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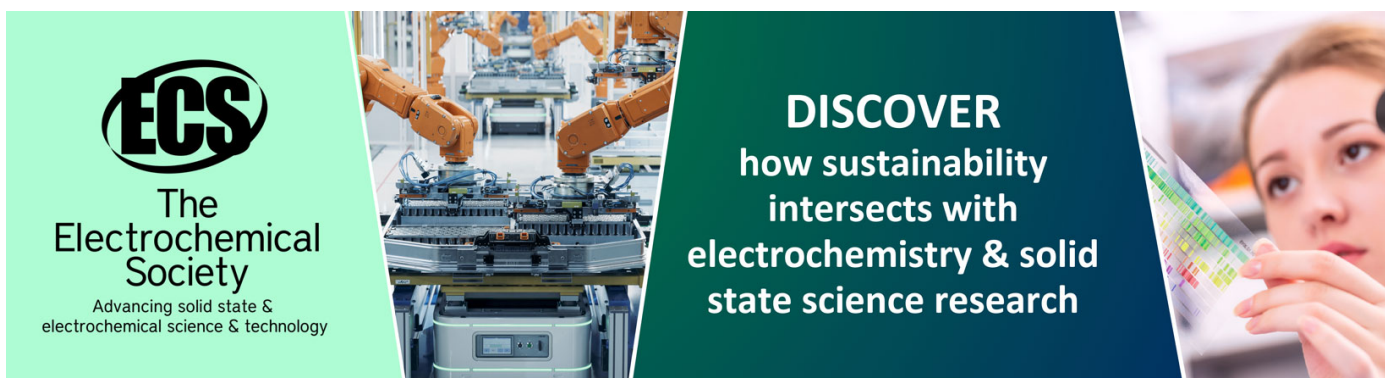
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Pressure pulse propagation in a double-layered elastic tube with viscoelastic liquid

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Abstract. Propagation of a pressure signal initially generated at the pipe end, in a double-layered elastic tube, filled with compressible viscoelastic liquid, is studied. It is supposed that liquid rheology follows the 3-parameter Oldroyd rheological model. Liquid dynamics in the wave is described within the quasi-one-dimensional approach. It is assumed that the bending stresses in the tube wall can be neglected, as compared with the membrane ones. The solution of the momentum balance equation for the layered shell, found in this approximation, is coupled with the transient solution, describing liquid flow in the wave, by appropriate boundary conditions. The resulting system of equations is solved by the operational method and propagation of a standard finite pressure signal, generated at the initial moment of time at the pipe end, is investigated numerically. The obtained data indicate that combination of elastic properties of the shell's layers and liquid viscoelasticity may influence essentially the pressure pulse propagation. The scale of the effect depends on the relative thickness of the layers, their elastic moduli and the dimensionless liquid relaxation time. The general conclusion, following from the data is that increase in the liquid relaxation time yields a marked decrease in the signal attenuation along the pipe.

1. Introduction

Layered systems composed of several layers of different materials are widely used for various purposes [1]. Among them, cylindrical layered structures have found especially important engineering and biomedical applications [2-4]. Acoustical properties of such structures may differ essentially from the homogeneous ones, giving rise to changes in guided waves propagation and signal dispersion and attenuation [5-9].

Propagation of sound waves in liquid-filled thin-walled cylindrical tubes is the subject of much investigation in hydraulics, power and chemical engineering, non-destructive pipe testing [10-13]. Wave propagation in such waveguides experiences the influence of a number of factors, among which elasticity of the wall, liquid compressibility, viscosity, and liquid-wall interactions (usually referred as fluid-structure interaction (FSI)) are the most important [14]. The wall thickness also has a great impact on the wave parameters. For negligibly thin and negligibly weak tubes, the process can be treated as wave propagation in a fluid jet, while for infinitely thick tubes it is just wave propagation in a fluid-filled borehole. Other limiting cases correspond to the rigid tube, ideal liquid etc. Despite approximate solutions of the problem and analysis of limiting situations have clear limitations, such studies are of

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great importance, because the obtained results provide an insight into the complex process of wave propagation in a coupled structure and help to understand better the impact of the system parameters.

