Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

The teaching and learning of addition and subtraction of integers through manipulative in Brunei Darussalam

To cite this article: N Sahat *et al* 2018 *J. Phys.: Conf. Ser.* **1088** 012024

IOP Conf. Series: Journal of Physics: Conf. Series **1088** (2018) 012024 doi:10.1088/1742-6596/1088/1/012024

The teaching and learning of addition and subtraction of integers through manipulative in Brunei Darussalam

N Sahat[1], K A Tengah[2] and R C I Prahmana[3]

[1]Rimba Secondary School, Ministry of Education, Perumahan Negara Kampong Rimba, Gadong, BE 3319, Brunei Darussalam
[2]Sultan Hassanal Bolkiah Institute of Education, Universiti Brunei Darussalam, Jalan Tungku Link, Gadong, BE 1410, Brunei Darussalam
[3]Universitas Ahmad Dahlan, Jl. Pramuka Kav. 5, Yogyakarta 55161, Indonesia

E-mail: khairul.tengah@ubd.edu.bn

Abstract. This action research studied on how the use of manipulative in the teaching and learning of integers could affect students' performances in adding and subtracting integers, involving Year 9 students from two mixed ability classes of the same government secondary school in Brunei. The intervention lessons used different colored counters to represent different signs of integers, followed by the adding and removal actions of these counters to mirror the addition and subtraction operations, with the added concept of zero-pair. Any improvement or beneficial outcome of the prior lesson was used to adapt the initially prepared following lesson as part of the Design Research process. The data collection consisted of pre-test and post-test. With the use of descriptive statistics and Wilcoxon Signed Ranked Test, this study concluded that there was the significant improvement on students' performances in the post-test and the effect size showed that the intervention gave big impact towards students' learning. Item analysis indicated that the intervention improved correct attempts in questions involving both addition and subtraction, with the more significant increase in the latter. The overall mean scores of correct attempts in the post-tests also showed that students scored higher on the addition of integers (89.7%) compared to subtraction of integers (81.0%).

1. Introduction

The poor performance of students in mathematics has become a source of worry to many. Many experts proposed several solutions to this problem, yet the problem persisted, believing that poor knowledge in basic mathematics among students is the key root to the problem [1-3].

One problem that students are facing when dealing with integers is that they are confused with signs and operations of the integers, which make them struggle in computing the integers [3]. Students taught using number line model can easily get confused with which direction to move when adding or subtracting integers since students always thought that adding means going forward and subtracting means going backward. It is not always true when the second addend involved negative integers [4]. However, it is unfortunate that this model has been widely used by teachers in Brunei [4]. Using algorithm as a teaching pedagogy is also popular yet ineffective, as some students could not remember the algorithm [4].

IOP Conf. Series: Journal of Physics: Conf. Series **1088** (2018) 012024 doi:10.1088/1742-6596/1088/1/012024

Researches revealed that students faced problems in understanding the concept in adding and subtracting integers [5-9]. A lack of foundation in mathematics, which includes the computations of integers, caused many Brunei upper secondary students unable to solve algebraic equations [10]. Moreover, a local study discovered that more than 50% of the Year 7 students in one government school failed in the computation of integers' pre-test [4].

The instructional approach that emphasised only on the mathematics procedural skills was one of the causes to the failure of computing integers among Bruneian students [11-13]. Brownell stated that repetition or computational drill does not lead to understanding [14]. Conceptual understanding plays a vital role in developing procedural skills. Through constructivist perspective, using manipulatives can actually enhance students' mathematics conceptual understanding. Through concrete representation that the manipulatives offer, it provides students the concrete experience in building the abstract concepts of mathematics [15].

Several researches, which made use of counters and zero-pair effect in the teaching and learning of integers, have proven to be effective in improving students' achievement [4, 16, 17]. Zero-pair effect involves neutralization or cancellation of positive with negative integers. Take red and blue counters as an example to represent positive and negative integers respectively. If a red counter and a blue counter come together, the positive and the negative signs cancel each other, creating zero. This means that if there are six red and six blue counters, it will result in zero, despite total twelve counters.

When doing subtraction, it involves the removal of positive or negative integers. If there are not enough positive or negative integers to be subtracted, zero-pair needs to be added in since zero pair will not cause any effect to the equation. For example, in $-3-(-9)$, there are initially three blue counters but since there are not enough blue counters to remove 9 of the blue counters, 6 zero-pairs need to be added in order to have enough blue counters to be removed. If students are already familiar with the manipulation of integers, they can advance themselves in computing the integers [18].

