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Measurement of the proton Zemach radius from the hyperfine splitting in muonic hydrogen atom

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Abstract. Muonic hydrogen is a bound state of a proton and a negative muon. Its Bohr radius is 200 times smaller than that of an electronic hydrogen atom. Therefore, a spectroscopy of the muonic hydrogen is highly sensitive to the finite size effect of proton. Recent years, the proton charge radius was determined by the laser spectroscopy of the Lamb shifts in muonic hydrogen atom. The experiment determined the proton charge radius significantly smaller than the results of past measurements. This anomaly is called "proton radius puzzle" and it has been an important unsolved problem in subatomic physics. Towards solving the puzzle, a new measurement of the ground-state hyperfine splitting in muonic hydrogen was proposed. The hyperfine splitting of muonic hydrogen derives the proton Zemach radius, which is defined as a convolution of the charge distribution with the magnetic moment distribution. This experiment aims to determine the proton Zemach radius with 1% precision by a measurement of the decay electron angular asymmetry. In order to test the feasibility of the laser spectroscopy, a preliminary experiment to measure the hyperfine quenching rate was proposed.

1. Proton Radius Puzzle

A proton is a subatomic particle with complex internal structure. It is a fundamental constituent of the universe and the structure of the proton has been studied from various aspects. Since the 2010s, an anomaly in the proton charge radius has become known as a result of the laser spectroscopy of the Lamb shift in muonic hydrogen atom [1]. Prior to the muonic hydrogen experiment, the proton charge radius has been determined by an electron-proton scattering and a spectroscopy of electronic hydrogen atom [2]. These two electronic measurements had provided consistent results, however, the muonic hydrogen spectroscopy gave a significantly discrepant result. This anomaly is known as the "proton radius puzzle" and it has been unsolved yet even though various interpretations were proposed to explain the discrepancy.



Alternatively, the size of the proton is defined by the Zemach radius, which is expressed as a convolution of the electric and magnetic distributions [3]

$$R_Z = \int d^3\mathbf{r} |\mathbf{r}| \int d^3\mathbf{r}' \rho_E(\mathbf{r}') \rho_M(\mathbf{r} - \mathbf{r}') \quad (1)$$

where ρ_E and ρ_M are the electric and magnetic distributions, respectively. The Zemach radius had been introduced to describe the hyperfine splitting shift of hydrogen atom. For the case of muonic hydrogen, the hyperfine shift in leading-order approximation is

$$\Delta E = E_F(1 - 2\alpha m_{\mu p} R_Z) \quad (2)$$

where E_F is the Fermi energy, α is the fine structure constant, and $m_{\mu p}$ is the reduced mass of muonic hydrogen.

Figure 1 summarizes the measurement results of the proton charge and Zemach radii [4, 5, 6, 7, 8]. For a case of the charge radius, the muonic result is 4% smaller than the electronic results and the deviation between two measurements is significantly larger than the uncertainties. On the other hand, the determination precision of the Zemach radius by the muonic hydrogen experiment is insufficient to discuss the consistency between measurements. This muonic result was indirectly derived with 3% precision from two Lamb shift frequencies via the 2S hyperfine splitting. We aim to improve the determination precision of the Zemach radius by a new direct measurement of the ground-state hyperfine splitting in muonic hydrogen atom.

