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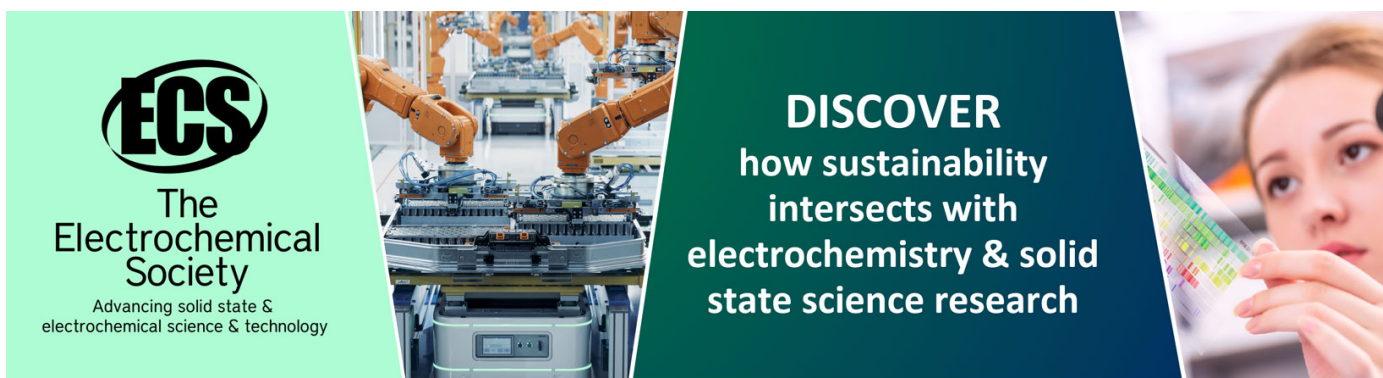
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A ground state and quantization of the energy of matter in the Friedmann universe

N N Gorobey, A S Lukyanenko, A A Shavrin

Peter the Great St. Petersburg Polytechnic University; 29 Politekhnicheskaya St., St. Petersburg, 195251, Russian Federation

n.gorobey@mail.ru, alex.lukyan@rambler.ru, shavrin.andrey.cp@gmail.com

Abstract. The energy of a closed Friedman universe is represented as a difference of two positive quantities – the energy of matter and the energy of expansion of space. They are equal according to the classical constraint equation. In Quantum theory, we study the problem of the minimum (extremum) of the average value of the energy of matter under an additional condition of equality of average values of both energies in a closed universe. As a result, we obtain the spectrum of admitted values of the energy of matter in quantum universe. The minimum value of energy corresponds to a ground state of the universe, which may be considered as its initial state [1]. A toy-model is proposed in which the degrees of freedom of matter and geometry are described by harmonic oscillators with one fitting parameter – the average volume of the universe, which is determined self-consistently by the corresponding stationary energy. Eigenstates and eigenvalues of the energy of matter are found in this self-consistent harmonic approximation. Since these states are not eigenstates of the Hamilton operator of the universe, the ground state will not be stationary and the dynamics of the universe expansion begins with it [2]. This dynamic was described with the use of the internal time formalism [3] in the case when the scalar field serves as a field of matter.

1. Introduction

The idea of the formation of the universe as a result of the Big Bang is one of the greatest ideas of the XX century. It appeared as a result of Einstein's General theory of relativity (GRT) development, initially in a simple homogeneous non-stationary solution of it's equation, described by Friedmann. Then it was confirmed experimentally by Hubble in the form of recession of galaxies. However, for classical GRT it has become a problem, which is called the cosmological singularity. It turns out that the universe arose from a point (in fact – from nothing), and in this initial state the density of matter is infinite. In order to describe the initial state (or avoid it) and the dynamics of the universe around it, it is necessary to modify the GRT. One of the possible modifications is the transition to quantum theory. Quantum theory once saved the classical planetary model of Rutherford's atom from the singularity-the electron falling into the nucleus. We can hope that this time we will find a suitable solution.

The options of the quantum description of the birth of the universe out of Nothing exist. This is the tunneling of the universe from the initial state with zero radius to the state of the final radius of A. Vilenkin [4], as well as the solution of the equations of the quantum theory of gravity in the form of the so-called no-boundary wave function of the Hartle and Hawking universe [5]. Both of these theories have problems with explanation, in particular, no-boundary wave function is not defined beyond the quasi-classical approximation. In this paper, we propose more regular (for the beginning of quantum mechanics) version of the quantum description of the universe near the Beginning in terms of states with a certain energy. This description is analogous to the theory of Bohr atom and is based on the principle of minimum (extremum) of the energy of matter.

