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What are the common errors made by students in solving logarithm problems?

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Abstract. This study analysed students' errors in solving problems related to logarithm properties and logarithm equations. It is a qualitative case study. The participants were 11 out of 66 public senior high school students in Yogyakarta of the class X MIPA 5 and X MIPA 6 who enrolled in Supplementary Mathematics course. They were purposively selected because when the result of the test was given back to the 66 students and after two weeks these 66 students were asked to recollect their result of the test, only 11 students who still keep their result. Miles & Huberman model of qualitative data analysis was used in this study. Four types of errors that adapted from a framework proposed by Movshovitz-Hadar, Zaslavsky, and Inbar [1] were used to categorize the errors made by students. This study found that the common errors made by students were technical error, distorted theorem or definition, unverified solution, and misused data with the number of errors of 24 (38.71%), 15 (24.19%), 12 (19.36%), and 11 (17.74%) respectively. This finding indicates that the students were careless in calculating or manipulating algebraic operation, tendentious to treat "log" as a variable, and deficient in mastering the concept of exponent and logarithm.

1. Introduction

One of the important things in mathematics learning is problem-solving. Problem-solving simply can be understood as an activity to achieve a goal by using an appropriate way [2]. Swartz and Perkins define problem-solving as ability in identifying, representing, selecting a strategy, applying the strategy and evaluating the process in order to gain the solution to the problem [3]. Moreover, according to Krulik and Rudnick [4], problem-solving is a process of finding the answer to an unfamiliar situation by using the knowledge, skills, and understanding. The unfamiliar situation then called as a problem. Therefore, problem-solving in mathematics learning context has a meaning as a process and skill that is required for obtaining the solution to mathematics problem.

Problem-solving may be difficult for many students due to the complexity of problem-solving process itself—identify the problem, activate what they already know and construct a solution to the problem [5]; difficulty in understanding and applying mathematical facts, concepts, principles, or procedures [6,7]; and lack of many mathematics skills such as number fact skill, arithmetic skill, information skill, language skill, and visual spatial skill [8]. The difficulties in solving mathematics problem can lead students to do some errors. Olivier states that errors are the result of planning in problem-solving and will be systematic if those errors appear regularly in the same situation due to student's existing conceptual structures [9]. This student's existing conceptual structures then known

as a misconception. Ashlock argues that misconceptions are the result of overgeneralizing and over-specializing towards mathematics concept [10]. Misconceptions can also as a result of gapping between assimilation and accommodation in the learning process [11]. The difficulties and errors in solving mathematics problem appear when mathematical concepts are not mastered by students [12]. In addition, Radatz [13] mentions that there are various causes of errors, i.e., language difficulties; difficulties in obtaining spatial information; insufficient mastery of prerequisite skills, facts, and concepts; incorrect associations or rigidity of thinking; application of irrelevant rules of strategies.

One of the very important aspects of the implementation of education is an assessment [14]. Recalling that the assessment is not merely for measuring students' learning achievement and the urgency for teacher to know common errors made by students in solving mathematics problem, such a crucial thing for the teacher to do one of the types of assessment, i.e., error analysis, in attempting to minimize students' misconceptions and errors in problem-solving. An error analysis is a diagnostic assessment which aimed to detect type(s) of errors and systematic errors made by students, identify students' misconceptions, and select an appropriate instructional method that facilitates students to overcome their errors and acquire the correct concept or procedure [15]. Therefore, through error analysis, the assessment is emphasized on both students' final answer and process of problem solving completion [16].

A logarithm is one of the topics that learned by 10th grade of senior high school students in the Supplementary Mathematics course on the first semester [17]. This topic divided into 4 subtopics, namely logarithm function, logarithm equation, logarithm inequality, and application of logarithm function in the real world situation. Unfortunately, logarithm has become the difficult topic for students and students often suffered by misconception when learning logarithm [18]. As a consequence, students oftentimes do errors in solving logarithm problem. Many studies have been done for exploring students' errors in solving logarithm problems [9, 18–21]. However, less attention has been focused on the common errors made by senior high school students in solving problems related to properties of logarithm, the application of logarithm function in daily life situation, and logarithm equation of the form $A(a \log f(x))^2 + B(a \log f(x)) + C = 0$ and ${}^{f(x)}\log g(x) = {}^{f(x)}\log h(x)$ simultaneously.

Considering the above situation, this study is aimed at analysing the common errors made by students in solving logarithm properties and logarithm equation problems by using the framework proposed by Movshovitz-Hadar, Zaslavsky, and Inbar [1]. This framework was modified by considering the type of problems. Knowing students' errors in solving logarithm problem could be beneficial for teachers to know students' understanding of logarithm, employ a proper intervention to reduce and overcome such errors and create a better mathematics learning quality.