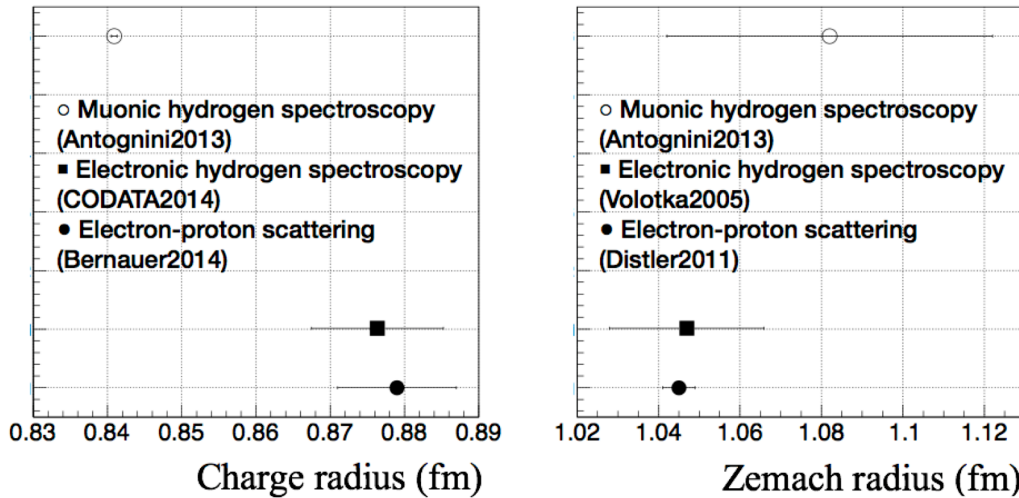


Figure 1. The proton radii obtained by the electronic and muonic measurements: (left) the charge radius [4, 5, 6]; (right) the Zemach radius [4, 7, 8]. The outlined circles correspond to the muonic results. The solid squares and circles correspond to the result of electronic hydrogen spectroscopy and electron-proton scattering, respectively. The original figure appeared in reference [9].

2. Ground-State Hyperfine Splitting in Muonic Hydrogen Atom

The theoretical expression of the ground-state hyperfine splitting in muonic hydrogen with higher-order correction terms is [10]

$$\Delta E_{\text{HFS}} = E_F(1 + \delta_{\text{QED}} + \delta_{\text{Proton}}) \quad (3)$$

where δ_{QED} is the higher-order electromagnetic correction and δ_{Proton} is the correction due to the strong interaction. The electroweak contribution is negligibly small [11]. The hadronic term is factorized

$$\delta_{\text{proton}} = \delta_{\text{Rec.}} + \delta_{\text{Pol.}} + \delta_{\text{HVP}} + \delta_{\text{Zemach}} \quad (4)$$

where $\delta_{\text{Rec.}}$, $\delta_{\text{Pol.}}$, and δ_{HVP} are the contributions arising from the recoil, the proton polarizability, and the hadronic vacuum polarization. The Zemach term is proportional to the Zemach radius as follows.

$$\delta_{\text{Zemach}} = -2\alpha m_{\mu e} R_Z. \quad (5)$$

The most recent theoretical calculation predicted $\Delta E_{\text{HFS}} = 182.638(62)$ meV and the Zemach contribution was estimated to be -1.362 meV [12]. The error was arising mostly from present experimental uncertainties of the proton form factors. A measurement of the ground-state hyperfine splitting in muonic hydrogen with the relative uncertainty of 2 ppm will derive the proton Zemach radius with 1% precision.

Furthermore, when the ground-state hyperfine splitting is precisely measured, the Sternheim interval [13]

$$\Delta E_{12} = 8\Delta E_{\text{HFS}}(2S) - \Delta E_{\text{HFS}}(1S) \quad (6)$$

will be obtained by combining the results of 1S and 2S hyperfine splitting intervals. This interval does not contain the proton structure and polarizability terms up to the order of α^5 . Hence, it provides a precision test of the bound-state quantum electrodynamics theory. For a case of muonic hydrogen, the Sternheim interval was theoretically estimated to be $\Delta E_{12} = -0.120$ meV with a precision of 10^{-6} [14].

3. Laser Spectroscopy of the Hyperfine Splitting in Muonic Hydrogen Atom

A conceptual scheme of the experiment is illustrated in Figure 2. The apparatus consists of a cryogenic hydrogen gas target, a circularly polarized transition laser, and a high-rate capable electron detector. The experiment was proposed to J-PARC MLF MUSE, where the world-highest intensity pulsed muon beam is provided [15]. The pulsed muon beam irradiates a gas cell filled with hydrogen at low temperature and pressure.

After muon stopping in the gas target, muonic hydrogen atoms form in highly excited states with a principle quantum number of more or less 14. These muonic hydrogens will be de-excited to the ground-state by radiative cascade transitions [16]. After the cascade de-excitation, muonic hydrogens will be in the spin-triplet state or the spin-singlet state. However, the atoms in the spin-triplet state will be de-excited due to the inelastic scattering with protons inside the target [17].

