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# Analysis of the terrestrial global digital model using fractal geometry and harmonic expansion into spherical functions

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**Abstract.** During the implementation of space missions on study of the Solar system a large amount of information on planets geophysics and their morphological properties has been obtained, that could be investigated using fractal geometry. The present paper describes the analysis of the GDEM terrestrial digital model built from the ASTER's observations. GDEM is global digital elevation model and ASTER is advanced spaceborne thermal emission and reflection radiometer. ASTER was installed on the platform of Terra (NASA) orbiter. In our study we used robust methods and fractal analysis. The fractal dimension values for the terrestrial surface, which has a heterogeneous structure, are obtained. The fractal dimensions are determined for geographical latitudes. Independent estimates of the Earth's macrostructure that could be used for a new interpretation of geophysical processes have been obtained as well.

## 1. Introduction

Analyzing structure and evolution of celestial bodies involves various methods for statistical multiparametrical analysis [1–3]. At the present time one of the promising directions of heterogeneous natural objects' structure, materials and their properties investigation is the fractal geometry. For instance, the fractal analysis of the Solar system bodies' parameters has been conducted in the works [4]. The fundamental property of fractal objects is similarity or scaling when zooming. The quantitative measure characterizing distribution of structure in space is the fractal dimension  $D$ . The fractal dimensions investigations allow to study not only structure but connection between structure and its formation processes as well. The fractal structures have been found in the dynamical systems too. The methods for the fractal structure recognition are used for heterogeneous surfaces' properties investigations, finding the similarity in certain parameters. In particular, the methods of the fractal analysis allow to describe quantitatively the models of celestial bodies' surfaces.

The investigations of the Earth figure and rotation, including the works on the development of a map of the Earth, are traditionally conducted on the basis of Engelhardt astronomical observatory (EAO) of Kazan federal university [5]. In particular, development of the inertial celestial reference system has been very successful recently. On the basis of space observations a coordinate system of proper motion and parallaxes of 118 218 stars from "Hipparcos" catalogue with the millisecond precision was obtained. According to the observations of 48 minor planet from the astrometric satellite "Hipparcos" an orientation of space coordinate system relatively to the dynamical was implemented. The disadvantage of this method is a poor conditionality of the system of conditional equations which has a great influence on the solution obtained with the least square method (LSM).

The digital topographic map of the Earth's surface with 99% coverage has been constructed by NASA. During the construction the most accurate measurements taken via Japanese ASTER (Advanced Spaceborne Thermal Emission and Reflection Radiometer) [6] set on the board of Terra (NASA) [7] spacecraft are used. The new map contains more than 1.3 million separate paired stereo-images. Until now the map constructed on the basis of Shuttle Radar Topography Mission (NASA) data and covering about 80% of the surface was considered the most detailed Earth's relief description. ASTER observations cover the territory from 83° of northern latitude to 83° of southern latitude. At the same time an interval between neighboring points is 30 meters. The present project is a part of Global Earth Observation System. NASA, METI, U.S. Geological Survey, and U.S. National Geospatial-Intelligence Agency have performed a study of the ASTER accuracy. NASA Land



Processes Distributed Active Archive Center and Earth Remote Sensing Data Analysis Center METI is responsible for distribution of the maps. ASTER is one of 5 tools for observing the surface of the Earth launched on Terra in December, 1999. The radiometer is capable of collecting data in the range of light frequencies from visible to infrared in spatial resolution from 50 to 300 feet (15-90 meters).

The terrestrial physical surface is a complex system. For the analysis of complex systems used methods of statistical physics. In this article fractal analysis was used to estimate the parameters of the Earth surface.