2. Methods

A qualitative case study approach was used in this study. A case study is used to gain an understanding of a case or multiple cases thoroughly [22]. Furthermore, the type of this case study was an intrinsic case study. It means that the study of the case, i.e., common errors made by students, is of primary interest and this study is not aimed to construct theory rather than to know more about the uniqueness of common errors made by students in solving logarithm problems [23].

The participants of this study were 11 out of 66 public senior high school students in Yogyakarta—2 males and 9 females—of the class X MIPA 5 and X MIPA 6 who enrolled in Supplementary Mathematics course. They were purposively selected because when the result of the test was given back to the 66 students and after two weeks these 66 students were asked to recollect their result of the test, only 11 students who still keep their result. However, the less participant was not a problem for this study because this study was not intended to generalize rather than to understand a unique phenomenon deeply.

A constructed response test was employed to gather data. To guarantee that there was no academic dishonesty, five different sets of test—set A, B, C, D, and E—with similar content were used in this test. Each set comprised of five questions related to the graph of logarithm and its characteristics (item

1), properties of the logarithm (item 2), application of logarithm in daily life (item 3), and logarithm equations (item 4a and item 4b). However, in order to meet the objective of this study, we just focused on the last three topics as well. The data that had been gathered then were analysed based on Miles and Huberman [24] model of qualitative data analysis. This model consists of three consecutive phases, namely data reduction phase; data display phase; and conclusion drawing and verification phase. In the data reduction phase, we examined every process of students problem solving and classified the right and the wrong processes (errors). Afterward, in the data display phase, we classified the type of errors made by students. In this study, we just concerned with four types of Movshovitz-Hadar, Zaslavsky and Ibar error analysis, namely misused data; distorted theorem or definition, unverified solution; and technical error (see Table 1). Finally, in the last phase, we derived a conclusion and verified this conclusion to ensure that the conclusion was suitable with the objective of this study.

Table 1. Description of errors that adapted from Movshovitz-Hadar, Zaslavsky, and Inbar [1].

Type of error	Description of error
Misused data	<ul style="list-style-type: none"> - Ignoring the given data that important to find a solution - Using a data which is different with the given data - Adding irrelevant or extraneous data - Using a numerical value of one variable for another variable
Distorted theorem or definition	<ul style="list-style-type: none"> - Applying a theorem or definition outside its condition - Applying a distributive property to a non-distributive function or operation - Incorrectly citing a definition, theorem, rule or formula
Unverified solution	<ul style="list-style-type: none"> - The final result is not the solution to the problem: error in examining the final result
Technical error	<ul style="list-style-type: none"> - Error in calculation due to carelessness - Error in manipulating algebraic symbol or operation - Error in applying an algorithm

3. Result and Discussion

Data from constructed response test revealed that the highest number of errors made by students was in item 4b, which was 23 (37.10%) in various type of errors compared to another items. On the other hand, the lowest number of errors made by students was in item 3, which was 6 (9.68%) in various type of errors compared to another items. Furthermore, according to the types of errors, the common errors made by students were technical error, distorted theorem or definition, unverified solution, and misused data with the number of errors of 24 (38.71%), 15 (24.19%), 12 (19.36%), and 11 (17.74%) respectively. The detailed results are summarized in Table 2.

Table 2. The total number of errors made by students in solving logarithm problems

Item	Misused Data	Distorted Theorem or Definition	Unverified Solution	Technical Error	Total
2	4 (6.45%)	5 (8.06%)	0 (0.00%)	7 (11.29%)	16 (25.81%)
3	0 (0.00%)	3 (4.84%)	0 (0.00%)	3 (4.84%)	6 (9.68%)
4a	1 (1.61%)	5 (8.06%)	5 (8.06%)	6 (9.68%)	17 (27.41%)
4b	6 (9.68%)	2 (3.23%)	7 (11.29%)	8 (12.90%)	23 (37.10%)
Total	11 (17.74%)	15 (24.19%)	12 (19.36%)	24 (38.71%)	62 (100%)

3.1. Students' errors in solving logarithm properties problems

This section discusses the errors made by the students in solving logarithm properties problems which represented by item 2 and 3 according to types of errors proposed by Movshovitz-Hadar, Zaslavsky, and Inbar [1]. From the Table 2, we can see that the common errors made by students in solving item 2 were a technical error, distorted theorem or definition and misused data with the number of errors of 7 (11.29%), 5 (8.06%) and 4 (6.45%) respectively. For an unverified solution, no error was founded. Moreover, the common errors made by students in solving item 3 were technical error and distorted theorem or definition with the number of errors of 3 (4.84%) for both types of errors. Unverified

solution and unused data were not found in this study for item 3. The detailed errors made by the student in solving logarithm properties problems are provided as follows.