The question is, what to understand by the energy of matter in the Friedmann universe (here we limit ourselves to this homogeneous model of the universe). In the paper [1] the Hamiltonian of a (closed) universe is represented as the difference of two positive values, one of which is called the energy of matter, and the other is the energy of expansion of space, or simply space. The Hamiltonian, as a result of Einstein's equations (so-called connections), must be zero. Thus, we come to the conditional principle of minimum (extremum) energy of the universe, which we formulate in the next section for the Friedmann universe. In the second section, within the self-consistent harmonic



approximation, we obtain a set of "stationary" solutions that are numbered by the quantum number of excitation of matter. They describe the "process" of the birth of matter (with space) from "nothing". This is the difference between the quantum model under consideration and the theory of "parametric" birth of quanta of matter on the classical background of the expanding universe [6].

2. Quantization and ground state of the Friedmann universe

The main equation that determines the state of the electron in an atom is the stationary Schrödinger equation:

$$\hat{H}\psi = E\psi \quad (2.1)$$

This equation can be obtained as a result of the extremum condition of the medium energy

$$W = \frac{\langle \psi | \hat{H} | \psi \rangle}{\langle \psi | \psi \rangle} \quad (2.2)$$

on the Hilbert space of states [7]. We are going to look for the ground state of the electron, limiting the variation to suitable States with a minimum set of parameters. Later we will apply this principle of minimum to the energy of matter in a closed Friedmann universe.

We shall confine ourselves here to consider one massive scalar field as a field of matter. The Lagrange function in this case will look like this [2]:

$$L = -\frac{1}{2g}a(\dot{a}^2 - 1) + 2\pi^2 a^3(\dot{\phi}^2 - m^2\phi^2) \quad (2.3)$$

here a – the radius of the universe, ϕ – scalar field of matter, $g = 2G/3\pi$, G – gravitational constant. Let us make a replacement $dx = \sqrt{a} da$:

$$L = -\frac{1}{2g}\dot{x}^2 + \frac{1}{2g}\left(\frac{3}{2}x\right)^{\frac{2}{3}} + \left(\frac{3}{2}x\right)^2 2\pi^2(\dot{\phi}^2 - m^2\phi^2) \quad (2.4)$$

and assume that $-\infty < x < +\infty$. By the help of Legendre transformation we find the matching Hamilton function:

$$H = -H_x + H_\phi, \quad (2.5)$$

Here

$$H_x \equiv \frac{2}{g}p_x^2 + \frac{1}{2g}\left(\frac{3}{2}x\right)^{\frac{2}{3}} \quad (2.6)$$

$$H_\phi \equiv \frac{4}{9\pi^2}\frac{1}{x^2}p_\phi^2 + \frac{9\pi^2}{4}x^2m^2\phi^2 \quad (2.7)$$

Let us move to the quantum theory. To do this, we replace canonical impulses with differential operators and pay attention to the choice of ordering of non-commuting factors. To avoid singularity, we use the same ordering in the spatial part of Hamiltonian $\hat{H}_x$, which arises in the Laplace operator for the radial variable of an electron in a hydrogen atom:

$$\hat{H}_x \equiv -\frac{\hbar^2 g}{2} \frac{1}{x^2} \frac{\partial}{\partial x} \left(x^2 \frac{\partial}{\partial x} \right) + \frac{1}{2g} \left(\frac{3}{2} x \right)^{\frac{2}{3}} \quad (2.8)$$

For this operator to be Hermitian, like a radial variable, the measure of integration in the scalar product must contain a multiplier x^2 :

$$\langle \psi_1 | \psi_2 \rangle \equiv \int_{-\infty}^{+\infty} \int x^2 \psi_1^* \psi_2 dx d\phi \quad (2.9)$$

Now let us formulate the principle of the energy of matter minimum for finding the ground and excited "stationary" states of the universe. We use inverted commas, because in the next section these states will be assigned with a dynamic meaning. The principle of minimum (extremum) in the case of a closed universe is conditional. Condition is that the total energy of the universe must be zero. In quantum theory, it is enough for us to turn the mean value to zero.

$$\langle \psi | \hat{H} | \psi \rangle = 0 \quad (2.10)$$

Thus, we come to the principle of extremum for the functional

$$W = \frac{\langle \psi | \hat{H} | \psi \rangle}{\langle \psi | | \psi \rangle} + \lambda \langle \psi | (-\hat{H}_x + \hat{H}_\phi) | \psi \rangle, \quad (2.11)$$

here λ is Lagrange multiplier. The variational parameters are $\psi, \bar{\psi}$ are elements of the Hilbert space of states and λ .

