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Convergence analysis of the extended Kalman filter used in the ultrasonic time-of-flight estimation

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Abstract. The ultrasonic Time-of-Flight (ToF) estimation may be achieved using different algorithms, such as signal processing techniques, artificial intelligence and, statistical estimators, among others. These algorithms have a highlight point, which is the guarantee of the estimation reliability through the convergence of the results and low estimation uncertainty. Hence, this work aims to perform the convergence analysis of the Extended Kalman Filter (EKF) algorithm to ToF estimation, with application in wind speed measurement. Therefore, a state space model of a delayed sine wave was constructed. The modeling used shows the influence of time-varying parameters, which determine the convergence of the states. By analyzing the convergence of the algorithm, it was possible to determine the range of variation of the model parameters to guarantee the final estimation results. Through the construction of a computational model using Matlab@Simulink are presented the simulation results to wind speed measurement.

1. Introduction

In the energy sector, there are numerous sources of energy available that can be used, standing out in the renewable ones, such as wind. Wind energy uses the strength of the wind to generate electricity through wind turbines installed in strategic locations. To determine the best locations for the installation of wind turbines, it is essential to monitor the wind speed, thus determining statistical parameters and constructing a model of it. However, it is required that the sensors used have characteristics of low uncertainty, less time constant for the detection of gusts of wind and with possibilities of operation in severe environments, such as rainfall, snow, and solar radiation. Thus, among the wind speed measurement systems that stand out are the ultrasonic anemometers. These instruments indirectly determine the wind speed by means of the ToF ultrasonic transit time, which is defined as the travel time of the ultrasonic wave from its emission on a transmitting transducer to its detection on a receiving transducer. In the bibliography, several methods are found for ToF estimation, such as threshold detection, phase difference, cross correlation, and Kalman filter.

In [1] and [2], the ToF estimation was approached using the Extended Kalman Filter (EKF) algorithm, by means of the estimation of states associated with a delayed sine wave. One of the limitations of this approach was the model's dependence on time-varying parameters. In this scenario, the objective of this work is to analyze the convergence of the EKF regarding the time-varying parameters of the model, with the purpose of adjusting the parameters of the model to a ToF estimation with low uncertainty.

2. Proposed procedure: State space model

Figure 1 shows a measurement configuration using ultrasonic transducers, positioned in a test section of a wind tunnel with laminar flow and Reynolds number less than 2000 [3]. In this configuration, the wind speed can be found by:

$$\vartheta = \frac{1}{\cos\theta}\left(\frac{L}{t_{AB}} - C\right) \tag{1}$$

where: L is the distance between the transducers, θ is the angle between the wind direction and the alignment of the transducers, t_{AB} is the time of flight, C is the speed of sound in the air that depends on the temperature (T_K in Kelvin) given by $C = 20.074\sqrt{T_K}$.

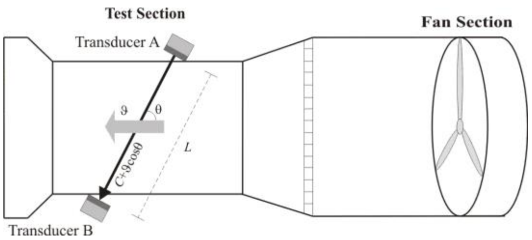

Figure 1. Wind tunnel – Test section for wind speed measurement.

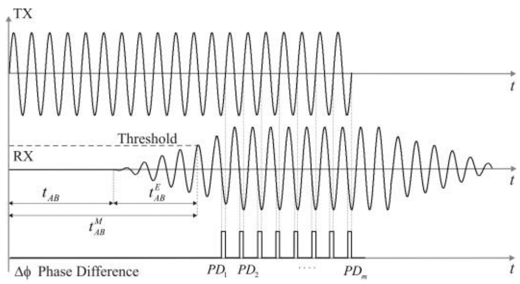

Figure 2. Measurement ToF using threshold detection and phase difference.