3.1.1. Misused data

The misused data occurs when students ignoring the given data which is crucial to gain the solution, adding irrelevant or extraneous data, even using a variable inappropriately. The examples of misused data are shown in Figure 1(a), Figure 1(b) and Figure 2. Figure 1(a) shows that student used a data which was different from given data. The given data were ${}^2\log(x^3y) = m$ and ${}^2\log(xy^2) = n$, but then the student used variable m and n to represent ${}^2\log x$ and ${}^2\log y$ respectively. Another example of misused data is shown in Figure 1(b). A student used variable x to represent ${}^2\log x$ and used variable y to represent ${}^2\log y$. As a consequence, this student may confuse towards the value of x and y —whether x and y as the arguments or the exponents of logarithm function.

Handwritten student work for Figure 1(a):

$$\begin{aligned} \text{Jawab : } {}^2\log(x^3y) = m &\Rightarrow {}^2\log x^3 + {}^2\log y = m \\ {}^2\log(xy^2) = n &\Rightarrow {}^2\log x + {}^2\log y^2 = n \\ \text{Misal : } {}^2\log x &= m \quad {}^2\log y = n \\ \begin{array}{r|l} 3m + n = m & \times 1 \\ m + 3n = n & \times 3 \end{array} & \begin{array}{l} 3m + n = m \\ 3m + 9n = 3n \end{array} \\ \hline & -8n = m - 3n \\ \hline & -5n = m \end{aligned}$$

Figure 1(a). Student's error in using a value of one variable for another variable.

Handwritten student work for Figure 1(b):

$$\begin{aligned} \text{misal. } x &= {}^2\log x \\ y &= {}^2\log y \end{aligned}$$

Figure 1(b). Student's error in supposing variable x and y .

Another variation of misused data is provided in Figure 2. In item 2, students were given data: ${}^2\log(x^4/y) = m$, ${}^2\log(x/y^2) = n$, and $p = {}^3\log\sqrt[3]{5^2} + 1/{}^{125}\log 9 - {}^7\log 5 / {}^7\log 3$. They were asked to represent ${}^2\log(x^3y) / [p({}^5\log 8)]$ in terms of m and n . But then, there was a student who used a data which was different with the given data. This student wrote $p({}^5\log 8)$ as p and then multiplied the last term of p — ${}^7\log 5 / {}^7\log 3$ —by ${}^5\log 8$ or ${}^5\log 2^3$. In the Figure 2, we can also see that this student also made a distorted theorem or definition. This student did not write $\sqrt[3]{5^2} = 5^{2/3}$; instead $\sqrt[3]{5^2} = 5^{2\frac{1}{3}}$. This finding indicates that the exponential concept is one of the important things that should be mastered by students in order to succeed in solving logarithm problems.

Handwritten student work for Figure 2:

$$\begin{aligned} p &= {}^3\log\sqrt[3]{5^2} + \frac{1}{125}\log 9 - \frac{{}^7\log 5}{{}^7\log 3} \times {}^5\log 2^3 \\ &= {}^3\log 5^{2\frac{1}{3}} + {}^3\log 5^3 - {}^3\log 5 \times {}^5\log 2^3 \\ &= {}^3\log(5^{2\frac{1}{3}} \cdot 5^3) - {}^3\log 2^3 \end{aligned}$$

Figure 2. Student's error in copying the given data.

The distorted theorem or definition is errors made by students due to applying incorrect theorem, definition, rule, or formula of logarithm inappropriately (see Figure 3(a) and Figure 3(b)). The Figure 3(a) shown that student tended to treat “log” as a variable. This student thought that $\log(2.928.200/2.000.000)$ is the multiplication between “log” and $(2.928.200/2.000.000)$. The same thing also holds for the denominator. Thus, this student cancels the “log” from both numerator and denominator. This finding is consistent with the finding of the previous studies that revealed students have a tendency to overgeneralize logarithmic expression as a variable or an object [18,19].

$$\frac{\log(2.928 \cdot 100)}{\log(\frac{100}{100} + \frac{10}{100})} = \frac{2.9282}{20.000} = \frac{11}{10}$$

Figure 3(a). Student's Error in applying logarithm property since treat "log" as a variable.

$$\frac{\log(149,01)}{\log(1,1)} = n$$

$$\log 135 = n$$

Figure 3(b). Student's error in applying logarithm property.

