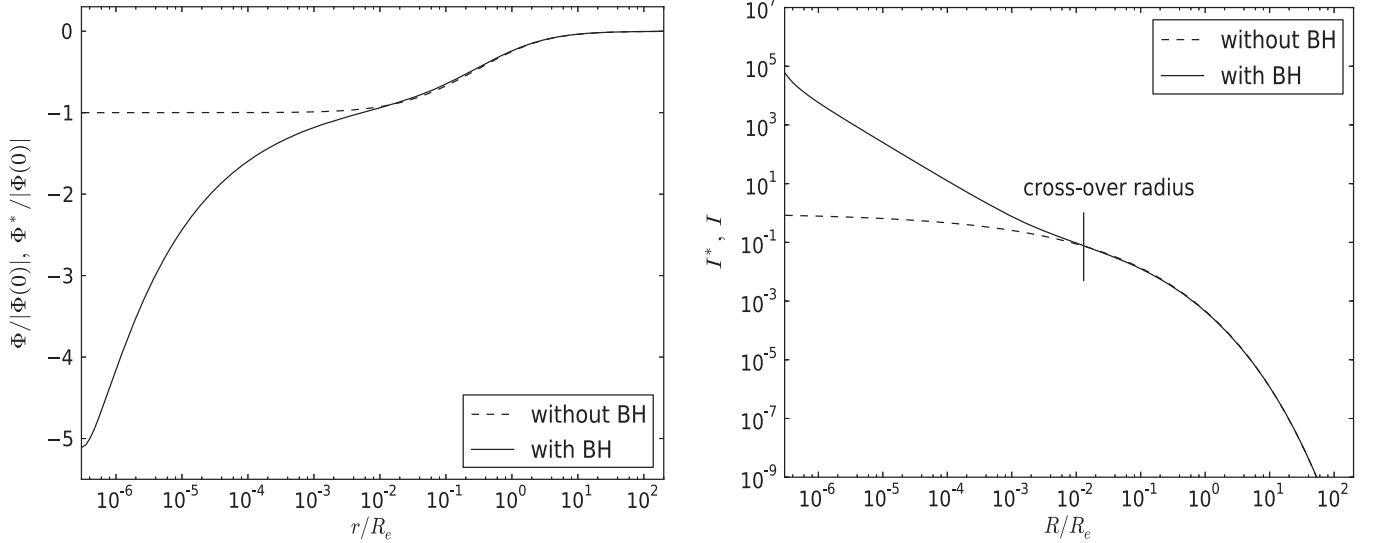


Figure 6. Typical potential and surface density of a galaxy with and without a BH. The Sérsic index $m = 4$ and the black hole mass relative to the galaxy mass is $M_*/M_G = 1.877 \times 10^{-3}$.

5. A BH of mass M_* is added at the center of the galaxy, thereby modifying the potential to $\Phi^*(s) = \Phi(s) - M_*/s$.
6. In this new potential we compute the angular momentum of circular orbits as a function of energy, $L_c^*(E^*)$.
7. A new energy-angular momentum grid $[E_j^*, x_k L_c^*(E_j^*)]$ is defined, in which the new energies E_j^* match the new potentials $\Phi_j^* = \Phi^*(s_j)$ at the radial grid points. The new action $I_{jk}^* = I_r^*(E_j^*, x_k)$ is computed on the new energy-angular momentum grid $[E_j^*, x_k L_c^*(E_j^*)]$.
8. Since actions are conserved, a star at (E, x) evolves during the growth of the BH to (E^*, x^*) where $I_r^*(E^*, x^*) = I_r(E, x)$, $x^* L_c^*(E^*) = x L_c(E)$. We solve for the original (E, x) at each point on the new energy-angular momentum grid $[E_j^*, x_k L_c^*(E_j^*)]$ and then compute the new DF as $f^*(E^*, x^* L_c^*) = f[E(E^*, x^* L_c^*)]$.
9. Using the DF f^* and potential Φ^* we compute the resulting density ρ^{**} from Equation (14) and the potential Φ^{**} from Poisson's Equation (8). We then replace ρ^* and Φ^* by ρ^{**} and Φ^{**} and return to step 6.

After about 20 iterations this algorithm converges, yielding a DF for the stellar system from which quantities of interest like density, surface brightness profile, and velocity dispersion can be computed.