Figure 2 shows the transmitted (TX) and received (RX) signals. In this figure, it is possible to identify the ToF and the techniques for its estimation (threshold detection and phase difference [2] [5]). Considering that the signal received on the transducer RX, $u(t)$, is a sum of sinusoidal signals composed by: a main ultrasonic signal, $y(t)$, reflected ultrasonic signals with attenuation and delay, and additive noise, given by:

$$\begin{aligned} u(t) &= y(t) + \sum_{i=1}^{\infty} A_i \sin\left(\omega_i (t - t_{ToFi}) + \phi_i(t)\right) + \eta(t) \\ y(t) &= A(t)\sin\left(\omega(t)(t - t_{ToF}(t)) + \phi(t)\right) \end{aligned} \tag{2}$$

Where: A, ω, ϕ and t_{TOF} are the amplitude, frequency, phase and time-of-flight of the main ultrasonic signal; A_i, ω_i, ϕ_i and t_{TOFi} are the amplitude, frequency, phase and time-of-flight of the reflected ultrasonic signal; t is the time and η is the additive noise.

From equation (2), we have built a nonlinear state space model [1], [4], given by:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}, t) + \mathbf{w}(t) \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}, t) + \mathbf{v}(t) \end{aligned} \tag{3}$$

Where: the states of the model are amplitude $x_1 = A$, frequency $x_2 = \omega$, phase $x_3 = \phi$ and the time-of-flight $x_4 = t_{ToF}$; $\mathbf{w}$ is the noise of the states with mean zero and standard deviation σ_w; $\mathbf{v}$ is the noise of the measurements with mean zero and standard deviation σ_v, and the nonlinear functions $\mathbf{f}(\mathbf{x}, t)$ and $\mathbf{g}(\mathbf{x}, t)$ are given by:

$$\mathbf{f}(\mathbf{x}, t) = \begin{bmatrix} \mu_1 (u - x_1 \sin\beta)\sin\beta \\ \mu_2 x_1 (u - x_1 \sin\beta)\cos\beta \\ \mu_3 x_1 (u - x_1 \sin\beta)\cos\beta \\ \dfrac{\mu_3 - \mu_2 x_4}{x_2 . x_1^{-1}} (u - x_1 \sin\beta)\cos\beta \end{bmatrix} \quad \text{and} \quad \mathbf{g}(\mathbf{x}, t) = \begin{bmatrix} x_1 \sin\beta \\ \hat{t}_{ToF} \end{bmatrix} \quad \text{and } \beta = x_2(t - x_4) + x_3 \tag{4}$$

In Equation (4) was defined the variables μ_1, μ_2, and μ_3, which are associated with the convergence of the Extended Kalman Filter (EKF) used for the estimation of the states from the measurements of the received signal amplitude and the ToF obtained by the phase difference technique [1]. In this work, it will be analyzed and defined the criteria to obtain the ranges of values of the variables which lead the estimator to a quick convergence and with the lowest standard deviation of the estimation.

3. Simulation results: ToF estimation using EKF

In the interest of implementing the Kalman Filter algorithm for ToF estimation, the Equation (4) was discretized considering a sampling period, T_s. Hence, a discrete state space system was obtained, as illustrated below:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{f}\left(\mathbf{x}_k, k\right) + T_s \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{g}\left(\mathbf{x}_k, k\right) + T_s \mathbf{v}_k \end{aligned} \tag{5}$$

Where: $\mathbf{x}_k$ is the n-dimensional vector of process states; $\mathbf{y}_k$ is the m-dimensional vector of current measurements; $\mathbf{w}_k \sim N(0, \mathbf{Q}_k)$ and $\mathbf{v}_k \sim N(0, \mathbf{R}_k)$ are the sequences of uncorrelated Gaussian noise; $\mathbf{f}(\mathbf{x}_k, k)$ and $\mathbf{g}(\mathbf{x}_k, k)$ are nonlinear functions in discrete-time kT_s, given by:

$$\mathbf{f}(\mathbf{x}_k, k) = \begin{bmatrix} x_1 + T_s u \sin\beta_k - T_s x_1 \sin^2\beta_k \\ x_2 + T_s x_1 u \cos\beta_k - T_s x_1^2 \sin\beta_k \cos\beta_k \\ x_3 + T_s x_1 u \cos\beta_k - T_s x_1^2 \sin\beta_k \cos\beta_k \\ x_4 + T_s x_1 \left(\dfrac{1 - x_4}{x_2}\right)(u - x_1 \sin\beta_k)\cos\beta_k \end{bmatrix} \quad \text{and} \quad \mathbf{g}(\mathbf{x}_k, k) = \begin{bmatrix} x_1 \sin\beta_k \\ x_4 \end{bmatrix} \tag{6}$$

