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Matrix WKB solution for electromagnetic waves in an inhomogeneous gyrotropic layer

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Abstract. A matrix solution 4x4 was obtained using the Wenzel- Kramers- Brillouin method for inclined incidence of a plane electromagnetic wave on an inhomogeneous gyrotropic torsion layer. The medium is affected by an external nonhomogeneous magnetic field reverses direction within the layer. Solutions for fields in the form of a fundamental matrix of the solution and the Cauchy matrix are obtained. The absolute values of the reflectance matrix coefficients and the reflection energy coefficients for the s and p polarized waves are calculated. The dependence of the cross- polarization coefficients of the reflection matrix of the total rotation angle of the external magnetic field vector within the layer is shown.

1. Introduction

Planar structures are used in the development of new devices, in the manufacture of various optical devices: polarizer's, light modulators, high and low reflection coatings, optical filters, phase compensators, VICSELs, waveguides, signal delay lines, optical splitters. Application of synthetic or natural materials with unusual optical properties can transmit the optical signal to control the characteristics of the light wave: the direction of propagation, localization waves in space, polarization and energy. We use the model of plasma in an external non-uniform time-constant magnetic field. Such a medium has unusual optical properties: gyrotropy and rotation of the optical axis. Usually, a layered models for isotropic [1] and anisotropic media [2]-[3] are used to calculate fields in inhomogeneous materials. The anisotropy and non-homogeneity of the medium complicate the problem considerably. Researchers use various methods of integrating ODE systems [4]. The advantage of matrix methods is that they provide the fulfillment of boundary conditions for projection fields. Such methods allow us to calculate the reflection and transmission coefficients of light and to calculate the field vectors in planar structures.

The use of electro-optical and magneto-optical effects makes it possible to control the optical properties of the medium, and, together with them, wave propagation modes [5], this is important when creating nonreciprocal thin-film devices of integral optics. To calculate fields in planar structures, the coupled mode method is also used. In [6], a planar structure with magnetization normal to the surface of the layer is investigated. It is shown that the waveguide gyro must consist of two different types of waveguide mode converters, the first must be anisotropic, and the second must be magnetically gyrotropic. The model of a layered medium is convenient for the calculation of photonic-crystal cells. The alternation of uniaxial anisotropic and gyrotropic layers makes it possible to create photonic crystal structures with magnetically induced gyrotropy. The external magnetic field controls the modes of wave propagation and the width of the forbidden band [7].

Gyrotropic materials are used to create magnetoplasmic rotators for active control and control of light polarization in devices of integral plasmonics [8]. The strip structure allows modifying the modal distribution of fields between the TE and TM modes. For optical applications it is important that the gyrotropic material has the form of a planar layer containing planar inclusions parallel to the boundaries of the layer [9]. At the moment created new optical device for changing the spatial distribution and temporal transformation optical signals [10], optical couplers [11] based on planar structures, converters polarization of optical beams [12].

Nanotechnology allows the creation of fundamentally new metamaterials with strong gyrotropy. They allow you to control the transmission of light and its polarization. The analytical solution gives an explicit form of the dependences of the field projections on the spatial coordinates and on the parameters of the medium. In [13], within the framework of the Yeh's formalism, a transfer matrix was obtained for a homogeneous birefringent magneto-optical crystal. An algorithm for wave vectors and eigen waves in an anisotropic medium is proposed.

However, for an inhomogeneous medium represented as a set of layers, artificial boundaries between homogeneous layers appear, which does not exist in reality. Field discontinuities appear on non-existent boundaries, artificial reflections and interference are taken into account. We have obtained a matrix mathematical tool to calculate fields in the material a continuously varying properties. It takes into account the inhomogeneity of the medium and gyrotropy. If the angle of torsion of the medium changes then the reflection coefficients of the cross-polarized waves must change. In this paper, we propose a matrix method 4×4 for calculating the vectors of electromagnetic-wave fields in a plane layer of a gyrotropic plasma.