A previous research on the teaching and learning on the four computations of integers (addition, subtraction, multiplication and division) was done to a lower secondary school in Brunei [4]. The study used jar model as a teaching pedagogy in computing integers, which is similar concept of using counters, and made use of zero-pair effect. The result revealed that students significantly improved in adding and subtracting integers but not for multiplying and dividing integers. This study, stemmed from that research, is fine-tuned and expanded as follows:

- The study involved upper secondary students concentrating only on addition and subtraction of integers.
- It focused on the computation of single and two digit integers.
- It differentiated and emphasised on the terms 'positive' and 'negative' for signs, and 'plus' and 'minus' for operations.

The general purpose of this study is to investigate the effectiveness of using manipulatives in the teaching and learning of integers by measuring students' performances on the addition and subtraction of integers. Hence, the research question used to guide this study: *How does the teaching and learning of integers using manipulatives affect students' performances on addition and subtraction of integers*?

2. Method

This study adopted quantitative method, with convenient sample of fifty students from two Year 9 classes from a co-ed Brunei government school in Brunei-Muara district. All students attempted the pre-test and post-test, and attended all the intervention lessons to be part of the data in this study.

Both pre-test and post-test consisted the same open-ended twenty-five items, covering different combinations of operation and signs of the two numbers (Table 1). However, addition and subtraction of zero integer were not included in the test item but covered in the intervention exercise. Calculators were not allowed in both tests and essential workings needed to be shown. A score of one would be given to correct response and zero for incorrect answer or unattempt question.

Table 1. Different combination of questions for addition and subtraction of integers

Operation	Sign of numbers	Item No.
Addition	positive & positive	1, 13
	positive & negative	2, 3, 4, 5, 14, 16, 17, 18
	negative & negative	6, 19
Subtraction	positive & positive	7, 8, 20, 21
	positive & negative	9, 10, 22, 23
	negative & negative	11, 12, 24, 25

The test instrument was analysed for reliability via test-retest to a group of Year 9 students of a mixed ability class not involve in the main study from the same school. Pearson's correlation to test for reliability obtained value of 0.866. This indicated that the test items were reliable, for correlation greater than 0.8, the test items were considered reliable [19]. The pre-test and post-test was also validate by three experienced teachers teaching that level. Since the test instrument was reliable and valid, no modification the items were necessary and all items used in main study.

The intervention lessons for this study consisted of seven lessons, focusing on introduction of and discovering rule of addition and subtraction of different combinations of integers using manipulatives (Table 2). Notes and supporting exercises were also included in each lesson. In addition, any improvement or useful outcome of the prior lesson was added or adapted to the following initially prepared lesson as part of the Design Research process. The lesson intervention lasted 5 hours in total.

Table 2. The intervention lessons conducted in this research study

Lesson	Focus	Materials used	Duration (minutes)
1	Differentiation and reinforce difference between Signs and Operations	Activity Worksheets	30
2	Introduce manipulatives (single and base ten) and reinforce concept of Zero Pairs Effect	Manipulatives, Activity sheets,	30
3	Introduction and activities using manipulative to Addition of Integers	Manipulatives, Task and Questions sheets.	60
4	Guided discovery of rule in Adding Integers	Manipulatives, Question sheets	30
5	Introduction and activities using manipulative to subtraction of integers	Manipulatives, Task and Questions sheets.	60
6	Guided discovery of rule in subtracting integers	Manipulatives, Question sheets	30
7	Reinforcement activities and exercises related to addition and subtraction of integers	Work sheet	60

The intervention lesson involved the adding and removal of manipulatives to represent the concept in this topic. Two different colored Lego (called counters) to represent the positive and negative signs of numbers, and single counter to represent unit and a 10-stacked counter to represent base-ten (Figure 1). For the activities, adding action to starting counter is used to represent addition, while removal represented subtraction. Zero-pair effect is used and reinforced throughout the intervention.

(a) (b)

Figure 1. Single cube counters (a) and base-ten cube counters (b)

Initially, manipulatives was planned to be used throughout the intervention lesson. However, some students attempted some questions using drawings instead of manipulatives in the addition and subtraction operation lessons, thus drawings were included and encouraged as transition phase of manipulatives to written work.

In this study, the pre-test results act as a baseline performance of students' knowledge prior to the intervention; while post-test measure the impact of interventions on students' performances. Paired sample t-test was initially intended to compare the pre-test and post-test mean scores. However, since the students' results did not satisfy one condition of t-test measure (difference between the pre-test and post-test scores' were not normally distributed), Wilcoxon Signed Ranked Test was used instead to determine whether the two mean scores of the pre-test and post-test were significantly different in students' overall performance. The effect size of the intervention was also measured. In addition, item-by-item analysis was also done to compare overall percentage mean scores of six different combinations of test items to determine which combination of questions were greatly improved by the intervention lessons.

