

PAPER • OPEN ACCESS

A fast Maximum Likelihood Estimation algorithm for demodulating Fiber White-Light Interferometry

To cite this article: Yi Qi *et al* 2018 *J. Phys.: Conf. Ser.* **1065** 252004

View the [article online](#) for updates and enhancements.

You may also like

- [Survival Analysis Competing Risk using Fine-Gray Subdistribution Model with the Maximum Partial Likelihood Estimation Method](#)
A Ainurrochmah, S W Purnami and Purhadi
- [Long-term Study of the Light Curve of PKS 1510-089 in GeV Energies](#)
Raj Prince, Pratik Majumdar and Nayantara Gupta
- [Gamma-Ray Flares in the Long-term Light Curve of 3C 454.3](#)
Avik Kumar Das, Raj Prince and Nayantara Gupta



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

A fast Maximum Likelihood Estimation algorithm for demodulating Fiber White-Light Interferometry

Yi Qi, Weimin Chen*, Juxia Xie, Xiaohua Lei

The Key Laboratory for Optoelectronic Technology & Systems of Ministry of Education, Sensors and Instruments Research Center, College of Optoelectronic Engineering, Chongqing University, Chongqing 400044, China

E-mail: wmchen@cqu.edu.cn

Abstract. Being introduced to upgrade the precision of demodulation for Fiber White-Light Interferometer, Maximum Likelihood Estimation faces the problem of low calculation speed. In this paper, a high-speed algorithm is proposed based on Newton Iteration for Maximum Likelihood Estimation to accelerate its fine-search process, and the process is precisely kept in the main peak of MLE spectrum with a numerical calculation. The simulation results show that the fast algorithm increase the speed for two orders of magnitude.

1. Introduction

As an absolute and accurate interferometer, optical Fiber White-Light Interferometry (FWLI), such as fiber F-P and fiber Michelson sensor, are widely used in many fields ^[1]. Signal demodulation is the key technology of FWLI, and many demodulation algorithms are presented. The period analysis in wavelength domain and FFT analysis in transform domain are the two typical representatives ^{[2][3]}. Although period analysis has been widely used in many applications, but its demodulation precision is not satisfied in some special fields. On the other hand, Cross-correlation Method, Total-phase Method and Maximum likelihood Method are reported in high precision demodulation of other interferometric signals ^{[4][5][6]}. Taking advantages of both frequency and phase of the signal, these algorithms have large computational volume which decreases its demodulation speed dramatically ^[7]. In some special applications of FWLI, both high precision and high speed demodulation is necessary. If the problem of computational volume can be addressed, fast maximum likelihood estimation is possible in high precision demodulation of FWLI.

2. Demodulation of FWLI with Maximum Likelihood Algorithm

Typical FWLI of Fiber F-P and fiber Michelson sensor are shown in Fig1. The output of FWLI is two-beam interference and can be written as Eq(1) after discrete sampling and DC filtering.

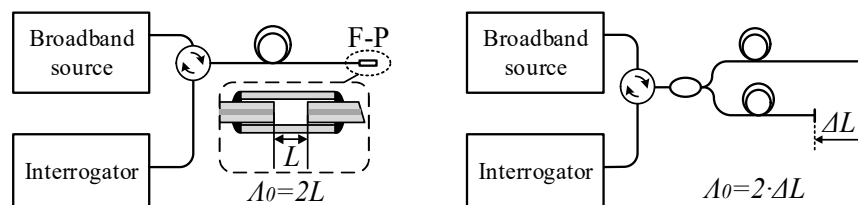


Fig 1 typical optical fiber White-Light Interferometry

$$S_n = A \cos[2\pi\Lambda_0(k_0 + n \cdot \delta k)] \quad (1)$$

Where, S_n is the discrete signal of interference spectrum, A , k_0 and δk are the amplitude and initial wavenumber and sampling interval of the signal respectively, Λ_0 is wanted parameter of optical-path-difference (OPD), $n=1,2,3,\dots$.

The Maximum Likelihood Estimator (MLE) $\hat{\Lambda}$ of the Λ_0 can be expressed as the following:

$$\hat{\Lambda} = \max \{ \ln p(\mathbf{S}_n, \Lambda) \} = \max \left\{ - \sum_{n=0}^{N-1} \left[\mathbf{S}_n - A \cos(2\pi\Lambda k_0 + 2\pi\Lambda n \cdot \delta k) \right]^2 \right\} \quad (2)$$

Because $(1/N) \cdot \sum \cos(2\pi\Lambda_0 k_n + \varphi_0) \approx 0$, Eq.(2) can be simplified into Eq.(3).