The major part of published results relates to the case when the liquid in the waveguide is ideal or pure viscous. The results relating to liquids with more complex rheology, particularly, viscoelastic fluids can be found in [15-17]. Here approach similar to [18] is used for modeling of pressure pulse propagation in a composite double-layered waveguide, made from two thin elastic cylindrical shells with different properties. It is supposed that the tube is filled with compressible viscoelastic liquid (solution of the high polymer in a low molecular solvent). Evolution of a finite pressure pulse, generated initially at the tube end, along the waveguide is studied numerically.

2. Model formulation

2.1 Dynamics of a double-layered cylindrical shell

Dynamic equations for the double-layered thin circular cylindrical shell are written below for the case when the layers are made from different isotropic elastic materials and the whole layered structure can be described within Kirchhof-Love approximation [19]. These equations in the axisymmetric case have the form:

$$\frac{C}{R^2} \frac{\partial^2 u}{\partial \zeta^2} + \frac{C_{12}}{R^2} \frac{\partial w}{\partial \zeta} - \frac{K}{R^3} \frac{\partial^3 w}{\partial \zeta^3} = (h_1 + h_2) \cdot \rho_s \cdot \frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$\begin{aligned} \frac{C w}{R^2} + \frac{C_{12}}{R^2} \frac{\partial u}{\partial \zeta} - \frac{K_{12}}{R^3} \frac{\partial^2 w}{\partial \zeta^2} + \frac{1}{R^3} \cdot \left(\frac{D_1}{R} \frac{\partial^4 w}{\partial \zeta^4} - K \frac{\partial^3 u}{\partial \zeta^3} - K_{12} \frac{\partial^2 w}{\partial \zeta^2} \right) \\ = -(h_1 + h_2) \cdot \rho_s \cdot \frac{\partial^2 w}{\partial t^2} + \Delta p. \end{aligned} \quad (2)$$

Here u and w are longitudinal and radial displacements of the shell at the liquid-shell interface, which is chosen for a coordinate surface. A cylindrical coordinate system with the origin on the tube axis is introduced, and x , r are the longitudinal and radial coordinates, correspondingly. In (1), (2) R is the radius of the coordinate surface (it is supposed that $R \ll L$ where L is the tube length); h_1 and h_2 - the thicknesses of the internal and external layers, respectively ($h_1, h_2 \ll R$); ζ is the non-dimensionless axial coordinate ($\zeta = x/R$); Δp - contact pressure, equal to normal stress in the liquid at the pipe wall; t - time; $C, C_{12}, K, K_{12}, D_1$ are parameters of the layered shell, defined by the formulas [19]:

$$C = B_{11}^{(1)} h_1 + B_{11}^{(2)} h_2, \quad C_{12} = B_{12}^{(1)} h_1 + B_{12}^{(2)} h_2, \quad K = 0.5 \cdot [B_{11}^{(1)} h_1^2 + B_{11}^{(2)} (2h_1 h_2 + h_2^2)] \quad (3)$$

$$K_{12} = 0.5 \cdot [B_{12}^{(1)} h_1^2 + B_{12}^{(2)} (2h_1 h_2 + h_2^2)], \quad D_1 = \frac{1}{3} \cdot [B_{11}^{(1)} h_1^3 + B_{11}^{(2)} (3h_1^2 h_2 + 3h_1 h_2^2 + h_2^3)]$$

$$B_{11}^{(i)} = \frac{E_i}{1 - \nu_i^2}, \quad B_{12}^{(i)} = \nu_i \cdot B_{11}^{(i)}, \quad i = 1, 2. \quad (4)$$

In (4) E_i , ν_i are Young modules and Poisson coefficients of the layers (index $i=1$ corresponds to the internal layer and $i=2$ - to the external one); ρ_s is the effective density of the layered shell, introduced by the formula:

$$\rho_s = \frac{\rho_1 h_1 + \rho_2 h_2}{h_1 + h_2}. \quad (5)$$

Here ρ_1 and ρ_2 are the layer's densities.