3. Results and discussions

Simple descriptive statistics (Table 3) confirmed there was difference in pre-test and post-test performance before and after the intervention. The mean score increased from 13.18 (*SD*=5.706) in the pre-test to 21.42 (*SD*=5.706) in the post-test.

Table 3. Overall pre-test and post-test scores of the sample based on total marks of 25

Test type	Total	Mean ($\overline{X}$)	Std. dev.(*SD*)	Lowest score	Highest score
Pre-test	50	13.18 (51.75%)	5.706	4	24
Post-test	50	21.42 (84.79%)	4.536	10	25

Out of fifty students in the sample, the highest pre-test mark was 24, scored by one student only. After the intervention, sixteen students obtained a full mark of 25. The lowest mark obtained in the pre-test was 4, whilst in the post-test, 10 was the lowest mark. Only four of the fifty participants scored lower in their post-test compared to pre-test, and one scored the same in both the pre-test and the post-test. Overall, the results suggest that students performed better in the post-test than the pre-test.

The Wilcoxon Signed Rank Test result (Table 4) revealed significance level, $p \leq 0.05$, which indicated that the difference in score between the pre-test and the post-test was significant, with large effect size (*ES* = 0.5788). For effect size, criteria of 0.1 indicates small effect, 0.3 indicates medium effect and 0.5 above indicates large effect [20]. Therefore, these tests provided evidences that students scored significantly better in their post-test as compared to pre-test, and the intervention had a large impact on students' performance and hence the intervention was suitable for all participants.

Table 4. Wilcoxon Signed Rank Test and effect size of the overall results

No. of students (N)	Median		Z	Asymp. Sig. (2-tailed) (p)	Effect size ($ES = \frac{\lvert z \rvert}{\sqrt{2N}}$)
	Pre test	Post-test			
50	12	23	– 5.788	0.000	0.5788

Bar graph (a) in Figure 2 indicated that addition of negative integer with negative integer improved the most (overall increase by 36.0%) followed by addition of positive integer with negative integer (from 55.1% to 89.1%). Most students did not have problem in adding the positive integers to begin with and after intervention, as reflected in high score in both pre-test and post-test scores. Overall, there was improvement in students' performance on the addition of integers in which the overall percentage increased significantly from average of 60.2% in the pre-test to average of 89.7% in the post-test.

(a) (b)

Figure 2. Percentage of correct attempts in the pre-test and post-test for the different combinations of integers in test items involving addition (a) and subtraction (b)

With respect to the subtraction of integers, shown in Figure 2(b), students improved most significantly when subtracting negative integer with negative integer, in which only 33.0% got the answers correct in the pre-test to 83.5% of the total participants who responded correctly in the post-test. The second most improved was when subtracting positive integer with negative integer, which increased by 44.0%. Small improvement was also observed when students subtracted positive integer with positive integer, which started relatively high in pre-test to begin with. Generally, there was a considerable improvement in students' performance in the subtraction of integers, from overall 46.5% (pre-test) to 81.0% (post-test).

Overall result revealed that there was an increased in the mean score of 50 participants from 51.75% in the pre-test to 84.79% in the post-test. The Wilcoxon Ranked Test Analysis of the overall result revealed that the significance level, $p = 0.000 \leq 0.05$ signified that the difference in scores between the pre-test and post-test was significantly different. The large effect size of r = 0.58 implied that the intervention lessons gave a huge impact in their post-test score.

This study also discovered that overall, the intervention in general improved on students' performance in any combinations of signs and operations. The overall means score of correct attempts in the post-tests showed higher on addition of integers (89.7%) compared to subtraction of integers (81.0%). Nonetheless, when comparing their performance improvement from pre-test to post-test, students improved more on the subtraction of integers (with percentage increase of 34.5%) than the addition of integers (with percentage increase of 29.5%).

The findings in this study cannot be generalized to the whole population (since based on relatively small sample size of only 50 students) but it could provide a guideline in planning for remediation. A significant improvement in students' achievement in the computation of integers was observed, largely due to the carefully designed instruction with meaningful use of manipulatives. With this positive indication, teachers are encouraged to use manipulatives in their teaching to promote students' understanding in mathematics [21-23]. This concrete representation truly aids in clarifying mathematical concepts [24]. It also provided meaningful learning experiences for students.