4. EXCESS MASS

We wish to quantify the excess light or stellar mass accumulated near the center of the galaxy as the result of the adiabatic adjustment of the stellar orbits to the growth of the BH. Since the excess has appeared at the center only because of the rearrangement of the stars, the total luminosity of the galaxy is unchanged: the density goes up substantially close to the center, and decreases by a much smaller amount at large radii. The definition of “excess mass” is therefore somewhat arbitrary, but most reasonable definitions will give very similar values. We define the “cross-over radius” as the value of the dimensionless radius $\eta = R/R_e$ at which the initial and final surface-density profiles cross (see Figure 6). Let $\Sigma(\eta)$ and $\Sigma^*(\eta)$ be the dimensionless initial and final surface-density profiles, then η_c is

defined by the equality $\Sigma^*(\eta_c) = \Sigma(\eta_c)$; we have verified that this solution exists and is unique. The excess mass is defined as

$$M_e = \frac{1}{2} \int_0^{\eta_c} \eta [\Sigma^*(\eta) - \Sigma(\eta)] d\eta. \quad (23)$$

Note that this definition cannot strictly be applied to observed galaxies, where we do not know the initial profile so different procedures must be used to define the excess mass (cf. Faber et al. 1997; Graham 2004; Merritt 2004).

The factor $\frac{1}{2}$ appears in the above equation so that the dimensional definition of the excess mass is $M_e = 2\pi \int_0^{R_c} [\Sigma^*(R) - \Sigma(R)] R dR$ (cf. Equation (8))—in other words it arises because the volume element is $4\pi s^2 ds$ but the area element is $2\pi \eta d\eta$.

5. RESULTS AND DISCUSSION

The surface-brightness profile $I(R)$ and anisotropy parameter $\beta(r)$ are plotted in Figure 7 for Sérsic index $m = 2.3, 3, 6$ and 9 for dimensionless BH masses varying from 10^{-5} to 5×10^{-3} . The BH masses are chosen such that they are 10^{-4} to few times 10^{-2} of the galaxy mass (M_G).

5.1. Surface-density Cusp

The prominence of the surface-brightness or surface-density cusps shown in Figure 7 varies with the Sérsic index m . For smaller m the cusp is more prominent, partly because the initial surface-brightness profile near the center is flatter. For large m , even with the largest BH masses, the cusp caused by the BH is hardly noticeable (see $m = 9$ case in Figure 7).

One can derive the approximate behavior of the surface-density cusp analytically (Quinlan et al. 1995; Gondolo & Silk 1999)

$$\rho^* \sim r^{-\Gamma}, \quad \Sigma^* \sim R^{1-\Gamma}, \quad \Gamma = 2 + \frac{1}{4-\gamma} = \frac{7m+2}{3m+1}. \quad (24)$$

where $\gamma = (m-1)/m$ is the exponent of the initial cusp (see Equation (5)). For the Sérsic index $m = 2.3$, $\rho^* \sim r^{-2.29}$ and