The equations (1), (2) are rewritten in dimensionless form, using the relations

$$\{u_1, u_2, \xi, \varepsilon_1, \varepsilon_2\} = R^{-1} \{w, u, r, h_1, h_2\}, \quad p_c = p_0^{-1} \Delta p, \quad \tau = t / t_0, \quad t_0 = R(\rho_s / p_0)^{1/2} \quad (6)$$

$$\{C^*, C_{12}^*\} = (p_0 R)^{-1} \{C, C_{12}\}, \quad \{K^*, K_{12}^*\} = (p_0 R^2)^{-1} \{K, K_{12}\}, \quad D^* = (p_0 R^3)^{-1} D_1,$$

where p_0 is the equilibrium pressure within the tube. The result has the form [20]:

$$C_{12}^* \frac{\partial u_1}{\partial \xi} - K^* \frac{\partial^3 u_1}{\partial \xi^3} + C^* \frac{\partial^2 u_2}{\partial \xi^2} = (\varepsilon_1 + \varepsilon_2) \frac{\partial^2 u_2}{\partial \tau^2} \quad (7)$$

$$C^* u_1 - 2K_{12}^* \frac{\partial^2 u_1}{\partial \xi^2} + D^* \frac{\partial^4 u_1}{\partial \xi^4} + C_{12}^* \frac{\partial u_2}{\partial \xi} - K^* \frac{\partial^3 u_2}{\partial \xi^3} = -(\varepsilon_1 + \varepsilon_2) \frac{\partial^2 u_1}{\partial \tau^2} + p_c. \quad (8)$$

2.2 Liquid dynamics in the wave

It is supposed that the waveguide is filled with viscoelastic polymeric liquid (solution of high-polymer in a low molecular solvent), following linear Oldroyd equation:

$$\tau_{ij} + \theta \frac{\partial \tau_{ij}}{\partial t} = 2\eta_p s_{ij} + 2\theta \eta_s \frac{\partial s_{ij}}{\partial t}, \quad s_{ij} = e_{ij} - \frac{1}{3}(\nabla \cdot \vec{v})I, \quad e_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right). \quad (9)$$

Here τ_{ij} , s_{ij} - deviators of stress and rate-of-strain tensors, respectively; θ - relaxation time; η_s , η_p - Newtonian viscosity of low-molecular solvent and polymeric solution, respectively; $\vec{v}$ - liquid velocity (it is supposed that originally the liquid is at rest). The model formulated below is based on quasi-one-dimensional approach [21], which implies:

$$v_r \ll v_x, \quad \frac{\partial^2 v_x}{\partial x^2} \ll \frac{1}{r} \frac{\partial v_x}{\partial r}, \quad \frac{\partial^2 v_x}{\partial x^2} \ll \frac{\partial^2 v_x}{\partial r^2}, \quad \dot{u} \ll v_x, \quad \frac{\partial v_r}{\partial x} \ll \frac{\partial v_x}{\partial r}. \quad (10)$$

Here v_x , v_r - liquid velocity components in longitudinal and transverse directions. After averaging of the balance equations for liquid along the tube cross-section with an account for relations (10) and the kinematic boundary condition $v_r = w$ at the tube wall, the following averaged equations of mass and momentum balance and state equation for liquid are obtained [18]:

$$\begin{aligned} \frac{\partial V^*}{\partial \tau} &= -\kappa \frac{\partial P^*}{\partial \xi} + 2\kappa \bar{\tau}_w, \quad \frac{\partial \bar{\rho}}{\partial \tau} + 2 \frac{\partial u_1}{\partial \tau} + \frac{\partial V^*}{\partial \xi} = 0, \quad P^* = \kappa^{-1} \bar{c}_f^2 \bar{\rho} \\ \bar{\tau}_w &= \frac{(\tau_{xr})_{r=R}}{p_0}, \quad V^* = \frac{V t_0}{R}, \quad P^* = \frac{P_f}{p_0} - 1, \quad \bar{\rho} = \frac{\Delta \rho}{\rho_{f0}}, \quad \kappa = \frac{\rho_s}{\rho_{f0}}, \quad \bar{c}_f = c_f t_0 / R. \end{aligned} \quad (11)$$