An intense pulsed mid-infrared laser beam induces the sub-level transition from the spin-singlet state to the spin-triplet state. A circular polarization of the laser beam enables a selective excitation to the particular spin-triplet state. The hyperfine state transition causes a muon spin flip and it results in the muon spin polarization as a function of time. In order to enhance the state transition efficiency, two highly-reflective mirrors are placed inside the gas target. These mirrors are arranged to be facing each other to construct a multi-pass cell for a confinement of the laser beam.

Muonic hydrogen atoms decay with the emission of an electron and neutrinos. Since the rate of nuclear capture by a proton is tiny, the lifetime of muonic hydrogen is $\tau = 2.194 \mu\text{s}$ which is approximately equivalent to the one of free muon [18]. The emission angle of the decay electron is correlated to the muon spin direction due to the parity violation. Therefore,

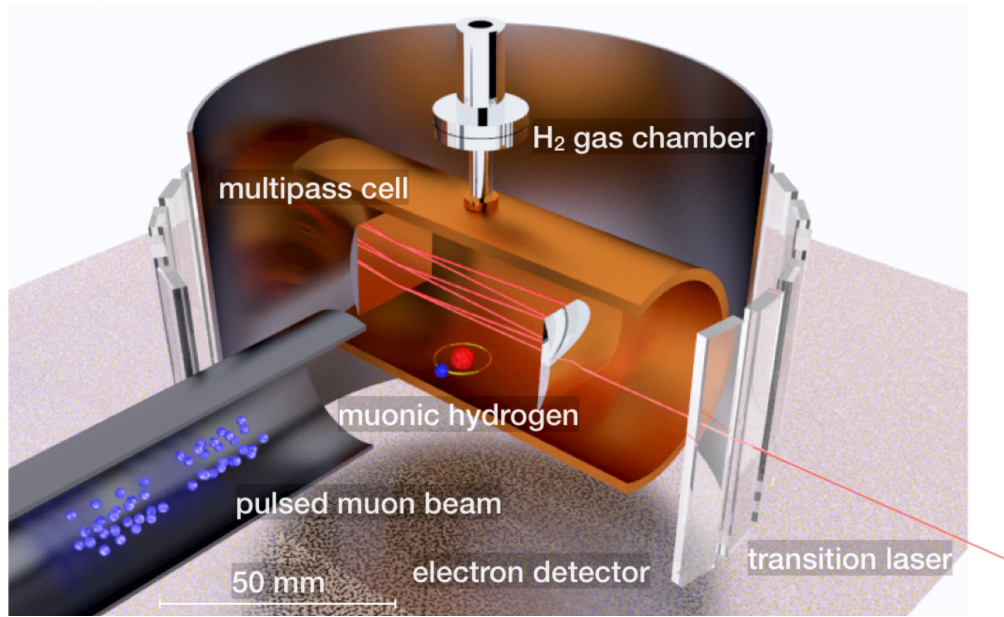


Figure 2. Experimental schematic. Negative pulsed muon beam irradiates the hydrogen gas target. Muon is captured by the proton and forms a muonic hydrogen atom. The hyperfine transition between the spin-singlet state and the spin-triplet state is induced by a circularly polarized mid-infrared laser light. Electrons from muonic hydrogen decay are detected by the electron counter around the gas chamber. The original figure appeared in reference [9].

an ensemble average of the muon spin polarization can be obtained by a measurement of the angular asymmetry in electron emission.

The dominant source of systematic uncertainty will be an accuracy of wave-meter, which monitors an absolute frequency of the transition laser. This uncertainty is possibly suppressed by precise calibration using a frequency-comb laser. The expected precision of spectroscopy was estimated to be 2 ppm in two weeks of measurement.

3.1. Hydrogen Gas Target

The target density is 1/1000 of the liquid hydrogen density (LHD). The low temperature and density are necessary to suppress the Doppler broadening, muonic molecule formation, and collisional hyperfine quenching. The Doppler broadening was estimated to be 57 MHz assuming a Maxwell distribution at the target temperature of 20 K. The molecular formation rate was measured by MuCap Collaboration and the result was $\lambda_{pp\mu} = (2.01 \pm 0.06 \pm 0.03)$ MHz at the LHD [19]. The first and second errors were statistical and systematic uncertainties, respectively. For a case of the low density gas target, the molecular formation will not be an obstacle to the experiment. The collisional hyperfine quenching rate has been predicted so high as to be a critical issue for a feasibility of the experiment. The details of this collisional quenching will be described in following section.