Through the equations (5) and (6) it concludes that the state space system is nonlinear; therefore, to perform the estimation of the states, the extended Kalman filter (EKF – Extended Kalman Filter) should be used. Table 1 shows the EKF algorithm to Time-of-Flight (ToF) estimation, where the states are defined by the amplitude (A), frequency (ω), phase (ϕ) and time-of-flight (ToF).

Table 1. EKF algorithm for the ToF estimation.

Initialization of states: $\mathbf{x}_0 = \begin{bmatrix} A_0 \\ \omega_0 \\ \phi_0 \\ ToF_0 \end{bmatrix}$ $\mathbf{P}_0 = \begin{bmatrix} \sigma^2_{\varepsilon A} & 0 & 0 & 0 \\ 0 & \sigma^2_{\varepsilon\omega} & 0 & 0 \\ 0 & 0 & \sigma^2_{\varepsilon\phi} & 0 \\ 0 & 0 & 0 & \sigma^2_{\varepsilon ToF} \end{bmatrix}$
State estimate propagation: $\hat{\mathbf{x}}_{k\|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1})$ Error covariance propagation $\mathbf{P}_{k,k-1} = \left[\dfrac{\partial \mathbf{f}(\hat{\mathbf{x}}_{k-1})}{\partial \mathbf{x}_{k-1}}\right] \mathbf{P}_{k,k-1} \left[\dfrac{\partial \mathbf{f}(\hat{\mathbf{x}}_{k-1})}{\partial \mathbf{x}_{k-1}}\right]^T + \mathbf{H}(\hat{\mathbf{x}}_{k-1})\mathbf{Q}_{k-1}\mathbf{H}^T(\hat{\mathbf{x}}_{k-1})$ Kalman gain matrix $\mathbf{G}_k = \mathbf{P}_{k,k-1}\left[\dfrac{\partial \mathbf{g}(\hat{\mathbf{x}}_{k\|k-1})}{\partial \mathbf{x}_k}\right]^T \cdot \left[\left[\dfrac{\partial \mathbf{g}(\hat{\mathbf{x}}_{k\|k-1})}{\partial \mathbf{x}_k}\right]\mathbf{P}_{k,k-1}\left[\dfrac{\partial \mathbf{g}(\hat{\mathbf{x}}_{k\|k-1})}{\partial \mathbf{x}_k}\right]^T + \mathbf{R}_k\right]^{-1}$ Error covariance update $\mathbf{P}_{k,k} = \left[\mathbf{I} - \mathbf{G}_k\left[\dfrac{\partial \mathbf{g}(\hat{\mathbf{x}}_{k\|k-1})}{\partial \mathbf{x}_k}\right]\right]\mathbf{P}_{k,k-1}$ State estimate update $\hat{\mathbf{x}}_{k\|k} = \hat{\mathbf{x}}_{k\|k-1} + \mathbf{G}_k\left[\mathbf{y}_k - \mathbf{g}(\hat{\mathbf{x}}_{k\|k-1})\right]$

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Figure 3 shows the result of the application of the EKF algorithm on a received signal $u(t)$ and the estimation of the parameters of the ultrasonic signal $y(t)$. The estimated parameters by the EKF algorithm are associated with the estimated signal given by equation (2), such as: $x_1 = A$, $x_2 = \omega$, $x_3 = \phi$ and $x_4 = ToF$. Even with satisfactory simulation results, the convergence of the EKF algorithm may depend on the values assigned to the variables μ_1, μ_2, e μ_3, which transform the system into a time variant system. It can be noticed that even if the received signal has the influence of noise, the Kalman filter estimates the states in a robust way.