The other distorted theorem or definition error made by student is shown in Figure 3(b). A student thought that $\frac{\log 149,01}{\log 1,1} = \log \frac{149,01}{1,1} = \log 135$; rather than $\frac{\log 149,01}{\log 1,1} = {}^{1,1}\log 149,01$. This error is categorized as distorted theorem or definition because it is the result of applying a distributive property to a non-distributive function or operation [1]. Furthermore, this finding in line with Liang and Wood [18] which found that a quite common misconception suffered by students was assume that $\log x / \log y = \log(x/y)$.

3.1.2. Technical error

A technical error is error due to careless in calculating. The technical error is illustrated in Figure 4. From Figure 4, we know that the objective of a student was eliminating variable b to obtain the value of variable a that was representing the value of ${}^3\log x$. But then, this student was careless in using a plus sign rather than a minus sign.

$$\begin{array}{l} \text{Jawab} \rightarrow {}^3\log x^3 - {}^3\log y^2 = m \quad {}^3\log x^2 - {}^3\log y^4 = n \\ {}^3\log x - 2{}^3\log y = m \quad {}^3\log x - 4{}^3\log y = n \\ {}^3\log x = a \quad 3a - 2b = m \quad \times 2 \quad 6a - 4b = 2m \\ {}^3\log y = b \quad 2a - 4b = n \quad 2a - 4b = n \quad \oplus \\ \hline 8a = 2m + n \end{array}$$

Figure 4. Student's error in doing algebraic calculation/operation.

3.2. Students' errors in solving logarithm equations problems

This section discusses students' errors in solving logarithm equation problems which represented by item 4a and 4b, based on types of errors proposed by Movshovitz-Hadar, Zaslavsky, and Inbar [1]. According to Table 2, the common errors made by students in solving item 4a were technical error, distorted theorem or definition, unverified solution, and misused data with the number of errors of 6 (9.68%), 5 (8.06%), 5 (8.06%), and 1 (1.61%) respectively. For item 4b, the common errors made by students were a technical error, unverified solution, misused data, and distorted theorem or definition with the number of errors of 8 (12.90%), 7 (11.29%), 6 (9.68%), and 2 (3.23%) successively. From this result, we can say that technical error was the type of errors with the highest frequent occurrence. Moreover, the number of errors made by students in solving the test items 4a and 4b indicate that this two test items are difficult for the students. Retnawati, Kartowagiran, Arlinwibowo, and Sulistyaningsih [25] argue that the reason behind the difficulty of the test items for the students is those test items "require complex completion steps or they should be completed through several phases" (p. 259). The detailed errors made by the student in solving logarithm equations problems are explained as follows.

3.2.1. Misused data

Based on the analysis towards students' work, the misused data in solving logarithm equation mostly occurred in form of using a numerical value of one variable for another variable (see Figure 5). In the beginning, a student supposed $y = 5x-2 \log(x^2-8)$. After obtained the values of y , this student substituted that value of y to $y = 5x-2$ not to $y = 5x-2 \log(x^2-8)$. It seems that this student added an irrelevant data. This student assumed that the base of logarithm was $y = 5x-2$ instead of $5x-2$.

$$\begin{aligned}
 & \textcircled{b} \quad [5x-2 \log(x^2-8)]^2 - 5x-2 \log(x^2-8) = 0 \\
 & \text{misal : } 5x-2 \log(x^2-8) = y \\
 & \Leftrightarrow y^2 - y = 0 \\
 & \Leftrightarrow y(y-1) = 0 \\
 & \Leftrightarrow y = 0 \vee y = 1 \\
 & \text{Cek. Bilok :} \\
 & y = 0 \Rightarrow 5x-2 = 0 \\
 & \quad \quad \quad 5x = 2 \\
 & \quad \quad \quad x = \frac{2}{5} > 0 \text{ ok} \\
 & y = 1 \Rightarrow 5x-2 = 1 \\
 & \quad \quad \quad 5x = 3 \\
 & \quad \quad \quad x = \frac{3}{5} > 0 \text{ ok}
 \end{aligned}$$

Figure 5. Student's error in using the obtained data.