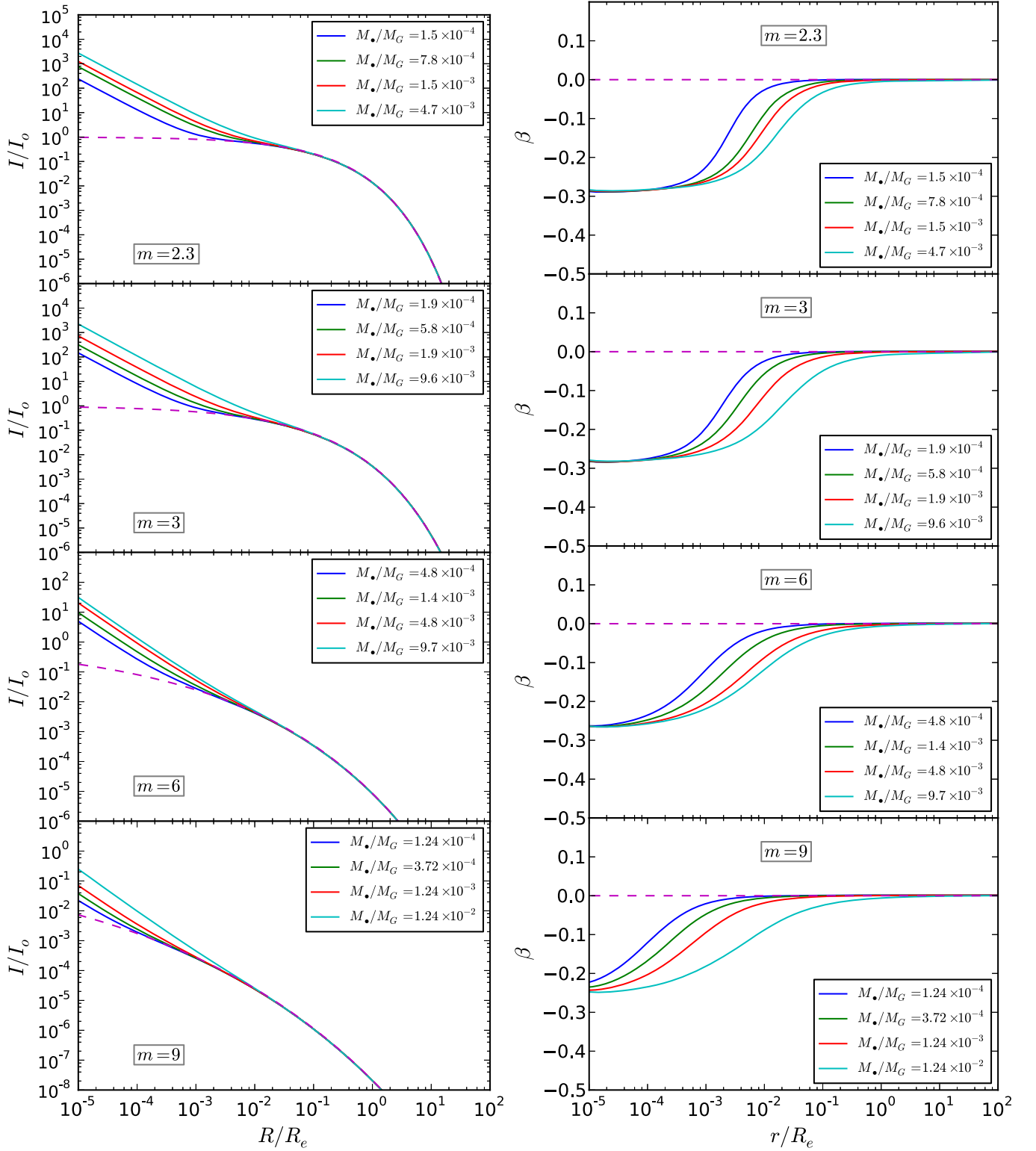


Figure 7. Surface-brightness profile (left) and anisotropy parameter (right) after the adiabatic growth of a BH in a galaxy with Sérsic index $m = 2.3$, $m = 3$, $m = 6$, and $m = 9$.

$\Sigma^* \sim R^{-1.29}$ and for $m = 9$, $\rho^* \sim r^{-2.32}$ and $\Sigma^* \sim R^{-1.32}$. We see that the exponent of the central surface-density cusp does not vary strongly with Sérsic index. Thus the cusp becomes less prominent as m increases and the initial profile becomes more cuspy.

5.2. Anisotropy and Velocity Cusp

The line-of-sight velocity dispersion (Equation (19)) is shown in Figure 8. Both the radial and tangential dispersions $\overline{v_r^2}$ and $\overline{v_t^2}$ scale as r^{-1} near the BH and this scaling is evident in Figure 8. An explicit formula giving the relation between the