4. Conclusion

In conclusion, in view of the importance of incorporating the use of manipulatives in mathematics teaching and learning as outline in the SPN21 mathematics curriculum framework, there is a need for continuous professional development to equip teachers with necessary content knowledge and pedagogical skills in using various manipulatives [25]. This will ensure that teachers become more competent in teaching using manipulatives, not only to integer topics but to other basic mathematics topics as well.

References

[1] Gningue S M, Menil V C and Fuchs E 2014 *The Electronic J. Math. Tech.* **8** 159
[2] Ismail S F Z, Shahrill M and Mundia L 2015 *Procedia – Soc. and Behav. Sci.* **186** 474
[3] Rubin R J, Marcelino J, Mortel R and Lapinid M R C 2014 Activity-based teaching of integer concepts and its operations *DLSU Research Congress.*
[4] Badarudin B R 2006 *Investigating teaching and learning of integers at form 1 level in Brunei Darussalam* (Brunei: Universiti Brunei Darussalam)
[5] Bofferding L 2010 Addition and subtraction with negatives: Acknowledging the multiple meanings of the minus sign *Proc. of the 32nd Annual Meeting of the North American Chapter of the Int. Grp. for the Psyc. of Math. Ed.* (Columbus, Ohio: Purdue University).
[6] Bolyard J J 2005 *A comparison of the impact of three virtual manipulatives on student achievement and conceptual understanding of integer addition and subtraction* (Fairfax, VA: George Mason University)
[7] Fagnant A, Vlassis J and Crahay M 2005 *Powerful environments for promoting deep conceptual and strategic learning*, eds L Verschaffel, E De Corte, G Kanselaar and M Valcke (Leuven, Belgium: Leuven University Press) p 81
[8] Terao A, Sawaki R, Hasegawa M and Murohashi H 2005 *Improving Skills of Addition and Subtraction Involving Negative Numbers Based on Cognitive Task Analysis and Assessment of Mental Representations of Negative Numbers: A Case Study of a Seventh-Grade Student.* (Troy, NY: Rensselaer Polytechnic Institute)
[9] Vlassis J 2004 *Learning and Instruction* **14** 469
[10] Hing L T 2000 *The teaching and learning of algebraic equations and factorisation in O-level mathematics: A case study* (Brunei: Universiti Brunei Darussalam)
[11] Clements M A 2002 Multiple perspectives and multiple realities of school mathematics *Seventh Annual International Conference of the Department of Science, Mathematics and Technical Education* (Brunei: Universiti Brunei Darussalam)
[12] Khoo S C 2001 *The teaching and learning of geometry* (Brunei: Universiti Brunei Darussalam)
[13] Lim T H 2000 *The teaching and learning of algebraic equations and factorization in O-level Mathematics: A case study* (Brunei: Universiti Brunei Darussalam)
[14] Brownell W A 1935 *The Teaching of Arithemetic, 10th Yearbbook of National Council of Teachers of Mathematics* 1
[15] Khalid M 2006 *Mathematical Thinking in Brunei Darussalam: Implementation Issues and Challenges. 2nd APEC-Tsukuba Int. Conf. Innovation of Classroom Teaching and Learning through Lesson Study II-Focusing on Math. Thinking, CRICED* (Japan: University of Tsukuba)
[16] Charalambous C Y, Hill H C and Mitchell R N 2012 *J. of Curriculum Studies* **44** 489
[17] Hunt A W, Nipper K L and Nash L E 2011 *Current Issues in Middle Level Education* **16** 1
[18] Tanner J and Hale K 2007 *Research and Teaching in Dev. Ed.* **23** 78

[19] Price P C, Jhangiani R S and Chiang I A 2015 Reliability and validity of measurement *Research Methods in Psyc.* (British Columbia, Canada: Kwantlen Polytechnic University)
[20] Cohen J 1992 A power primer *Psychological Bulletin* **112** 155
[21] Khalid M and Tengah M K A 2007 Communication in mathematics: The role of language and its consequences for English as second language students *3rd APEC-Tsukuba International Conf. Innovation of Classroom Teaching and Learning through Lesson Study III—Focusing on Math. Communication, CRICED* (Japan: University of Tsukuba)
[22] Zainil M, *et al* 2018 *J. Phys.: Conf. Ser.* **943** 012046
[23] Runisah, *et al* 2017 Using the 5E learning cycle with metacognitive technique to enhance students' mathematical critical thinking skills *International Journal on Emerging Mathematics Education* **1** 87
[24] Setyawan F, *et al* 2018 *J. Phys.: Conf. Ser.* **943** 012004
[25] Curriculum Development Department 2011 *Framework and guidelines for curriculum and assessment: Mathematics year 7 and year 8* (Brunei Darussalam: Ministry of Education)