2. The matrix solution for the inhomogeneous gyrotropic layer

Let us consider an oblique incidence of a plane electromagnetic wave (EMW) on a layer of plasma with a constant concentration of charge carriers in thickness. We consider a plane layer. The plasma is in an external magnetic field $\vec{H}_{ext}(z)$ and is gyrotropic; the angle τ between the plane of incidence of the wave and the direction of the vector $\vec{H}_{ext}$ can be arbitrary. In our case, it is a function of the coordinate z, as shown in Figure 1. Function $\tau(z)$ - is a slowly varying function at distances of the order of the wavelength of light. The permittivity tensor of inhomogeneous plasma in an external magnetic field also depends on the z coordinate and has the form [14]. Rotation of the coordinate system relative to the 0Z axis leads the tensor to the form:

$$\hat{\varepsilon}=\begin{pmatrix} 1-\frac{\omega_p^2}{\omega^2-\omega_H^2}\sin^2\tau-\frac{\omega_p^2}{\omega^2}\cos^2\tau & \frac{-\omega_p^2\omega_H^2}{2\omega^2\left(\omega^2-\omega_H^2\right)}\sin 2\tau & i\frac{\omega_p^2\omega_H}{\left(\omega^2-\omega_H^2\right)\omega}\sin\tau \\ \frac{-\omega_p^2\omega_H^2}{2\omega^2\left(\omega^2-\omega_H^2\right)}\sin 2\tau & 1-\frac{\omega_p^2}{\omega^2-\omega_H^2}\cos^2\tau-\frac{\omega_p^2}{\omega^2}\sin^2\tau & i\frac{\omega_p^2\omega_H}{\left(\omega^2-\omega_H^2\right)\omega}\cos\tau \\ -i\frac{\omega_p^2\omega_H}{\left(\omega^2-\omega_H^2\right)\omega}\sin\tau & -i\frac{\omega_p^2\omega_H}{\left(\omega^2-\omega_H^2\right)\omega}\cos\tau & 1-\frac{\omega_p^2}{\omega^2-\omega_H^2} \end{pmatrix}. \tag{1}$$

Here $\omega_p=\sqrt{\frac{4\pi e^2 N_e}{m}}$ – is the plasma frequency, $\omega_H=\frac{eH_{ext}}{mc}$ – is the gyroscopic frequency. The components of the permittivity tensor $\hat{\varepsilon}$ depend on z because the direction of the magnetic field vector depends on the coordinate z.

The change in the projections of the vectors $\vec{E}$ and $\vec{H}$ in a medium having optical properties (1) will be determined by a system of four ordinary differential equations (ODE) 4×4, which follows from the Maxwell equations:

$$\frac{d}{dz}\begin{pmatrix} E_y \\ H_x \\ H_y \\ E_x \end{pmatrix} = ik_0 \begin{pmatrix} 0 & -\mu & 0 & 0 \\ -\left(\varepsilon_{22}-\frac{\varepsilon_{23}\varepsilon_{32}}{\varepsilon_{33}}-\frac{\alpha^2}{\mu}\right) & 0 & \frac{\alpha\varepsilon_{23}}{\varepsilon_{33}} & -\left(\varepsilon_{21}-\frac{\varepsilon_{23}\varepsilon_{31}}{\varepsilon_{33}}\right) \\ \left(\varepsilon_{12}-\frac{\varepsilon_{13}\varepsilon_{32}}{\varepsilon_{33}}\right) & 0 & -\frac{\alpha\varepsilon_{13}}{\varepsilon_{33}} & \left(\varepsilon_{11}-\frac{\varepsilon_{13}\varepsilon_{31}}{\varepsilon_{33}}\right) \\ -\frac{\alpha\varepsilon_{32}}{\varepsilon_{33}} & 0 & \mu-\frac{\alpha^2}{\varepsilon_{33}} & -\frac{\alpha\varepsilon_{31}}{\varepsilon_{33}} \end{pmatrix} \begin{pmatrix} E_y \\ H_x \\ H_y \\ E_x \end{pmatrix}. \quad (2)$$