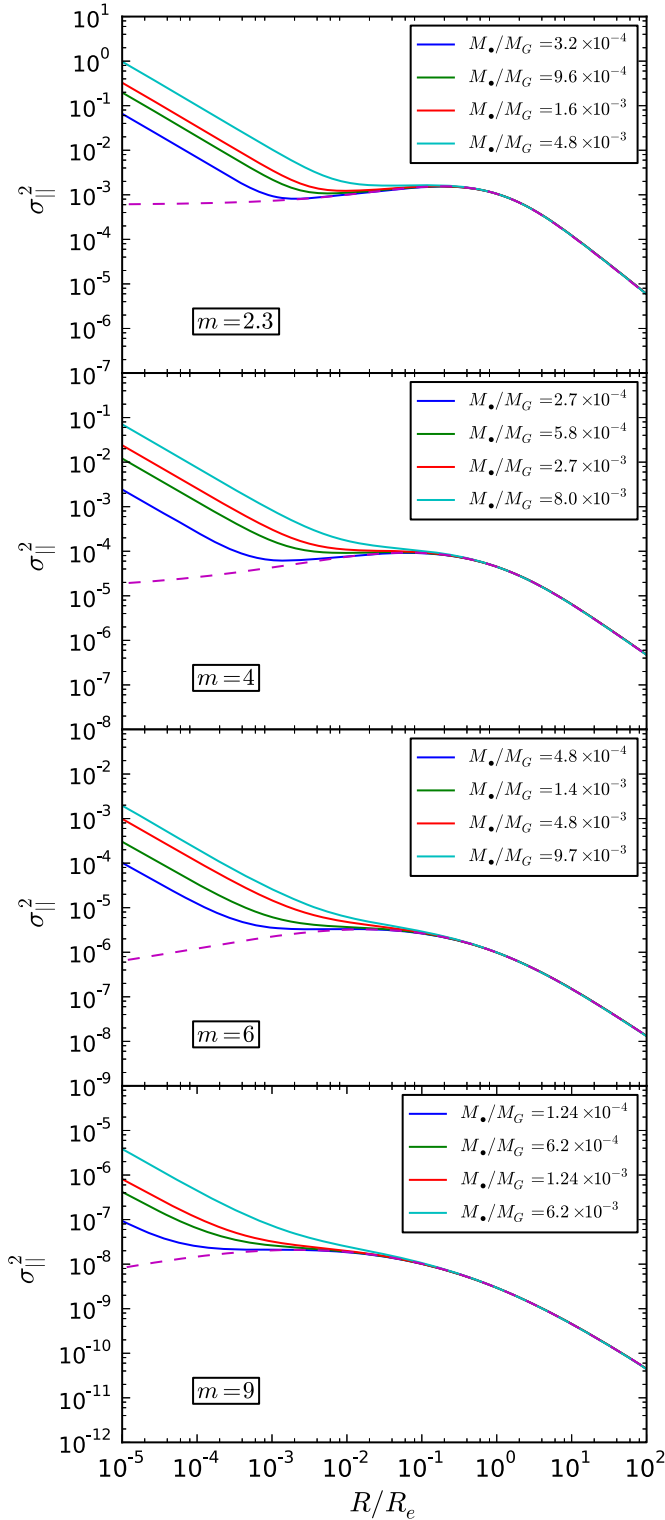


Figure 8. Line-of-sight velocity dispersion after the adiabatic growth of a BH in a galaxy with Sérsic index $m = 2.3, m = 4, m = 6, m = 9$. The dashed line is for the initial model without a BH.

line-of-sight velocity dispersion and BH mass as $r \rightarrow 0$ is given in Appendix B. In contrast, the velocity dispersion goes to zero at the center for all values of the Sérsic index m for a galaxy without a central BH (see Figure 5).

The anisotropy parameter β is plotted for $m = 2.3$ to $m = 9$ in Figure 7. The plots show that as a result of the BH growth

the stellar DF develops a tangential anisotropy $-\beta \simeq 0.24$ – 0.28 near the center. The anisotropy depends only weakly on the Sérsic index, and is independent of the BH mass: as the BH mass grows the anisotropy simply extends to larger distances.

Using an approximate expression for the radial action near the center of a Sérsic model due to Gondolo & Silk (1999), we compute in the appendix the ratio of the radial and tangential velocity dispersions and thus the anisotropy parameter β (Equation (20)). When sufficiently close to the center, the anisotropy ratio is independent of the BH mass and the radius, and is determined solely by the initial Sérsic index m . The values of β for $m = 2.3$ and $m = 9$ obtained from Equation (31) are -0.313 and -0.306 , respectively, and the corresponding values from Figure 7 are -0.285 and -0.247 .

The constant tangential anisotropy near the center contrasts with the situation for initial galaxies having analytical cores as described in Goodman & Binney (1984) and Quinlan et al. (1995): with analytic cores the initial DF is constant as $r \rightarrow 0$ so the ratio $\overline{v_r^2}/\overline{v_t^2}$ tends to $\frac{1}{2}$ and β tends to zero. However, Quinlan et al. (1995) and Merritt (2004) also examine adiabatic growth of a central black hole in galaxies with central power laws and show that the tangential anisotropy is constant near the center, consistent with our finding.

5.3. Excess Mass

In Figure 9, we plot the excess mass (as defined in Section 4) with respect to the BH mass. We observe that the excess mass scales approximately linearly with the BH mass, although with some offset and curvature in the relation. To make these features more apparent we plot in Figure 10 the ratio of excess mass to BH mass as a function of the BH mass.