## 2. Making the GDEM model

As a model describing the Earth relief we use expansion of an altitude function in a series of spherical harmonics in the form of regression [5]:

$$h(\varphi, \lambda) = \sum_{n=0}^N \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \cdot \bar{P}_{nm}(\cos \varphi) + \varepsilon, \quad (1)$$

where:

$\varphi, \lambda$  – latitude, longitude – known parameters of the Earth objects;

$\bar{C}_{nm}, \bar{S}_{nm}$  – normalized harmonic amplitudes;

$\bar{P}_{nm}$  – normalized associated Legendre functions;

$\varepsilon$  – random error of the regression.

Unfortunately the series (1) is slowly convergent. For instance, to describe relief details changing through  $1^\circ$  an order of expansion of about 180 is required which causes the necessity of estimating  $(180+1)^2$  coefficients (amplitudes) of expansion. Practically dimension of the model (1) and therefore an order  $n$  should be set based on the amount of objects approximately equally distributed on a sphere. Their number should exceed the number of estimated objects 5÷15 times.

In terms of the regression modeling approach we solved the overdetermined system (1) for various sources of hypsometric information. The approach involves aside from the usual stages (postulating the model (1) and the amplitudes  $\bar{C}_{nm}, \bar{S}_{nm}$  estimation) using a number of quality statistics including external measures:

- the diagnostics of observance of basic LSM conditions. As LSM computational schemes Gauss-Jordan and Householder schemes were used;

- adaptation in case of their violation.

The main violation of LSM application conditions to processing (1) is the presence of:

- surplus (noise) harmonics which lead to prediction of single altitudes as well as isohypses precision decrease;

- harmonic amplitudes correlating with each other when applied (1) to describe relief on a sphere segment or when objects distribution is highly heterogeneous; in this case the digital model (a set of amplitudes LSM-estimates) should be considered incorrect.

The adaptation to these 2 violations was made possible by the application of step by step regression – well known procedure of regression analysis. It appears to be rather efficient for the model's (1) parameters if:

- a) the objects are distributed over the entire sphere although heterogeneously;

- b) the order of expansion defined by the amount of points should be relatively small ( $n < 15$ ); otherwise the calculation time increases rapidly. In case of objects' homogenous distribution over the entire sphere it is sufficient to eliminate statistically insignificant harmonics and conduct the calculations again.

To obtain the expansion (1) in spherical harmonics in order to form a digital Earth model an automated research system "SFERA" has been used. "SFERA" is created for distribution of various properties (relief, gravitational, magnetic and other kind of potential fields) description on the sphere and its parts by their values measured in points with known coordinates. Using this software the models of (1) kind can be formed, predictions in the form of section, isoline, tone and three-

dimensional representation of property values distributions may be implemented. The models (1) formation is accompanied by their quality control and observance of LSM conditions. In case of their violation the corresponding adaptation methods were applied. “SFERA” software in “split” mode may be used for models of high order when parallel processing. Owing to the fact of using the expansion in harmonic functions and other properties described above the application of “SFERA” allows providing description and prediction precision increase over 40% compared with the popular software package “SURFER”.

The corresponding internal criteria characterizing the estimation accuracy and statistical significance of single coefficients and the whole model in general were determined simultaneously with the harmonic coefficients. The step by step regression procedure (inclusion–exclusion principle) at significance level  $\alpha=0.05$  (statistically significant dependence is used for observance of regression analysis and LSM estimation for the postulated and optimal model) was used to form an optimal structure of a model according to t-criterion (Student’s criterion). It was noted that coefficients values at  $N=40$  almost coincided with the values at  $N=70$ . That confirms the correctness of basic calculations. The regression analysis assumptions (LSM) compliance led to the following conclusions: the model contained about 30% of statistically insignificant terms; paired correlation coefficients  $r_{ij}<0.3$  which indicates the practical orthogonality of the expansion; on the basis of residue analysis it was concluded that there had been some violation of normality condition; Durbin-Watson criterion was 0.58 which meant there had been some autocorrelation of the first order. As a result the digital Earth model was created.