Here V , P_f , $\Delta\rho$ are averaged axial velocity of liquid, pressure and density disturbances in the wave, respectively; ρ_{f0} - equilibrium liquid density, c_f - sound speed in liquid. It was shown in [15] that transient friction $\bar{\tau}_w$ at the tube wall can be defined by the relation:

$$\bar{\tau}_w = \int_{-\infty}^{\tau} \bar{G}(\tau - \bar{t}_1) \left(\frac{\partial \bar{v}_x}{\partial \xi} \right)_{\xi=1} d\bar{t}_1 + \bar{\eta}_s \left(\frac{\partial \bar{v}_x}{\partial \xi} \right)_{\xi=1}, \quad (12)$$

where $\bar{G}(\tau - \bar{t}_1)$ is relaxation function, corresponding to the rheological equation (9):

$$\bar{G}(\tau - \bar{t}_1) = \frac{\bar{\eta}_p - \bar{\eta}_s}{\bar{\theta}} e^{-\frac{\tau - \bar{t}_1}{\bar{\theta}}}, \quad \bar{\theta} = \theta / t_0. \quad (13)$$

In order to close the model, it is necessary to find the relation between the pressure in liquid and deformation in the tube wall. Similar to [21], the bending stresses in the composite shell hereafter are neglected, as compared with the membrane ones, the same as longitudinal deformations and the shell inertia [22]. In the case of a double-layered shell, these assumptions correspond to the following inequalities: $K^* \ll C_{12}^*$, $D^* \ll K_{12}^* \ll C^*$ [20]. As a result, it follows from the momentum balance equations (7), (8):

$$u_1 = P^* / C^*. \quad (14)$$

3. Problem solution by the operational method

The equations (11) – (14) are solved by the operational method. After applying Laplace transform $L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$, the following expressions for the images $\hat{P}$, $\hat{V}$ of pressure and liquid velocity disturbances in the wave, generated at the moment $\tau = 0$ by a pressure pulse $g(\tau)$ ($g(\tau) = 0$ for $\tau < 0$) in the tube cross section $\zeta = 0$ ($P^*(0, \tau) = g(\tau)$), are obtained:

$$\begin{aligned} \hat{P} &= G(s)e^{-\Lambda\zeta}, \quad \hat{V} = \frac{\kappa s G(s)}{\Lambda \bar{c}_k^2} e^{-\Lambda\zeta}, \quad G(s) = L\{g(\tau)\} \\ \Lambda^2 &= s(s + 2\kappa\hat{\eta}\Phi(\mu))\bar{c}_k^{-2}, \quad \bar{c}_k^{-2} = \bar{c}_f^{-2} + \bar{c}_s^{-2}, \quad \hat{\eta} = \bar{\eta}_s + (\bar{\eta}_p - \bar{\eta}_s)(1 + s\bar{\theta})^{-1} \\ \bar{c}_s^2 &= \frac{1}{2}\kappa C^*, \quad \Phi(\mu) = -\frac{\mu T(\mu)}{1 - 2\mu^{-1}T(\mu)}, \quad T(\mu) = \frac{J_1(\mu)}{J_0(\mu)}, \quad \mu^2 = -\frac{s}{\kappa\hat{\eta}}. \end{aligned} \quad (15)$$

Here s is the Laplace variable, $G(s)$ - Laplace image of the initial pressure disturbance, and J_0 , J_1 are Bessel functions of the first kind of zero and first order, respectively. Relations (15) describe pressure and velocity time evolution in the s -domain at a fixed tube cross-section, defined by the value of the dimensionless axial coordinate ζ . For inviscid liquid ($\hat{\eta} = 0$) the expression for $\hat{P}$ takes the form:

$\hat{P} = G(s)e^{-\frac{s}{\bar{c}_k}\zeta}$. The meaning of the result is that, in the case of ideal liquid, the initial pulse $g(\tau)$ propagates with the modified Korteweg velocity [23] $\bar{c}_k$, corresponding to the layered shell, and retains its form:

$$P^*(\zeta, \tau) = g(\tau - \zeta / \bar{c}_k). \quad (16)$$

4. Numerical results

Relations (15) were inverted numerically. To find the function $P^*(\zeta, \tau)$, the algorithm [24] based on accelerating the convergence of the Fourier series obtained from the inversion integral, was used. The code was preliminarily tested for the case of ideal liquid, described by the analytical solution (16). In order to avoid numerical instabilities at the Bessel functions calculations with large μ values in the case of viscous and viscoelastic liquid, the asymptotic expansion $\Phi(\mu) \approx \left(\frac{s}{\kappa \hat{\eta}}\right)^{1/2}$ at $|\mu| \gg 1$ was used [23].