We see that the excess mass M_e is always smaller than the BH mass $M_\bullet$ for all values of the BH mass and all values of the Sérsic index m . There is a peak in the excess mass as a function of BH mass for all values of the Sérsic index, which occurs when the mass of the BH is between about 10^{-3} and $10^{-1.5}$ times the mass of the galaxy. The maximum of the ratio $M_e/M_\bullet$ increases as m decreases, consistent with the stronger visual appearance of the cusp in Figure 7 at smaller Sérsic index, but this maximum varies by less than a factor of two over a wide range of Sérsic indices.

The dependence of the ratio of the excess mass to BH mass shown in Figure 10 can be fitted to a cubic polynomial,

$$y = \frac{M_e}{M_\bullet} = a \chi^3 + b \chi^2 + c \chi + d; \quad (25)$$

where $\chi = \log\left(\frac{M_\bullet}{M_G}\right)$

where the coefficients a, b, c and d are functions of m alone. Table 2 gives the fitted values of the coefficients of the cubic polynomial; the ratio of black hole mass to the galaxy mass at the maximum of $M_e/M_\bullet$ ($M_\bullet^{\max}/M_G$); and the maximum deviation of the fitting function y_{fit} from the numerically calculated values y_{num} , that is, $\max(|y_{\text{fit}} - y_{\text{num}}|)$.

6. COMPARISON TO OBSERVATIONS

We now ask whether our results are consistent with observations of excess light in elliptical galaxies. Kormendy et al. (2009) estimate that the excess light is 0.3%–13% of the total light in the galaxy (median 2.3%); for comparison the

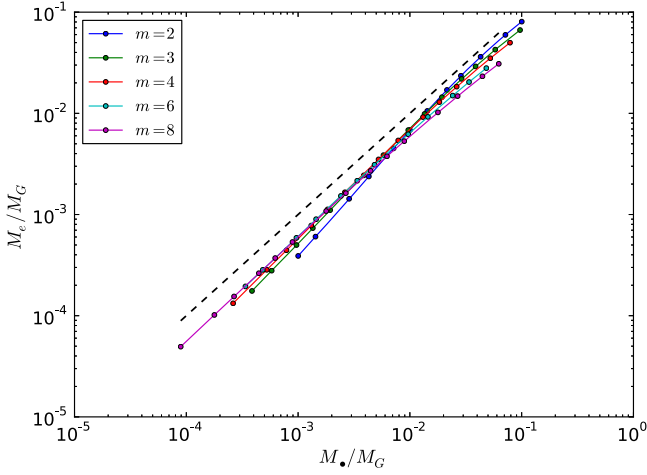


Figure 9. Excess mass vs. BH mass, both scaled to the mass of the galaxy. The dashed line is $M_* = M_e$.

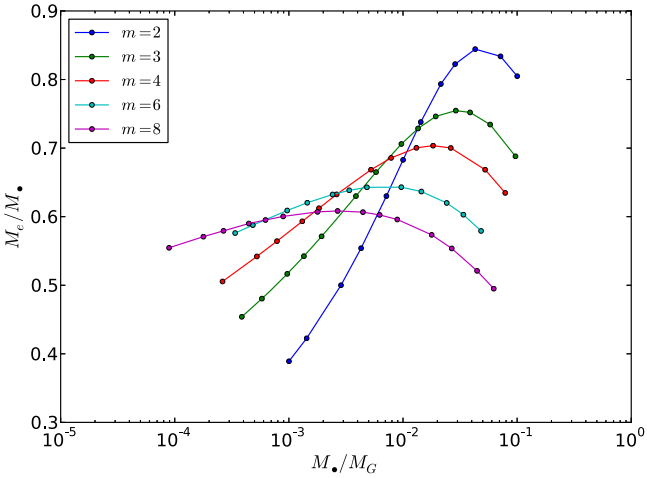


Figure 10. Ratio of the excess mass to the BH mass, as a function of BH mass. The abscissa is plotted on a log scale.

typical ratio of BH mass to galaxy mass (in stars) is $M_*/M_G \simeq 0.5\%$ (Kormendy & Ho 2013). Figure 10 then implies $M_e/M_* \simeq 0.7$ so $M_e/M_G \simeq 0.3\%–0.4\%$, at the lower end of the observed range. This result is consistent with the top panel of Figure 42 of Kormendy et al. (2009), which shows that M_e/M_* ranges between 1 and 50, with typical values around 10. In other words the adiabatic model correctly predicts that the excess mass should be roughly proportional to the galaxy or BH mass, but the predicted magnitude is too small by a factor 5–10. We conclude that the excess light in ellipticals does not arise from adiabatic growth of a central BH. This conclusion is consistent with the observation (Kormendy et al. 2009) that the excess light often has disk isophotes, suggesting that it formed via dissipative processes rather than by adiabatic ones—most likely late star formation due to gas that has collected in the central region of the galaxy (e.g., Hopkins et al. 2009a).