### 3. Fractal analysis of the GDEM model

Methods for objects’ fractal structure and their properties determination are of great interest [8, 9]. Investigations of this kind allow to describe quantitatively the following systems: polymers, colloidal complexes, rough and porous surfaces, branching structures, plots of the Earth and planets, biological objects [10].

The investigation started with building the large Earth segments of surface for each  $45^\circ$  in longitude and finding the number of their coverings by small segments in latitudes. Then for each structure at the initial stage according to the classic formula the fractal dimension was defined which is sufficient to conduct a comparative analysis [9]:

$$D = \lim_{\sigma \rightarrow 0} \frac{\ln N(\sigma)}{\ln \frac{1}{\sigma}}, \quad (2)$$

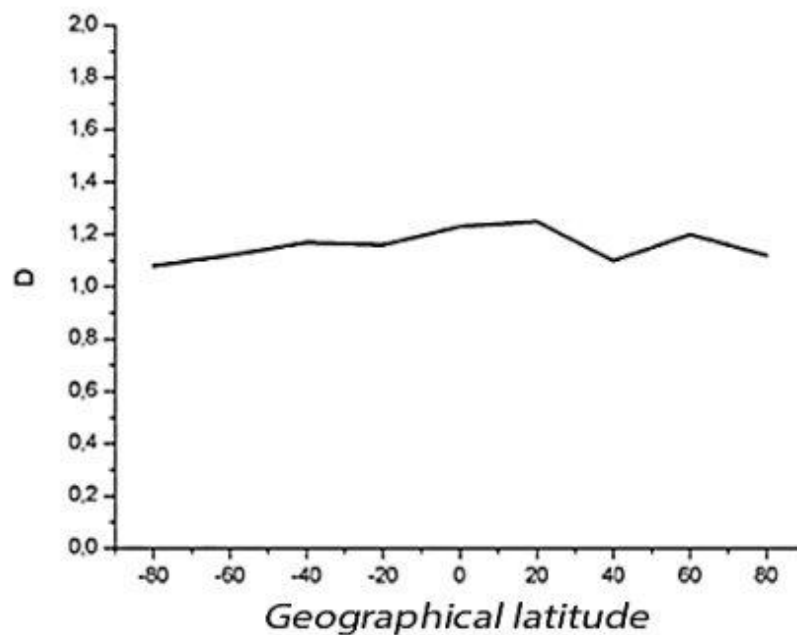
where:

$D$  – fractal dimension;

$\delta$  – small segments dimension;

$N$  – number of small segments.

Figure 1 shows the values of the fractal dimensions of the Earth surface model at different geographical latitudes. From the figure 1 we see a fairly good agreement of fractal dimensions at various Earth geographic latitudes. It can be noted that the results within the error limits are consistent with each other. Considering the fact that in the present work the models of the Earth displaying the same relief were taken, we may conclude the method of fractal dimension calculation allows to obtain reliable maps similarity estimates. In addition the values of fractal dimension shows the reference surfaces for altitude data have no influence on the models’ similarity estimates.



**Figure 1.** Values of the fractal dimensions of the Earth surface segments with respect to geographical latitude.

#### 4. Summary and conclusions

In this work the fractal dependences for Earth surface model have been defined; for geographical latitudes have been obtained the averaged fractal dimension value  $\langle D \rangle = 1.178$ .

It should be noted that analysis of complex physical systems using the fractal method makes it possible to estimate their similarity. Thus, the use of a fractal geometry for studying physical celestial objects and processes is an important direction [8–10]. In these studies, the invariant properties of the fractal method are very important too. The nonuniformity of nonlinear processes and complex topographic systems can be investigated by obtaining fractal dimensions of values.

Also, physical processes, including those related to the Earth, can be successfully studied with the use of the fractal method. In further the fractal comparative analysis use for space measurements data reducing will surely bring some interesting results which will allow to solve certain problems of space geodesy. The further use of the fractal comparative analysis at space measurement data processing will certainly bring the results allowing to solve some problems of space geodesy [11–14].

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