Propagation of the following finite pulse was studied:

$$g(\tau) = \frac{1}{2} u(\tau) \cdot u(2\tau_0 - \tau) \left[1 + \cos\left(\frac{\pi(\tau - \tau_0)}{\tau_0}\right) \right], \quad G(s) = \frac{\pi^2(1 - e^{-2s\tau_0})}{2s\tau_0^2(s^2 + \pi^2\tau_0^{-2})}. \quad (17)$$

The parameter τ_0 in (17) has the meaning of the pulse half-width, $u(\tau)$ is the unit step function.

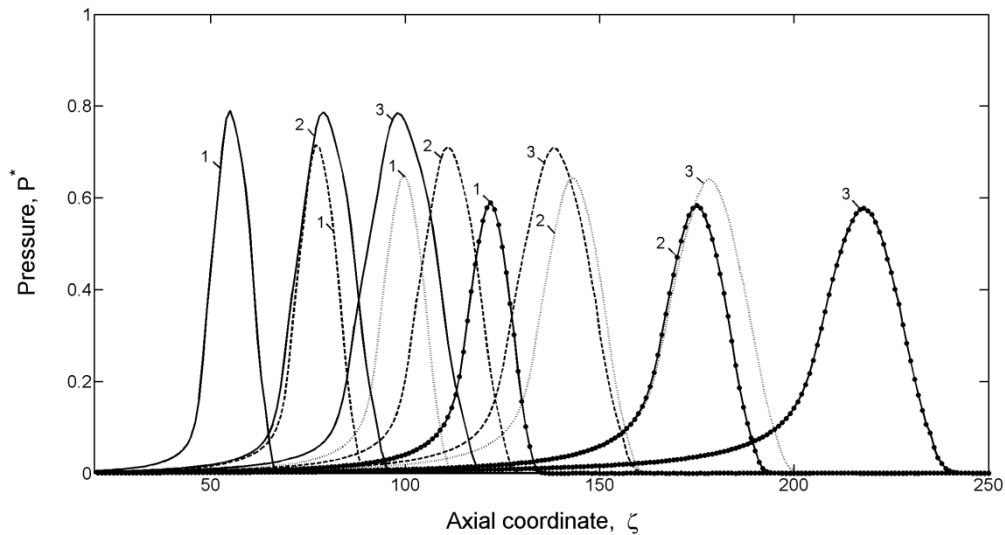


Figure 1. Pressure pulse evolution along the pipe. Effect of the shell structure (pure viscous liquid, $\theta = 0$). 1, 2, 3 - $\varepsilon_1/\varepsilon_2 = 1/9, 1, 9$; — - $\tau = 3$, ---- - $\tau = 4$, - $\tau = 5$, - $\tau = 6$.

Results of simulations are presented on the figures 1, 2 for a composite shell with the following parameters: $E_1 = 3 \text{ GPa}$, $E_2 = 0.7 \text{ GPa}$, $\rho_1 = 1.1 \cdot 10^3 \text{ kg/m}^3$, $\rho_2 = 950 \text{ kg/m}^3$ (the elastic moduli and densities, specified above, correspond approximately to an internal layer made from polyamide and the external one - from polypropylene). The data describe the pressure pulse propagation along the semi-infinite tube; four sets of plots on each figure correspond to different moments of time. It is supposed that $R = 10^{-2} \text{ m}$ and the dimensionless thickness of the shell was chosen equal to 0.1. For all plots $\varepsilon_1 + \varepsilon_2 = 0.1$, while individual values of ε_1 , ε_2 were varied (figure 1). The rest of the parameters were chosen as follows: $p_0 = 10^5 \text{ Pa}$, $\rho_{f0} = 850 \text{ kg/m}^3$, $c_f = 1500 \text{ m/s}$, $\eta_p = 0.5 \text{ Pa} \cdot \text{s}$, $\eta_s = 5 \cdot 10^{-3} \text{ Pa} \cdot \text{s}$. The liquid relaxation time θ was changed in a certain range in order to illustrate the viscoelasticity

effect (figure 2). Note that the liquid parameters, specified above, at $\theta = 0.01$ s correspond approximately to 2.5% solution of polystyrene in toluene [25].