Nuclei or nuclear star clusters are a different form of excess light at the centers of galaxies. Kormendy et al. (Section 9.7) make a distinction between the “excess light” in elliptical galaxies and the “nuclei” in spheroidal galaxies. In Figure 11

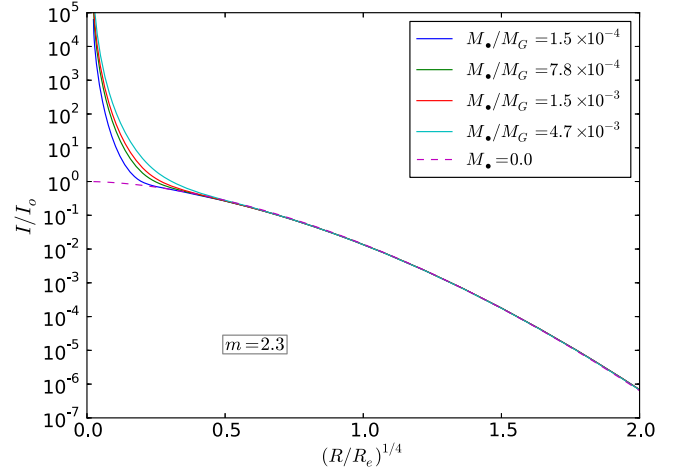


Figure 11. Surface-brightness profile of the galaxy with Sérsic index $m = 2.3$, plotted with respect to $(R/R_e)^{1/4}$. The dashed line is $M_* = 0$.

the surface brightness profile for a galaxy with Sérsic index $m = 2.3$ is plotted using the same $(R/R_e)^{1/4}$ abscissa as in Kormendy et al. Comparing this plot to Figures 25, 26, and 28 of that paper, we notice that the slope of the surface-brightness profile associated with nuclei in spheroidal galaxies, which is much steeper than the profile associated with excess light in ellipticals, is similar to what we obtain in the adiabatic-growth model. Moreover, the median light fraction of nuclei in spheroidal galaxies is 0.3% (Kormendy et al. 2009), much smaller than the fractional excess light in ellipticals and consistent with our model if the nuclei contain typical BHs with mass $M_*/M_G \simeq 0.5\%$ (Kormendy & Ho 2013).

Spheroidal galaxies typically have Sérsic indices $m \lesssim 2$ (Kormendy et al. 2009, Figure 33). Simulations suggest that the Sérsic index could be a indicator of the number of mergers a galaxy have gone through: the galaxies produced in a single merger typically have $m = 3–4$ (Naab & Trujillo 2006) and with additional (minor) mergers the index grows (Bournaud et al. 2007). Thus the properties of spheroidals are unlikely to be determined mainly by mergers (Robertson et al. 2006; Hopkins et al. 2009b). In this case internal processes such as the adiabatic growth of the central BH may be more relevant for shaping the central surface-brightness profile.

The observational relation between BHs and nuclear star clusters remains unclear. Côté et al. (2006) and Wehner & Harris (2006) have argued that nuclear star clusters and central BHs define a single smooth relationship between mass (of the cluster or BH) and galaxy luminosity, with clusters dominating at low luminosities and BHs at high luminosities. Kormendy et al. (2009) argue that this correlation is accidental, but our understanding of the correlation between BH masses and their host galaxy properties, particularly in low-luminosity and late-type galaxies, is too limited for a definite conclusion.

One attractive alternative to the formation of nuclear star clusters by the adiabatic growth of a BH is that the clusters form through in-spiral of globular clusters (Tremaine et al. 1975; Antonini 2013; Gnedin et al. 2014). In this case the BH could form before or after the bulk of the cluster formation.