To approximate the state with minimal energy, we limit the variation to a two-parameter family of functions

$$\psi(x; \phi) = A e^{-\frac{x^2}{2\chi^2} - \frac{\phi^2}{2\sigma^2}}, \quad (2.12)$$

here χ, σ are parameters. In this case, the average energy of the scalar field is

$$W_\phi = \frac{1}{2} \frac{2}{9\pi^2} \hbar^2 \frac{1}{\sigma^2} \frac{1}{\chi^2} + \frac{3}{2} \frac{9\pi^2}{2} m^2 \frac{\chi^2}{2} \frac{\sigma^2}{2}. \quad (2.13)$$

Let us find the extremum of the parameter σ . According to the condition $\partial W_\phi / \partial \sigma^2 = 0$

$$\sigma^2 = \frac{2}{9\pi^2} \frac{\hbar}{m} \sqrt{\frac{4}{3}} \frac{1}{\chi^2}. \quad (2.14)$$

Variation of (1.11) for λ gives the equation

$$-3 \frac{g}{2} \hbar^2 \frac{1}{\chi^2} - \frac{1}{\sqrt{\pi}} \frac{1}{2g} \left(\frac{3}{2}\right)^{\frac{2}{3}} \Gamma\left(\frac{11}{6}\right) \chi^{\frac{2}{3}} + \frac{1}{2} \frac{2}{9\pi^2} \hbar^2 \frac{1}{\sigma^2} \frac{1}{\chi^2} + \frac{3}{2} \frac{9\pi^2}{2} m^2 \frac{\chi^2}{2} \frac{\sigma^2}{2} = 0, \quad (2.15)$$

according to equation (1.14) we write the equation for χ :

$$3 \frac{g}{2} \hbar^2 \frac{1}{\chi^2} + \frac{1}{\sqrt{\pi}} \frac{1}{2g} \left(\frac{3}{2}\right)^{\frac{2}{3}} \Gamma\left(\frac{11}{6}\right) \chi^{\frac{2}{3}} - \frac{3}{4} m \hbar \sqrt{\frac{4}{3}} = 0. \quad (2.16)$$

For non-dimensional quantity $\tilde{\chi}^2 = \chi^2 / l_{pl}^3$, where $l_{pl} = \sqrt{\hbar g}$ is Planck length, this equation takes the form

$$\frac{6}{\tilde{\chi}^2} + \frac{4}{\sqrt{\pi}} \left(\frac{3}{2}\right)^{\frac{2}{3}} \Gamma\left(\frac{11}{6}\right) \tilde{\chi}^{\frac{2}{3}} - 4\sqrt{3}m = 0, \quad (2.17)$$

where parameter m is also expressed in Planck units of mass. The solution of this equation depends on the value of parameter m . There are two solutions when the parameter is set to $m > m_0$, in this case, the problem is the choice of solution. It is interesting question for additional research. There is no solution when $m < m_0$. As we will see later, in this case, the energy balance can be achieved by the presence of the real quanta of the field in the ground state. Here we set limits to the boundary value of mass m , in which there is a single solution:

$$\tilde{\chi}^2 = \left(\frac{2}{9\sqrt{\pi}} \left(\frac{3}{2}\right)^{\frac{2}{3}} \Gamma\left(\frac{11}{6}\right) \right)^{-\frac{3}{4}} \quad (2.18)$$

$$m_0 = 6 \frac{\left(3^{-\frac{3}{4}} + 3^{\frac{1}{4}}\right)}{4\sqrt{3}} \left(\frac{2}{9\sqrt{\pi}} \left(\frac{3}{2}\right)^{\frac{2}{3}} \Gamma\left(\frac{11}{6}\right) \right)^{\frac{3}{4}} \quad (2.19)$$