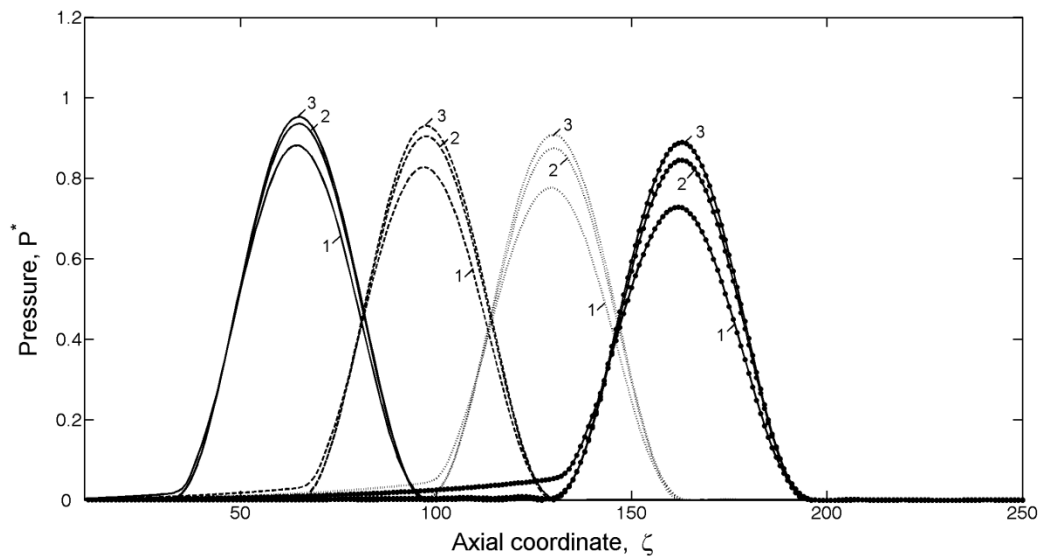


Figure 2. Pressure pulse evolution along the pipe. Effect of liquid viscoelasticity. $\varepsilon_1/\varepsilon_2 = 1$; 1, 2, 3 - $\theta = 0.001, 0.005, 0.01$; — - $\tau = 3$, ---- - $\tau = 4$, - $\tau = 5$, - $\tau = 6$.

As it follows from the data on figure 1, the relative thickness of the layers influence mainly the wave speed – the thicker is the internal layer, characterized by a larger Young module, the more is the pulse propagation velocity. The plots indicate that increase in the value of the parameter $\varepsilon_1/\varepsilon_2$ yields also a slight increase of the pulse width at the corresponding moments of time. The pressure amplitude decreases with time as a result of viscous losses at the wave propagation along the tube.

The plots in figure 2 are aimed to illustrate the liquid viscoelasticity effect on the wave propagation in a double-layered tube with the same layers thickness ($\varepsilon_1 = \varepsilon_2$). It can be seen that the liquid rheology influence mainly the signal amplitude while its velocity remains almost the same for different values of the relaxation time θ . The increase of θ yields also an increase in the maximum pressure value in the wave, which is explained by the effective reduction of viscous dissipation in a viscoelastic liquid flow in the wave. The effect becomes more pronounced with the pulse propagation along the tube.

5. Conclusions

The results of the study have revealed the essential impact of the double-layered shell and viscoelastic properties of liquid on the finite pulse propagation in the waveguide. It is shown that the relative thickness of the shell layers with different elastic properties influence mainly the signal speed, and in a much less extent its shape. The increase in the thickness of a more rigid layer yields an increase in the wave velocity and vice versa. It is important to note that the relative thickness of the layers has almost no effect on the pressure in the wave. On the other hand, liquid viscoelasticity has pronounced impact on the pressure amplitude, causing its increase with the increase of the relaxation time. The result is explained by the effective reduction of viscous dissipation at viscoelastic liquid flow in a fast dynamic process.

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