3.2.2. Distorted theorem or definition

The Figure 6 shows that the student had a tendency to treat "log" as a variable or common factor. This student treated ${}^x \log 6^2 - {}^x \log x^2$ as ${}^x \log(6^2 - x^2)$. This finding in line with Liang and Wood [18] that commonly found students factorizing ${}^a \log x + {}^a \log y$ into ${}^a \log(x+y)$, just like factorizing $2x+2y$ into $2(x+y)$. The same finding also obtained by Aziz, Puri, and Purnomo [19] where through their research, they found that there was a student who factorizing ${}^2 \log(xy) - {}^2 \log y^2 + {}^2 \log 2$ into ${}^2 \log(xy - y^2 + 2)$. Moreover, Figure 7 shows a student's process in solving logarithm equation problem (item 4a). In this problem, students were asked to find the solution of ${}^x \log(3+12/x) = 4 {}^x \log 3 - {}^x \log x^2$. In the first row of the process, this student thought that ${}^x \log(3+12/x) = {}^x \log 3 \times {}^x \log(12/x)$.

$$\begin{aligned}
 & \textcircled{a} \quad {}^x \log\left(2 + \frac{12}{x}\right) = 2 {}^x \log 6 - {}^x \log x^2 \\
 & {}^x \log\left(2 + \frac{12}{x}\right) = {}^x \log 6^2 - {}^x \log x^2 \\
 & \left(2 + \frac{12}{x}\right) = (36 - x^2)
 \end{aligned}$$

Figure 6. Student's error in treating "log" as a variable.

$$\begin{aligned}
 & {}^u \log 3 \times {}^u \log \frac{12}{u} = 4 \cdot {}^u \log 3 - {}^u \log u^2 \\
 & {}^u \log 3u \times {}^u \log \frac{12}{u} = 4 \cdot {}^u \log 3 - 2 \cdot {}^u \log u \\
 & {}^u \log(3u+12) - 4 \cdot {}^u \log 3 + 2 = 0
 \end{aligned}$$

Figure 7. Student's error in applying logarithm properties and algebraic calculation.

3.2.3. Unverified solution

Oftentimes when students have found the final result, they automatically use that final result as the solution of the problem without verifying whether that final result satisfies the condition of base and arguments of the logarithm. This phenomenon is represented when a student solve logarithm equation problem ${}^x \log(3+24/x) = 3 {}^x \log 3 - {}^x \log x^2$. This students got a final result $x = -9$ or $x = 1$ and wrote this final result as his final answer. But then, unfortunately, this student did not verify that final result first. This student might not realize that the base of logarithm must be positive and not equal to one and the argument of logarithm must be positive as well.

3.2.4. Technical error

From Table 2, we know that the number of technical error in item 4a and 4b is greater than other types of errors. This finding indicates that students were careless in doing a calculation or manipulating algebraic operation. These types of errors are shown in Figure 8(a) and Figure 8(b). Figure 8(a) shows that a student does error in factorizing a quadratic form and for the Figure 8(b) shows that students were confused in determining the value of x which satisfies $x^2 = 4$.

Handwritten student work for Figure 8(a) showing errors in factorizing a quadratic form. The work is as follows:

$$\begin{aligned} 7x+3 \mid \log(x^2-15) &= 0 \\ (7x+3)^0 &= x^2-15 \\ 1 &= x^2-15 \\ 0 &= x^2-16 \\ 0 &= (x-8)(x+8) \\ x &= 8 \quad \vee \quad x = -8 \end{aligned}$$

Figure 8(a). Student's error in factorizing quadratic form.

Handwritten student work for Figure 8(b) showing errors in determining the solution of $x^2 = 4$. The work is as follows:

$$\begin{aligned} 5x+3 \mid \log(x^2-3) &= 0 \\ x^2-3 &= (-5x+3) \\ x^2-3 &= 1 \\ x^2 &= 4 \\ x &= 2 \quad \text{at} \end{aligned}$$

Figure 8(b). Student's error in determining the solution of $x^2 = 4$

The existed errors which was made by students indicate that teacher should implement a new learning paradigm, such as focuses on a role of teacher to help their students in developing and making an interconnection among mathematical concepts through discovery activity [26], train problem solving and higher order thinking skills [16,26,27], and using technology in teaching and learning[28]. Moreover, teachers is expected to consider learning trajectory [29] in learning about logarithm.

4. Conclusion

The technical error, distorted theorem or definition, unverified solution, and misused data respectively are common errors made by the student in solving problem-related to logarithm properties and logarithm equations. This finding indicates that there is a tendency for students to see "log" as a variable and students do not mastery concept of algebra, exponent, and logarithm. Indeed, knowing students errors could be beneficial for the teacher to design an intervention in order to overcome and reduce such errors. Moreover, the teacher can use students' errors as an opportunity for facilitating the other students to get a deeper understanding of logarithm, i.e., presenting incorrect worked example (erroneous example).

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