Table 2
The Fitted Values of the Coefficients a, b, c, d in Equation (25) and $M_*^{\max}/M_G$, the BH Mass at Which M_e/M , is Largest

m	$-a$	$-b$	$-c$	d	$M_*^{\max}/M_G$	$\max(y_{\text{fit}} - y_{\text{num}})$
2	1.21×10^{-2}	1.84×10^{-1}	0.77	-0.140	5.21×10^{-2}	6.34×10^{-3}
3	5.25×10^{-3}	9.33×10^{-2}	0.46	0.0598	3.03×10^{-2}	5.183×10^{-3}
4	3.23×10^{-3}	6.36×10^{-2}	0.356	0.0897	1.76×10^{-2}	3.107×10^{-3}
5	2.47×10^{-3}	5.13×10^{-2}	0.312	0.0784	1.09×10^{-2}	1.115×10^{-3}
6	1.88×10^{-3}	4.14×10^{-2}	0.272	0.0810	6.78×10^{-3}	0.568×10^{-3}
7	1.39×10^{-3}	3.31×10^{-2}	0.236	0.0934	4.21×10^{-3}	0.761×10^{-3}
8	1.03×10^{-3}	2.64×10^{-2}	0.205	0.1070	2.65×10^{-3}	0.968×10^{-3}
9	9.19×10^{-4}	2.41×10^{-2}	0.194	0.0970	1.83×10^{-3}	0.574×10^{-3}

7. CONCLUSIONS

We have studied the effect of the adiabatic growth of a central black hole on the $R^{1/m}$ or Sérsic model of spherical galaxies, following the methods described by Young (1980). The black hole induces a surface-brightness cusp at the center of the galaxy, which can be described as “excess light” or “excess mass” above the inward extrapolation of the best-fit Sérsic profile for the outer galaxy. At the smallest radii the surface brightness is found to vary with radius as $I(R) \sim R^{-1.28}$ to $I(R) \sim R^{-1.32}$ for m varying from 2 to 9. Increasing the BH mass increases the strength of the cusp without changing the logarithmic slope near the center. The line-of-sight velocity dispersion is found to scale as $R^{-1/2}$, which is the expected behavior in a Keplerian potential; the anisotropy parameter β near the center is between -0.24 and -0.28 (tangential anisotropy).

We calculated the excess mass (defined as the mass interior to the radius on the sky where the initial and final surface densities were the same) for Sérsic models with varying indices m . The excess mass is generally between 0.4 and 0.85 times the black hole mass.

If the typical black hole mass is $\sim 0.5\%$ of the stellar mass in the galaxy, the excess mass produced by adiabatic contraction is ~ 5 – 10 times smaller than the excess mass determined from Sérsic fits to the photometry of elliptical galaxies, but similar to the masses of nuclear star clusters in low-luminosity galaxies of all Hubble types. If black holes form by slow accretion of gas then adiabatic contraction may play an important role in determining the properties of nuclear star clusters.

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APPENDIX A

VELOCITY ANISOTROPY NEAR THE CENTER OF THE GALAXY

The goal of this appendix is to determine the velocity anisotropy near the center of the cusp formed by the adiabatic growth of a central BH.

The density for a spherically symmetric Sérsic profile of index m can be approximated near the center (see also Baes &

Gentile 2011) as (Equation (5)):

$$\rho_m(s) = \rho_0 s^{-\gamma} \quad \text{where} \quad \gamma = (m - 1)/m, \quad (26)$$

and the corresponding potential is

$$\Phi_m(s) - \Phi_m(0) = \frac{\rho_0}{(3 - \gamma)(2 - \gamma)} s^{2-\gamma} = \phi_0 s^{2-\gamma}. \quad (27)$$

The phase-space DF for the above density and potential pair is given by Equation (13):

$$f_m(E) \sim [E - \Phi_m(0)]^{-\alpha}, \quad \alpha = \frac{6 - \gamma}{2(2 - \gamma)} = \frac{5m + 1}{2(m + 1)}. \quad (28)$$

An approximate expression for the radial action (Gondolo & Silk 1999) accurate to 8% for all $m > 1$ is

$$I_r(E, L) = \frac{2\pi}{d} \left\{ -\frac{L}{\lambda} + \sqrt{2} \phi_0 \left[\frac{E - \Phi_m(0)}{\phi_0} \right]^{(4-\gamma)/(4-2\gamma)} \right\}, \quad (29)$$

where

$$\lambda = \frac{2^{1/(2-\gamma)}(2 - \gamma)^{1/2}}{(4 - \gamma)^{(4-\gamma)/(4-2\gamma)}}; \quad d = \pi \frac{(2 - \gamma)}{B[1/(2 - \gamma), \frac{3}{2}]}$$