The gas cell is a cylinder with the length of 8 cm and the diameter of 3 cm. It is made of tungsten in order to suppress background events arising from muons stopped on the wall. A muon stopped in heavy materials decays with short lifetime due to the high rate of the nuclear capture. For a case of tungsten, the lifetime was measured by J. C. Sens and the result was $\tau = 81 \pm 2$ ns [20]. In our experiment, a transition laser is delivered after a microsecond from muon beam pulse arrival. Therefore, the background from wall-stopped muons will be negligible.

3.2. Transition Laser

The hyperfine splitting interval in the ground-state muonic hydrogen is 183 meV, hence, a coherent light having the wavelength of $6.8 \mu\text{m}$ is required for the experiment. The hyperfine

transition is optically forbidden by the selection rule. Therefore, a high pulse energy is necessary for the transition laser. The transition probability is [21]

$$P = 2 \times 10^{-5} \frac{E}{S\sqrt{T}} \quad (7)$$

where E is the laser pulse energy, S is the cross-section of the laser beam, and T is the target temperature. The multi-pass cell enhances the transition probability by the number of laser light reflection N . The experimental design parameters are $E = 20$ mJ, $S = 4$ cm², $T = 20$ K, and $N = 2000$.

The transition laser system consists of three stages: a Tm³⁺, Ho³⁺ co-doped YAG ceramic laser; an optical parametric oscillator (OPO) using a ZnGeP₂ (ZGP) nonlinear optical crystal with a quantum cascade laser (QCL); a ZGP optical parametric amplifier (OPA). The ZGP-OPO is pumped with the Tm,Ho:YAG ceramic laser and the OPO oscillates at 6.8 μ m. The QCL is adopted as a narrowband seeder for the OPO. The output beam is amplified by the ZGP-OPA. The details of the laser system were described in references [9, 22].

3.3. Electron Detector

The electron detector consists of a segmented scintillation counter with silicon photomultiplier (SiPM) readout. The high-intensity pulsed muon beam at J-PARC is beneficial for the accumulation of statistics, however, a high-rate capability is required for the particle detector. At J-PARC MLF, the muon beam intensity of more than 1×10^5 μ^- /s is expected at the beam momentum of 20 MeV/c and the accelerator operation power of 1 MW. The full width at half maximum (FWHM) of the beam pulse is 100 ns and the repetition frequency is 25 Hz.

To deal with the high-intensity pulsed muon beam, a new positron counting system was developed for the muonium spectroscopy experiment [23]. The detector has been smoothly operated at the beam intensity of 3×10^6 μ^+ /s without significant systematic uncertainty arising from high beam intensity [24].

4. Collisional Hyperfine Quenching

One of the obstacles in the experiment is the short lifetime of the spin-triplet state. The quenching of the spin-triplet state occurs by an inelastic scattering between a muonic hydrogen atom and a proton consisting hydrogen

$$\mu p(F = 1) + p \rightarrow \mu p(F = 0) + p \quad (8)$$

where F is the total angular momentum of the muonic atom. On the cross-section of this process, only theoretical predictions are known and no measurement had been performed. On the other hand, the cross-section of similar process for muonic deuterium

$$\mu d(F = 3/2) + d \rightarrow \mu d(F = 1/2) + d \quad (9)$$

was obtained both experimentally and theoretically. However, there is approximately 40% of discrepancy between the experimental result and theoretical calculation [25]. The spread of wave functions in a muonic protium is different from ones in a muonic deuterium, whereas it is of importance to experimentally observe the collisional hyperfine quenching of muonic protium. In order to measure the hyperfine quenching rate, we proposed an experiment using muon spin rotation method.

The cross-section of hyperfine quenching is collision energy dependent, *i.e.*, it depends on a target temperature. At the target temperature of 20 K, the collision energy is approximately 2 meV and the cross-section was estimated to be 600×10^{-20} cm² [17]. At higher target temperature, the scattering cross-section becomes smaller, however, a frequency of collision increases. The fast quenching of spin-triplet state demands the high pulse energy of transition laser and low density of the hydrogen gas target.