Or
$$\frac{d\vec{Q}}{dz} = ik_0\hat{A}(z)\vec{Q}\,. \quad (3)$$

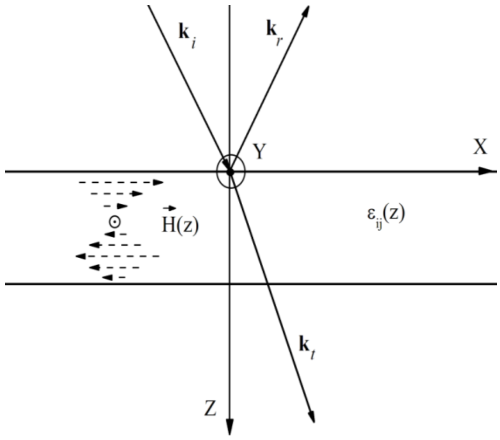

Figure 1. The plane of incidence of an EMW on an inhomogeneous gyrotropic plasma layer.

The value of α is introduced for a brief description of the Snell's law $\alpha = \frac{k_\parallel}{k_0} = n(z)\cdot\sin\theta(z) = const$ in the equations; $k_\parallel$ - is the projection of the wave vector on to the interface z = 0; It retains its value for any z. The components of $\varepsilon_{ij}(z)$ are functions of frequency ω and coordinates z. They also depend on the electron concentration N_e, and on the absolute value of the vector $\vec{H}_{ext}$. The vector $\vec{Q}$ is composed of the projections of the electric and magnetic field strengths: $\vec{Q}(z) = \left(E_y(z) \quad H_x(z) \quad H_y(z) \quad E_x(z)\right)^T$. We obtain the solution of (2) by the WKB method. We seek the proper solutions of the system in the form: $\exp\left(ik_0\sigma(z)\right)$, where $\sigma(z) = \sigma_0(z) + \frac{\sigma_1(z)}{ik_0} + \dots$. The analysis of tensor (1) and system (2) shows that $\varepsilon_{12}\varepsilon_{33} - \varepsilon_{13}\varepsilon_{32} = \varepsilon_{21}\varepsilon_{33} - \varepsilon_{23}\varepsilon_{31}$, $\varepsilon_{23} = -\varepsilon_{32}$, $\varepsilon_{13} = -\varepsilon_{31}$. Therefore, the matrix $\hat{A}(z)$ is transformed to the form:

$$\hat{A}(z) = \begin{pmatrix} 0 & -b & 0 & 0 \\ -c & 0 & p & -h \\ h & 0 & -d & e \\ p & 0 & f & d \end{pmatrix}. \quad (4)$$

The parameters *b, c, d, e, f, h, p* in (4) are functions of the *z* coordinate. The matrix $\hat{A}(z)$ for arbitrary z has four eigenvalues λ_i:

$$\lambda_{1,2} = \pm\sqrt{\frac{d^2+ef+bc+\sqrt{\left(d^2+ef-bc\right)^2+4bh\left(fh-dp\right)-4bp\left(dh+ep\right)}}{2}}, \quad (5)$$

$$\lambda_{3,4} = \pm\sqrt{\frac{d^2+ef+bc-\sqrt{\left(d^2+ef-bc\right)^2+4bh\left(fh-dp\right)-4bp\left(dh+ep\right)}}{2}}. \quad (6)$$

These values correspond to four independent waves. Each eigenvalue λ_i of the system (2) corresponds to the eigenvector $\vec{Q}$ have the form:

$$\vec{Q}_i = e^{ik_0\int_0^z \lambda_i(\xi)d\xi}\begin{pmatrix} E_{y,i} \\ H_{x,i} \\ H_{y,i} \\ E_{x,i} \end{pmatrix}. \quad (7)$$