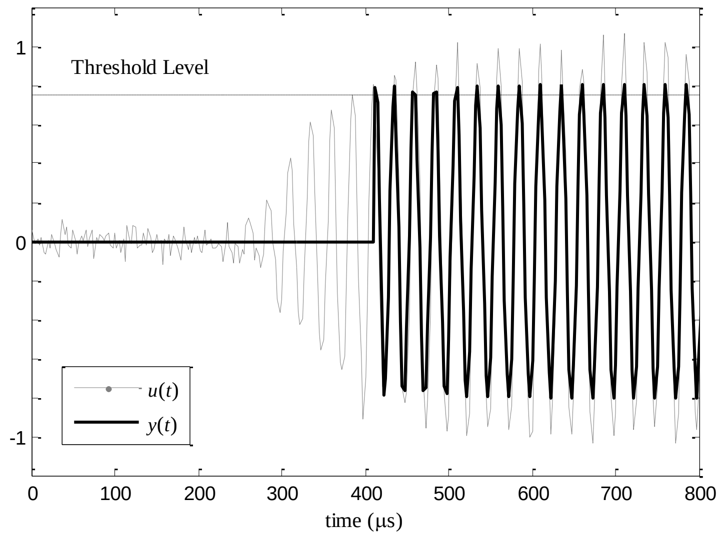

Figure 3. Parameters estimated of the ultrasonic signal using the EKF algorithm.

4. Convergence analysis of the EKF algorithm for ToF estimation

To perform the convergence analysis of the EKF with respect to the model parameters in state spaces, μ_1, μ_2, and μ_3, a computational model was constructed to measure wind speed using Matlab@Simulink. The ultrasonic transducers were modeled by means of bandpass Butterwoth filters. An additive noise of the Gaussian type with a standard deviation of 0.01 mv was considered. ToF represents the delay of the ultrasonic wave from the emission until its detection. In the simulations, a ToF equal to 228.931 μs was used (Equation (1)), for a wind speed of 10 m/s, temperature of 40°C, distance between the transducers of 83 mm, and angle of inclination of $\pi/4$.

Figure 4 the results of the ToF estimations are illustrated by the EKF algorithm. The variations of μ_1, μ_2, and μ_3 were considered in the range of 1 to 10000. It is observed that the ToF estimates do not change even with the variations of μ_1 and μ_2. In these simulations, an uncertainty was obtained in the ToF estimation of 0.5 μs, defining a confidence interval in which the result of the theoretical ToF is contained [6].

On the other hand, in the same figure, it was observed that the ToF estimates undergo changes in their estimation in relation to the increase of μ_3, increasing the uncertainty in the estimation as well. In this simulation, an uncertainty of 2 μs was obtained in the ToF estimation. From this result, it can be verified that the convergence of the EKF algorithm depends on the values of the model parameters (Equations (5) and (6)). Consequently, the uncertainty associated with the state estimate $x_4 = ToF$ can be compromised for certain values of these parameters.

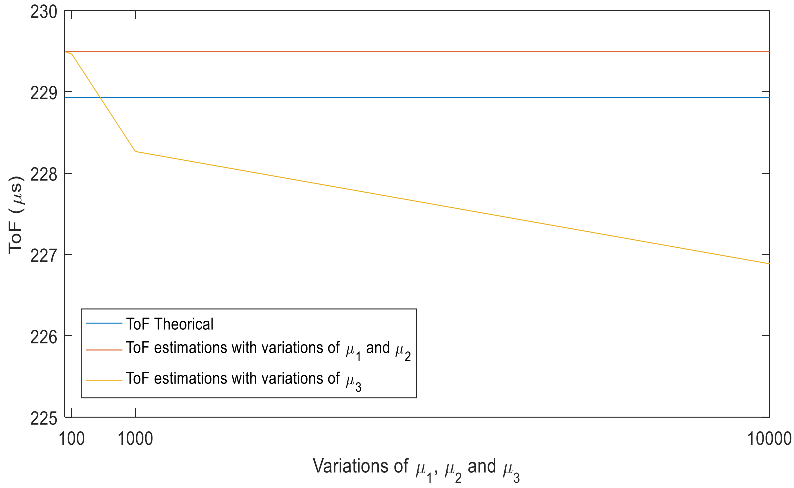

Figure 4. Estimation of ToF by EKF for different variations of μ_1, μ_2, and μ_3.