5. Measurement of the Collisional Hyperfine Quenching Rate

Figure 3 is a drawing of the apparatus for the hyperfine quenching rate measurement. Helmholtz coils generate a static magnetic field of 0.06 T in the transverse direction. The experiment was proposed to RIKEN-RAL muon facility [26]. Pulsed negative muon beam with the momentum of 20 MeV/c irradiates the hydrogen gas target confined inside the aluminum gas cell. For background suppression, inner walls of the gas cell are covered with silver plates. The beam has the double-pulsed timing structure with the repetition frequency of 50 Hz.

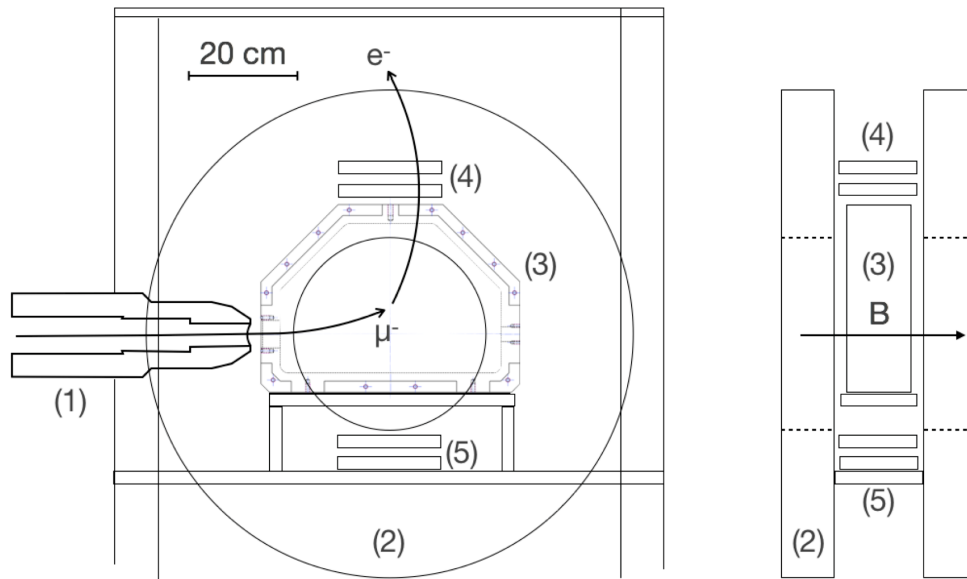


Figure 3. Experimental setup of the hyperfine quenching measurement. The left figure shows a cross-sectional view, the right one shows a view from downstream: (1) muon beam collimator made of lead; (2) Helmholtz coils for a magnetic field; (3) target gas cell; (4) upper electron detectors; (5) lower electron detectors. The electron counters on the CHRONUS is not shown.

The CHRONUS spectrometer at Port4 [27] is employed to generate a magnetic field and detect electrons from muon decays. Additional electron detectors are placed on the top and bottom of the gas cell for larger acceptance.

The frequency of muon spin rotation depends on the muon gyromagnetic ratio in nuclei and the hyperfine state of muonic atoms as follows [28]

$$\gamma_+ = \frac{1}{I + 1/2}(\mu_\mu + \mu_N) \quad (10)$$

$$\gamma_- = -\frac{1}{I + 1/2}(\mu_\mu - \frac{I + 1}{I}\mu_N) \quad (11)$$

where I is the nuclear spin, μ_μ and μ_N are the magnetic moment of muon and nuclei, respectively. The first formula corresponds to the state with the total spin $F = I + 1/2$, the second corresponds to the state with $F = I - 1/2$. For a case of the spin-singlet state with $I = 1/2$ nuclei, the latter formula is invalid and no spin precession occurs. The muon spin precession frequencies were summarized in Table 1.

Table 1. Muon spin precession frequencies in muonic hydrogen atoms.

Muonic atom species	$F = I + 1/2$	$F = I - 1/2$
Muonic protium (μp)	46.5 MHz/T	No spin precession
Muonic deuterium (μd)	54.7 MHz/T	41.0 MHz/T

In the experiment, typical target density is 0.05% LHD, which corresponds to 0.5 atm at 300 K. Therefore, the expected lifetime of the spin-triplet state is about 100 ns for the muonic protium. Figure 4 shows a simulated muon stopping distribution projected on the beam axis at the hydrogen gas pressure of 0.5 atm and the transverse magnetic field of 0.06 T. The muon stopping efficiency in the gas target was evaluated to be approximately 18%.