$$\hat{\Lambda} \approx \max_{\Lambda} \left\{ \sum_n \mathbf{S}_n \cos(2\pi\Lambda k_0 + 2\pi\Lambda n \cdot \delta k) \right\} = \max_{\Lambda} \left\{ \text{Re} \left[e^{-2\pi j k_0 \Lambda} \right] \cdot \text{Re} \left[\mathcal{F}(\mathbf{S}_n) \right] \right\} \quad (3)$$

According to Eq(3), the MLE is the FT spectrum of $\mathbf{S}_n$ modulated by a high-frequency factor $\text{Re}[exp(-jk_0\Lambda)]$, which is different from those of Discrete Fourier Transform (DFT) and Zero-Padded DFT (ZPDFT) as shown in Fig.2(b). According to Eq(3), $\hat{\Lambda}$ is the maximum location of the spectrum and MLE which can be realized in peak-seeking in Fig2. According to Fig2, MLE has a higher precision of $\hat{\Lambda}_{MLE}$ due to its sharp waveform, while $\hat{\Lambda}_{DFT}$ and $\hat{\Lambda}_{ZPDFT}$ of DFT and ZPDFT have relative lower precision due to their gentle waveform as shown in Fig2(c). Otherwise, high precise demodulation of MLE is a time-consuming process in its in maximum peak-seeking due to multiple peaks screening^[6].

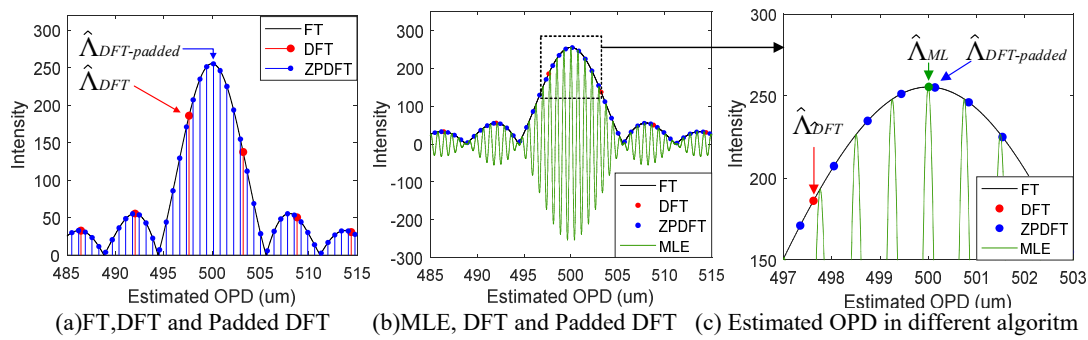


Fig 2. Transform domain results of different algorithms

In a deep investigation of Fig 2(c), it is clear that $\hat{\Lambda}_{ZPDFT}$ is close to $\hat{\Lambda}_{MLE}$. This implies that it is possible to cut the process of multiple peaks screening dramatically if the $\hat{\Lambda}_{MLE}$ is limited in the section of its main peak by ZPDFT. In addition, if there is a fast algorithm in seeking $\hat{\Lambda}_{MLE}$ from $\hat{\Lambda}_{ZPDFT}$, it could further accelerate the demodulating process of MLE.

Because the Newton iteration method has the ability to quickly find the peak of a function within its iterative convergence range, it is possible to be introduced in the process of locating MLE spectrum within its main peak and speeds up the peak-finding process. In order to carry out this fast iterative algorithm to the estimation of OPD, two problems must be addressed. Firstly, the convergence interval of OPD must be determined. Secondly, the iteration converges must be kept in the main peak in the iterative process, instead of other peaks.