Waves propagating in the forward direction correspond to the eigenvalues of $\lambda_{1,3}$, and to waves propagating in the opposite direction of the value - $\lambda_{2,4}$. The waves corresponding to $\lambda_{1,2}$ are extraordinary, and the waves corresponding to $\lambda_{3,4}$ - are ordinary. In the particular case when $\tau = 90^0$ and the vector $\vec{H}_{ext}$ are perpendicular to the plane of incidence of the EMW, then: $\lambda_{1,2} = \pm\sqrt{d^2+ef}$, $\lambda_{3,4} = \pm\sqrt{bc}$, and the solutions of the ODE system (2) are waves of s- and p-polarization. The matrix solution for these two waves in an inhomogeneous anisotropic plasma was obtained in [15] by the Wentzel–Kramers–Brillouin (WKB) method. Of the four functions, the form (7), and the equations of system (1) for the general case, there follows the fundamental matrix of the solution (FMS) $\hat{Y}(z)$, of the form [16]:

$$\hat{Y}(z) = \hat{F}\cdot diag\left[e^{ik_0\int_0^z \lambda_1(\xi)d\xi},\ldots,\ e^{ik_0\int_0^z \lambda_4(\xi)d\xi}\right], \quad (8)$$

and the Cauchy matrix: $\hat{N}(z,z_0) = \hat{Y}(z)\hat{Y}^{-1}(z_0)$ [17]. The matrix factor $\hat{F}(z)$ in the formula (8) is determined by substituting the form of the solution in system (2) Formula (8) gives the initial approximation in the WKB method, each eigenvalue (5)-(6) corresponds to the function $\sigma_{0,k} = \int_0^z \lambda_k(\xi)d\xi$. To find the FMS, we first write down the solutions for the projections H_x and E_x with considering (8):

$$H_x = S_1 e^{ik_0\int_0^z \lambda_1(\xi)d\xi} + S_2 e^{ik_0\int_0^z \lambda_2(\xi)d\xi}, \quad (9)$$

$$E_x = S_3 e^{ik_0\int_0^z \lambda_3(\xi)d\xi} + S_4 e^{ik_0\int_0^z \lambda_4(\xi)d\xi}. \quad (10)$$

We substitute these functions in system (2). For the projections E_y and H_y from the system (2) we obtain system of linear equations:

$$\begin{cases} ik_0 cE_y - ik_0 pH_y = -\frac{d}{dz}H_x + ik_0 hE_x, \\ ik_0 pE_y + ik_0 fH_y = \frac{d}{dz}E_x - ik_0 dE_x. \end{cases} \quad (11)$$

Then $\hat{Y}(z)$ takes the form:

$$\hat{Y}(z)=\begin{pmatrix} -\frac{S_1 f\lambda_1}{\Delta(z)}e^{ik_0\int_0^z\lambda_1(\xi)d\xi} & \frac{S_2 f\lambda_1}{\Delta(z)}e^{-ik_0\int_0^z\lambda_1(\xi)d\xi} & S_3\frac{hf+p(\lambda_3-d)}{\Delta(z)}e^{ik_0\int_0^z\lambda_3(\xi)d\xi} & S_4\frac{hf-p(\lambda_3+d)}{\Delta(z)}e^{-ik_0\int_0^z\lambda_3(\xi)d\xi} \\ S_1 e^{ik_0\int_0^z\lambda_1(\xi)d\xi} & S_2 e^{-ik_0\int_0^z\lambda_1(\xi)d\xi} & 0 & 0 \\ \frac{S_1 p\lambda_1}{\Delta(z)}e^{ik_0\int_0^z\lambda_1(\xi)d\xi} & \frac{S_2 p\lambda_1}{\Delta(z)}e^{-ik_0\int_0^z\lambda_1(\xi)d\xi} & S_3\frac{c(\lambda_3-d)-ph}{\Delta(z)}e^{ik_0\int_0^z\lambda_3(\xi)d\xi} & -S_4\frac{c(\lambda_3+d)+ph}{\Delta(z)}e^{-ik_0\int_0^z\lambda_3(\xi)d\xi} \\ 0 & 0 & S_3 e^{ik_0\int_0^z\lambda_3(\xi)d\xi} & S_4 e^{-ik_0\int_0^z\lambda_3(\xi)d\xi} \end{pmatrix}. \quad (12)$$