The electron asymmetry is obtained by taking the ratio between time spectra measured by the upper and lower electron detectors. Figure 5 shows simulated electron asymmetry as a function of muonic hydrogen age. The double-pulsed beam structure with the pulse interval of 320 ns and the FWHM of 75 ns was considered. The residual polarization after the cascade de-excitation of 12% [16], the muon lifetime of 2.2 μ s, the initial population of the triplet-state of 75%, and the Larmor frequency of 2.79 MHz were assumed. The proposed experiment was approved and will be performed in 2018 autumn. The determination precision of the quenching rate was estimated to be several percents in a day of measurement. The measurement uncertainty of 10% or less is sufficient to determine the specification of the transition laser and the hydrogen gas density.

6. Summary

The conflict between the electronic and muonic measurements of the proton charge radius is known as the "proton radius puzzle". In order to shed some light on the puzzle, a new measurement of the ground-state hyperfine splitting in muonic hydrogen atom for a determination of the proton Zemach radius was proposed. The experiment is a laser spectroscopy by a measurement of the decay electron's angular asymmetry. A cryogenic hydrogen gas target, an intense mid-infrared pulsed laser, and a high-rate capable electron detector will be employed. Development of the laser system is in progress and the electron detector is in operation with high-intensity pulsed muon beam. The experimental proposal was submitted to J-PARC and approved as a stage-1 project. One of the difficulties in the experiment is the collisional hyperfine quenching due to the inelastic three-body collision. The cross-section of this process has not been measured yet and only theoretical predictions are known. For an experimental determination of the hyperfine quenching rate, a muon spin precession measurement was proposed.

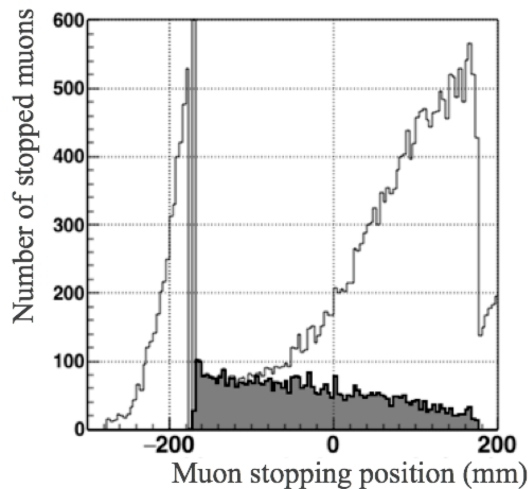


Figure 4. Simulated muon stopping distribution projected on the beam axis. The center of the gas cell was at zero on the horizontal axis. The muon momentum of 20 MeV/c with $\pm 4\%$ spread and the target pressure of 0.5 atm were assumed. The shaded histogram indicates the stopped muons in the hydrogen gas target.

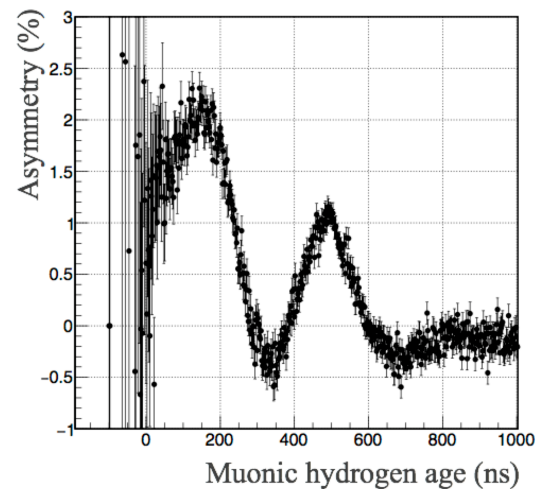


Figure 5. Simulated electron asymmetry by taking a ratio between the decay electron time spectra which were obtained by the upper and lower detectors. The second muon pulse arrives at 320 ns. A damping constant in the oscillating asymmetry provides the collisional hyperfine quenching rate. See text for the details.

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