3. The interval of convergence in MLE

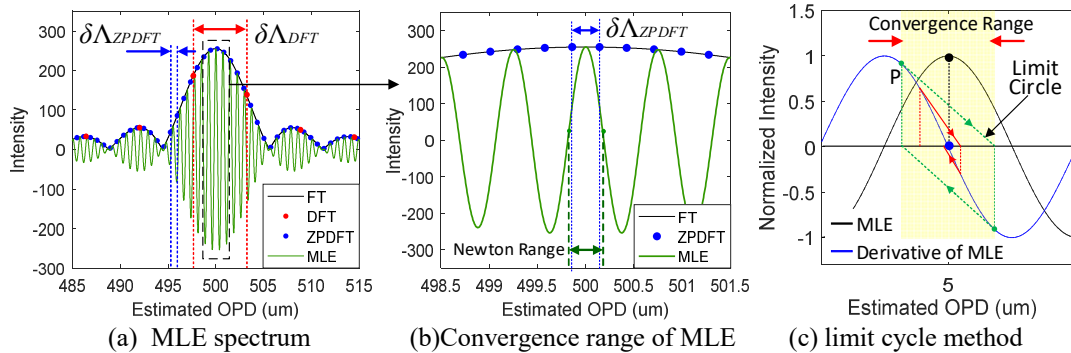


Fig 3 calculation of Newton range by limit cycle method

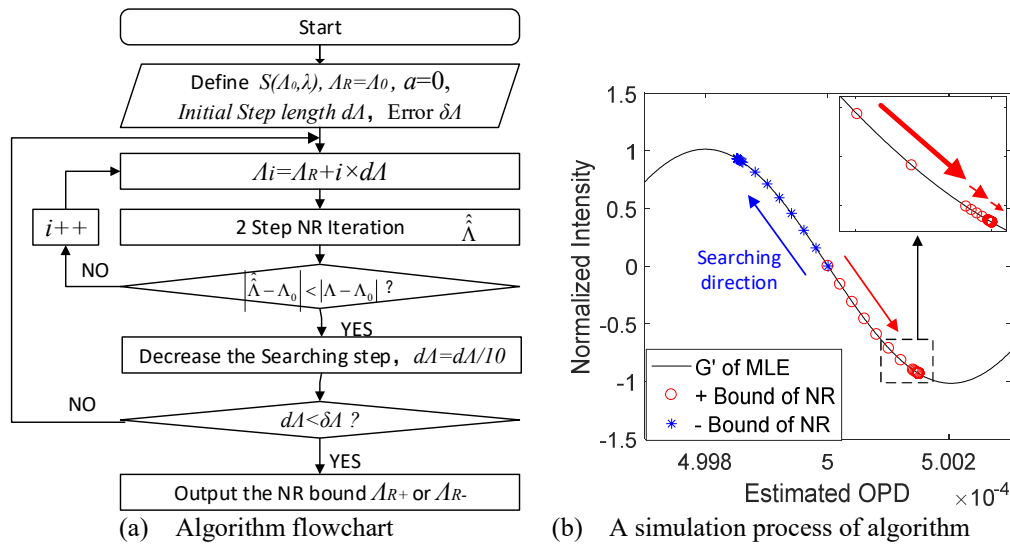
The convergence range of Newton method is the range which keeps the iteration process in converging at the local peak. In MLE spectrum, as long as the Spectral resolution of ZPDFT $\delta\Lambda_{padded-DFT}$ is smaller than Newton Range (ΔNR) of the main peak, the $\hat{\Lambda}_{ZPDFT}$ will be located

within the ΔNR as shown in Fig 3 (b). Then the $\hat{\lambda}_{MLE}$ can be accurately calculated by Newton method starting from $\hat{\lambda}_{ZPDFT}$.

Because the convenient way to find the maximum value of a function is correspondent to locate its derivation in zero, both MLE (black dot line and spot) and its derivative function (blue line and spot) are plotted in Fig3 (c) comparatively.

Been limited in the main peak of MLE, locating its derivation in zero can be quickly done from an initial location “P” within the Newton convergence interval range. The Newton Range (NR) can be determined by finding the range of limit cycle as the green loop of the infinite iterative circle in Fig3(c) shows. However, unlike the calculation of NR in traditional frequency estimation problems^[8], the MLE of interference spectrum is modulated by a high-frequency factor $\text{Re}[\exp(-jk_0A)]$, which makes the Analytical expression of the NR cannot be obtained in OPD ML estimation. Therefore, this paper presents a numerical solution of Newton convergence interval range.

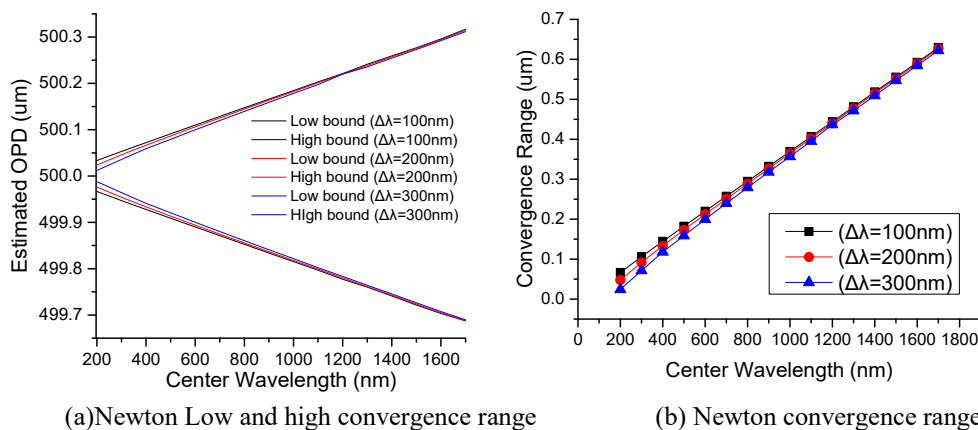
The calculation process of finding the high-precision Newton convergence interval range contains two circulations. The first is finding the Newton range under accuracy dA and the second is decrease the accuracy by $dA = dA/10$. The numerical calculation stops until the accuracy is high enough. A complete process of calculation shown in fig4 (b).