Here we introduce the notation for the determinant of the system (11): $\Delta(z)=c(z)f(z)+p^2(z)$. We find the matrix $\hat{Y}^{-1}(0)$:

$$\hat{Y}^{-1}(0)=\begin{pmatrix} -\frac{c(0)}{2S_1\lambda_1(0)} & \frac{1}{2S_1} & \frac{p(0)}{2S_1\lambda_1(0)} & \frac{h(0)}{2S_1\lambda_1(0)} \\ \frac{c(0)}{2S_2\lambda_1(0)} & \frac{1}{2S_2} & -\frac{p(0)}{2S_2\lambda_1(0)} & -\frac{h(0)}{2S_2\lambda_1(0)} \\ \frac{p(0)}{2S_3\lambda_3(0)} & 0 & \frac{f(0)}{2S_3\lambda_3(0)} & \frac{d(0)+\lambda_3(0)}{2S_3\lambda_3(0)} \\ -\frac{p(0)}{2S_4\lambda_3(0)} & 0 & -\frac{f(0)}{2S_4\lambda_3(0)} & \frac{-d(0)+\lambda_3(0)}{2S_4\lambda_3(0)} \end{pmatrix}. \quad (13)$$

In the formulas (12) - (13), the values of the variable coefficients of the matrix of the system (2) in the plane z = const are assumed to be the values *c, d, f, h, p,* λ_i, Δ(z) and the values *c(0), d(0), f(0), h(0), p(0),* $\lambda_i(0)$ – are the values of the same quantities in the z = 0 plane. The coefficients n_{ij} of the Cauchy matrix are calculated by the formula: $\hat{N}(z,0)=\hat{Y}(z)\hat{Y}^{-1}(0)$. Matrix methods allow the boundary conditions for the fields at the media interfaces to be stitched, the fields in thin films and waveguides can be calculated [17]. The Cauchy matrix $\hat{N}(z,0)$ makes it possible to stitch solutions on the boundary of the layer and calculate the reflection matrix from the values of n_{ij} [18]:

$$\hat{R}=\begin{pmatrix} R_{ss} & R_{sp} \\ R_{ps} & R_{pp} \end{pmatrix}. \quad (14)$$

When the inhomogeneously gyrotropic layer is reflected at the boundary, the polarization and amplitude of the wave vary according to the rule:

$$\begin{pmatrix} E_{s,r} \\ E_{p,r} \end{pmatrix}=\begin{pmatrix} R_{ss} & R_{sp} \\ R_{ps} & R_{pp} \end{pmatrix}\begin{pmatrix} E_{s,i} \\ E_{p,i} \end{pmatrix}. \quad (15)$$

3. Calculations

For a plane layer of thickness $d=5\lambda_0$, with parameters: $\mu=1$, whose dielectric permittivity tensor has the form (1), the modues of the coefficients of the reflection matrix (14) were calculated. The ratio of the plasma frequency to the radiation frequency: $u=\omega^2{}_p/\omega^2=2.5$, the ratio $W=\omega_H/\omega=0.03$, the angle τ depends on the coordinate z: $\tau=\mathrm{T}_0\cdot z/d$, the angle T_0 takes the values: 0^0, 15^0, $,90^0$. The absolute values of $|R_{ss}|$ and $|R_{pp}|$ are close to unity. The dependencies of $|R_{sp}|$ and $|R_{ps}|$ are shown in the Figure 2. With increasing rotation angle T_0 in the medium, the values of $|R_{sp}|$ and $|R_{ps}|$ increase. The energy reflection coefficients for s- and p-polarization waves were calculated from the formulas:

$$\Re_S=(R_{SS}+R_{SP})(R_{SS}+R_{SP})^*, \quad (16)$$

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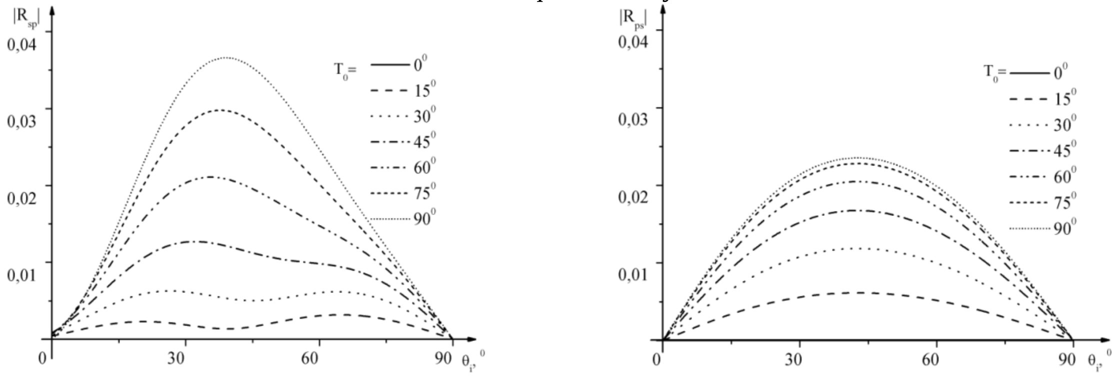
$$\Re_P = \left(R_{PS} + R_{PP}\right)\left(R_{PS} + R_{PP}\right)^* \tag{17}$$

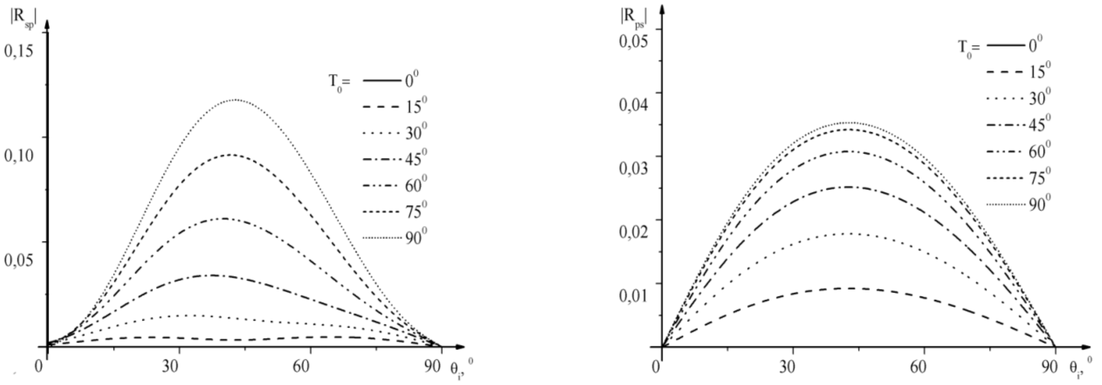
The calculation showed that their values are equal to unity.



a) b)

Figure 2. Absolute values of the reflection coefficients for $W = 0.03$: a) R_{sp}, b) R_{ps}.

For a layer, thickness $d = 5\lambda_0$ with $u = \omega^2{}_p / \omega^2 = 2.5$ and $W = \omega_H / \omega = 0.045$ the modules of the coefficients of the matrix $\hat{R}$ were calculated for a linear change in the angle τ. For a given u y in the case of an isotropic plasma, a complete reflection occurs at its boundary. The angle T_0 assumed the values: 0^0, 15^0, 90^0. The dependencies of $|R_{sp}|$ and $|R_{ps}|$ are shown in the Figure 3. With increasing T_0 the angular dependences of $|R_{sp}|$ and $|R_{ps}|$ increase. The calculation showed that for the given parameters, complete internal reflection is performed. The calculation showed that for the given parameters, total internal reflection is performed. This is in good agreement with the known results: in both cases the frequency ω was less than the plasma frequency ω_p.