3. "Stationary" states of the universe

The variational principle is suitable for finding the ground state with minimal energy. We consider this state as the beginning of the universe. To find the excited "stationary" states we use the so-called self-consistent harmonic approximation, in which the quantum dynamics of the universe is

described by a system of two harmonic oscillators. These oscillators are connected only by the condition of equality of total energy of the universe to zero, and one parameter of self-matching. The corresponding Hamilton operator may be taken in the form:

$$\hat{H} = \frac{g\hbar^2}{2} \frac{\partial^2}{\partial x^2} - \frac{1}{2g\langle x^2 \rangle^{\frac{2}{3}}} \left(\frac{3}{2} \right)^{\frac{2}{3}} x^2 - \frac{2\hbar^2}{9\pi^2 \langle x^2 \rangle} \frac{\partial^2}{\partial \phi^2} + \frac{9\pi^2}{2} \langle x^2 \rangle m^2 \phi^2, \quad (3.1)$$

where $\langle x^2 \rangle$ is matching parameter, which has a physical meaning of the average volume of the universe in a given state. In doing so we use the usual ordering since the introduction of the matching parameter eliminates the singularity.

Let's look for "Stationary" states in the form of $\psi(x; \phi) = \psi_l(x) \cdot \psi_n(\phi)$, here l, n are quantum numbers of the corresponding degrees of freedom:

$$\hat{H}_x \psi_l = \hbar \omega_l \left(l + \frac{1}{2} \right) \psi_l \quad (3.2)$$

$$\hat{H}_\phi \psi_n = \hbar \omega_n \left(n + \frac{1}{2} \right) \psi_n \quad (3.3)$$

where

$$\omega_l = \frac{1}{\langle x^2 \rangle^{\frac{1}{3}}} \quad \omega_n = m. \quad (3.4)$$

At this stage, the parameter $\langle x^2 \rangle$ must be matched with the degree of excitation of the geometry. According to the law of energy distribution for a harmonic oscillator, the medium potential energy is equal to half of its total energy, so:

$$\langle U \rangle_l = \frac{1}{2} W_l, \quad (3.5)$$

$$\langle x^2 \rangle^{\frac{1}{3}} = \left(\frac{2}{3} \right)^{\frac{1}{3}} l_{pl} \sqrt{l + \frac{1}{2}}. \quad (3.6)$$

According to energy balance, excitation energy of geometry is equal to the excitation energy of matter

$$\frac{\hbar}{\langle x^2 \rangle^{\frac{1}{3}}} \left(l + \frac{1}{2} \right) = m \left(n + \frac{1}{2} \right), \quad (3.7)$$

so, according to (3.6)

$$\left(n + \frac{1}{2} \right) = \left(\frac{3}{2} \right)^{\frac{1}{3}} \frac{1}{m} \sqrt{l + \frac{1}{2}}, \quad (3.8)$$

where m is expressed in Planck's units. This equation, that bounds quantum numbers n и l , in general, can only be performed with fractional values of the lasts. However, we will continue to use this oscillator model, given her approximate character.

According to equation (3.8) the birth of n quants of excitation of matter is accompanied by the corresponding excitation of space. In doing so the density of energy of emerging matter has the following dependence on the degree of excitation of the space:

$$\rho = \frac{W_\phi}{\langle x^2 \rangle} = \text{const} \frac{1}{l + \frac{1}{2}}. \quad (3.9)$$

It can be seen that the density decreases with increasing degree of excitation of the space. Despite the fact that this process has the character of quantum jumps, we can interpolate it by a smooth function of the form

$$\rho \sim \frac{\rho_0}{a^2}. \quad (3.10)$$

In comparison, the density of energy of dusty matter in the classical Friedmann's model is proportional to $\sim a^{-3}$, the energy density of radiation is $\sim a^{-4}$. In our case, the speed of decreasing the density of

energy with the expansion of the Universe is less due to the birth of matter. In addition, unlike classical cosmology, we are "protected" from singularity, because we have

$$V_{min} \sim a^3 \sim \langle x^2 \rangle_{l=0} \sim l_{pl}^3. \quad (3.11)$$

Conclusion

Thus, the principle of minimum (extremum) of energy in application to the energy of matter in a closed universe makes it possible to determine not only its initial state, corresponding to the absolute minimum of energy, but also the following "stationary" states. These "stationary" states are numbered by the quantum number of excitation of matter (the number of quanta of matter), which is uniquely related to the degree of excitation of space. In other words, the birth of quanta of matter from the vacuum is accompanied by the birth of the space they need. This is the difference between the considered quantum model and the "parametric" excitation of matter on the classical background of the expanding universe [6]. In doing so the energy density of matter falls from the maximum (but finite) Planck value with the increase of the quantum number of excitation. This "process" can be interpolated by a smooth function (2.10), similar to such relations in classical cosmology. However, in our quantum model, the density decreases with expansion more slowly due to the birth of matter.

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