Once the black hole grows adiabatically, the galaxy potential at the center is approximately Keplerian. The radial action is now given by $I_r^*(E^*, L) = 2\pi (M_*/\sqrt{-2E^*} - L)$. The final DF $f^*(E^*, L)$ is obtained by solving $I_r(E, L) = I_r^*(E^*, L)$ for E as a function of E^* and L . Using the equation $E^* = \frac{1}{2}v^2 - M_*/s$ and $L = s v_t$ to eliminate E^* and L in favor of the total and tangential speeds v and v_t , we find that the dependence of the final DF on velocity at a given radius can be written as

$$f^* \sim \left[\left(\frac{1}{\lambda d} - 1 \right) x \sin \psi + \frac{x_m^2}{2\sqrt{x_m^2 - x^2}} \right]^{-\delta}, \quad (30)$$

where

$$\delta = \frac{6 - \gamma}{4 - \gamma}; \quad x = sv; \quad x_m = \sqrt{2M_*} s; \quad v_t = v \sin \psi.$$

The quantities x and x_m are defined following Goodman & Binney (1984).

Multiplying f^* by v_r^2 and v_t^2 and integrating over all velocities we obtain the ratio of velocity dispersions and the anisotropy parameter β :

$$\frac{\overline{v_r^2}}{\overline{v_t^2}} = \frac{1}{2(1-\beta)} = \frac{\int_0^1 dy y^4 \int_0^\pi d\psi \sin \psi \cos^2 \psi \left[\left(\frac{1}{\lambda d} - 1 \right) y \sin \psi + \frac{1}{2\sqrt{1-y^2}} \right]^{-\delta}}{\int_0^1 dy y^4 \int_0^\pi d\psi \sin^3 \psi \left[\left(\frac{1}{\lambda d} - 1 \right) y \sin \psi + \frac{1}{2\sqrt{1-y^2}} \right]^{-\delta}}, \quad (31)$$

where $y = x/x_m$. The only approximation in deriving this result is the approximate form for the radial action, Equation (29). The quadratures are straightforward to evaluate numerically.

APPENDIX B

LINE-OF-SIGHT VELOCITY DISPERSION NEAR THE CENTER OF THE GALAXY

We derive the velocity dispersion near the center of the cusp formed by the adiabatic growth of a central BH. Near the center we can neglect the self-gravity of the stellar system compared to the gravitational field from the BH. Using the notation and results from Appendix A, we first calculate the density $\rho(s)$ and the radial and tangential dispersions $\overline{v_r^2}(s)$, $\overline{v_t^2}(s)$ as

$$\rho(s) \sim (2M_\bullet s)^{(-\delta+3)/2} s^{-3/2} \times \left\{ \int_0^1 dy y^2 \int_0^\pi d\psi \sin \psi g_m(y, \psi) \right\}, \quad (32)$$

$$\rho(s) \overline{v_r^2} \sim (2M_\bullet s)^{(-\delta+5)/2} s^{-5/2} \times \left\{ \int_0^1 dy y^4 \int_0^\pi d\psi \sin \psi \cos^2 \psi g_m(y, \psi) \right\}, \quad (33)$$

$$\rho(s) \overline{v_t^2} \sim (2M_\bullet s)^{(-\delta+5)/2} s^{-5/2} \times \left\{ \int_0^1 dy y^4 \int_0^\pi d\psi \sin^3 \psi g_m(y, \psi) \right\}, \quad (34)$$

where

$$g_m(y, \psi) = \left[\left(\frac{1}{\lambda d} - 1 \right) y \sin \psi + \frac{1}{2\sqrt{1-y^2}} \right]^{-\delta},$$

and all of the above integrals have the same proportionality constant.

The surface density and line-of-sight dispersion are then given by Equations (18) and (19). After a few straightforward integrals we find

$$\sigma_{\parallel}^2(\eta) = f(m) \frac{M_\bullet}{\eta} \quad \text{where} \quad f(m) = \frac{B\left(\frac{7m+2}{2(3m+1)}, \frac{3}{2}\right)}{B\left(\frac{4m+1}{3m+1}, \frac{1}{2}\right)} \times \frac{U_r(m) + \frac{7m+2}{2(3m+1)} U_t(m)}{\chi(m)}. \quad (35)$$

where $\eta = R/R_e$ and $\chi(m)$, $U_r(m)$ and $U_t(m)$ are the quantities in braces in Equations (32), (33) and (34) respectively. We have $f(2) = 0.21667$ and $f(10) = 0.21665$. Note that the value of $f(m)$ is almost independent of m .

In dimensional variables the above result can be written as

$$\sigma_{\parallel}^2(R) = f(m) \frac{GM_\bullet}{R}.$$

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