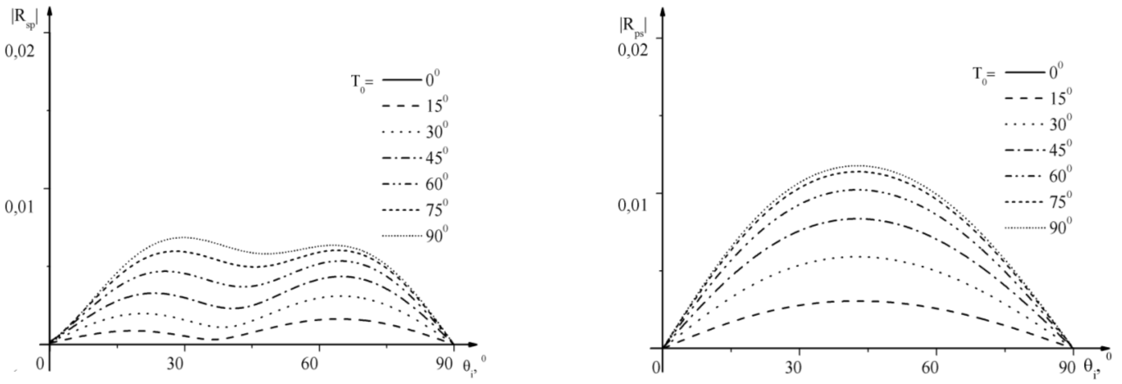


a) b)

Figure 3. Absolute values of the reflection coefficients for $W = 0.045$: a) R_{sp}, b) R_{ps}.

For a layer with the same thickness d at a ratio of squares of the frequencies $u = 2.5$ was performed calculation modules reflection coefficient matrix. The full torsion angle T_0 assumed the values 0^0, 15^0, 90^0.The calculation showed that the absolute values of the $|R_{ss}|$ and $|R_{pp}|$ are close to unity, and the coefficients of $|R_{sp}|$ and $|R_{ps}|$ grow in absolute value with increasing torsion angle T_0.

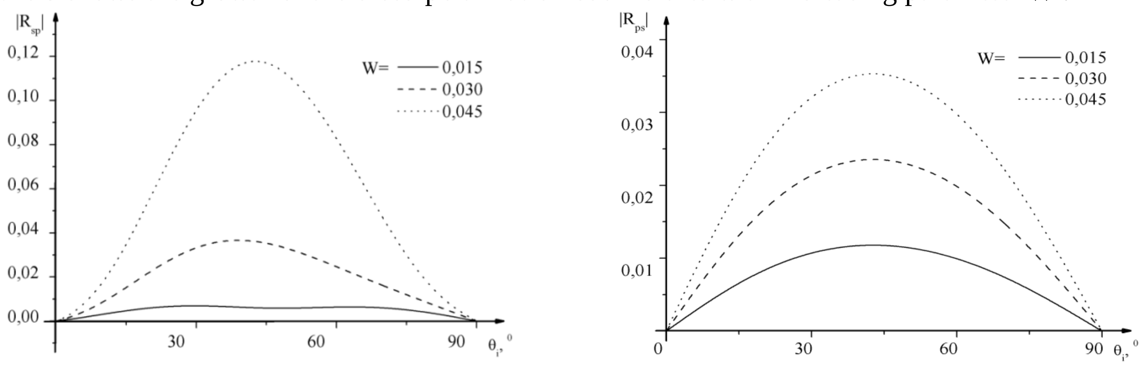
The values of the energy reflection coefficients $\Re_S$ and $\Re_P$ are equal to one.



a) b)

Figure 4. Absolute values of the reflection coefficients for $W = 0.015$: a) R_{sp}, b) R_{ps}.

The calculated dependences for the values of the $|R_{sp}|$ and $|R_{ps}|$ are affected by the change in direction and the value of the field strength, which determines the gyroscopic frequency of the ohm. Figure 5 shows the growth of the cross-polarization coefficients with increasing parameter W .



a) b)

Figure 5. The dependence of the absolute values of the coefficients R_{sp} and R_{ps} with a change in W: a) R_{sp}, b) R_{ps}.

4. Conclusions

A matrix method for calculating fields in an inhomogeneous gyrotropic layer is obtained. The coefficients of the reflection matrix for the gyrotropic plasma layer are calculated, the optical properties of which depend on the transverse coordinate inside the layer. It is shown that when light is reflected from a gyrotropic medium with torsion, cross-polarized components appear. With an increase in the torsion angle from 0 to 90^0, the moduli of the reflection coefficients for cross-polarized components reach maxima at 90^0. The cross-polarization increases with increasing magnetic field strength.

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