In figure 5, the convergence of the EKF for the estimation of the state associated with the ToF is shown. Considering $\mu_1 = 1$, $\mu_2 = 1$ and variations of $\mu_3 = \{1, 10^3, 10^4, 10^6, 10^7\}$. It is possible to observe that the standard deviation decreases according to the number of EKF iterations; however, for greater values of μ_3 it is realized that the EKF algorithm does not converge to a minimum value of the state variance x_4 (ToF). It may also be noted that the minimum standard deviation value was obtained for μ_3=1.

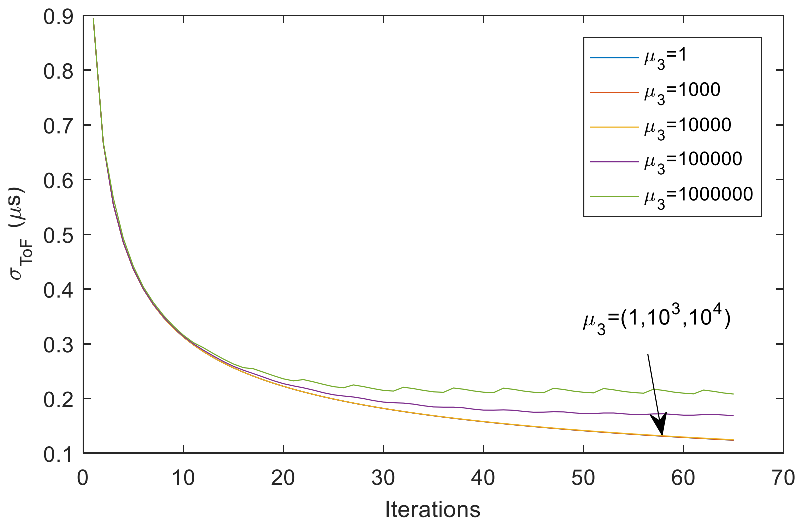

Figure 5. Standard deviation of ToF estimation through EKF.

5. Conclusions

In this work, the algorithm for the estimation of the ultrasonic transit time based on the extended Kalman filter (EKF) was developed. It was observed that the state model is nonlinear and time variant. With respect to the nonlinear characteristic, the extended Kalman filter allows an approximation by means of a linearization process. Regarding the time-varying characteristic, the model depends on the variables μ_1, μ_2, and μ_3, inducing the result obtained in the ToF estimation process, as well as showing the increase in the uncertainty of the estimated value. This result can be explained by the fact that the EKF algorithm does not converge for certain values of these variables, making it unstable. As result of this analysis is the determination of the possible variations of variables μ_1, μ_2, and μ_3, in which the EKF algorithm presents convergence to a ToF value and with low uncertainty.

Acknowledgments

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References

[1] Villanueva J, Catunda S, Freire R, Costa M and Ingaroca 2013 Wind speed measurement based on time-of-flight estimation using Extended Kalman Filter *IEEE Intern. Inst. and Meas. Technology Conf.* pp 1182-7

[2] Villanueva J, Catunda S and Tanscheit R 2009 Maximum-Likelihood Data Fusion of Phase-Difference and Threshold-Detection Techniques for Wind Speed Measurement. *IEEE Transactions on Instrumentation and Measurement*. **58** 2189-7

[3] Pereira M 2009 Flow Meters: Part 1 *IEEE Instrumentation and Measurement Magazine*. **12** 18-9

[4] Ziarani A and Konrad A 2004 A method of extraction of nonstationary sinusoids *Signal Processing* **84** 1323-14

[5] Duarte R, Villanueva J, Costa M, Freire R, Catunda S and Costa W 2013 Ultrasonic time-of-flight estimation for wind speed measurement based on time-frequency domain using STFT *19th IMEKO TC 4 Symposium* (*Barcelona, Spain, 18-19 July 2013*) pp 72-6

[6] JCGM 200:2012 *International vocabulary of metrology – Basic and general concepts and associated terms* (*VIM*).