(a) Algorithm flowchart

(b) A simulation process of algorithm

Fig 4 calculation process of Newton convergence interval range



(a) Newton Low and high convergence range

(b) Newton convergence range

Fig 5 The relationship between optical spectrum parameters and the convergence range

In OPD estimation problem, parameters of the optical spectrum will influence the Newton convergence range. Center wavelength (λ_c) and wavelength range ($\Delta\lambda$) are two most important parameters. The convergence range in different λ_c and $\Delta\lambda$ conditions are shown in fig 5 by simulation. The results show that the convergence interval is approximately linear with the center wavelength. The $\Delta\lambda$ has only a small influence on the result.

4. Fast iteration method of OPD Maximum Likelihood Algorithm

Similar to the traditional MLE frequency estimation of a sinusoid algorithm, the fast OPD algorithm contains coarse and fine-search processes^[7]. The coarse search uses zero-padded M-point FFT and fine-search using Newton iterative method. As mentioned before, the resolution of zero-padded $\delta\lambda_{\text{padded-DFT}}$ must smaller than Newton Range to make ensure that coarse-search result falls into the NR. Assume that the original optical spectrum contains N points, the sampling interval in wavenumber domain is δk and the Newton range is ΔNR . The value of M must follow Eq.(5) which can be calculated by digital Fourier transform principle easily.

$$M > \frac{2}{\delta k \cdot \Delta NR} + 1 \quad (5)$$

In order to compare the algorithm speed before and after using Newton method, table.1 shows the total calculation time of 1000 times demodulation.

Table 1. total time of 1000 times demodulation

parameters	$\delta k=5\text{cm}^{-1}$, $\Delta\lambda=100\text{nm}$, $\lambda_c=1550\text{nm}$, $\text{OPD}=500\mu\text{m}$, $\Delta NR=0.593\mu\text{m}$					
fine-search methods	Interpolation method			NR iteration method		
accuracy of MLE Estimated OPD	0.1um	0.01um	0.001um	0.1um	0.01um	0.001um
fine-search calculation load of MLE spectrum	1000	2000	3000	3	3	4
Total time (s)	0.952	1.901	3.049	0.0097	0.0096	0.0151

5. Conclusion

In summary, a fast Maximum Likelihood Algorithm for Optical-Path-Difference Estimation in Spectral Interferometry is proposed. By using Newton iteration method in fine-search of estimated OPD, The algorithm increases the speed of the algorithm by two orders of magnitude in the simulation. The Newton convergence interval range is calculated by limit cycle method to ensure that the fine-search process convergence to the main peak of MLE spectrum. The research in this paper paves the way for the engineering application of MLE of OPD algorithm.

References

- [1] B. H. Lee, Y. H. Kim, K. S. Park, J. B. Eom, M. J. Kim, B. S. Rho, *et al.*, "Interferometric Fiber Optic Sensors," *Sensors*, vol. 12, pp. 2467-2486, Mar 2012.
- [2] B. Qi, G. R. Pickrell, J. C. Xu, P. Zhang, Y. H. Duan, W. Peng, *et al.*, "Novel data processing techniques for dispersive white light interferometer," *Optical Engineering*, vol. 42, pp. 3165-3171, Nov 2003.
- [3] X. H. Lei, W. M. Chen, P. Zhang, and Y. Zhu, "Demodulation method based on Fourier transform with cubic spline interpolation for optical fiber Fabry-Perot sensors," *Guangzi Xuebao/Acta Photonica Sinica*, vol. 37, pp. 705-708, 2008.
- [4] J. H. Xie, F. Y. Wang, Y. Pan, J. J. Wang, Z. L. Hu, and Y. M. Hu, "High resolution signal-processing method for extrinsic Fabry-Perot interferometric sensors," *Optical Fiber Technology*, vol. 22, pp. 1-6, Mar 2015.
- [5] M. Han, Y. Zhang, F. B. Shen, G. R. Pickrell, and A. B. Wang, "Signal-processing algorithm for white-light optical fiber extrinsic Fabry-Perot interferometric sensors," *Optics Letters*, vol. 29, pp. 1736-1738, Aug 2004.
- [6] C. Li, S. Chen, and Y. Zhu, "Maximum Likelihood Estimation of Optical Path Length in Spectral Interferometry," *Journal of Lightwave Technology*, vol. 35, pp. 4880-4887, 2017/11/01 2017.
- [7] T. Abatzoglou, "A fast maximum likelihood algorithm for frequency estimation of a sinusoid based on Newton's method," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 33, pp. 77-89, 1985.
- [8] T. J. Abatzoglou, "Fast Maximum Likelihood Joint Estimation of Frequency and Frequency Rate," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-22, pp. 708-715, 1986.