

PAPER • OPEN ACCESS

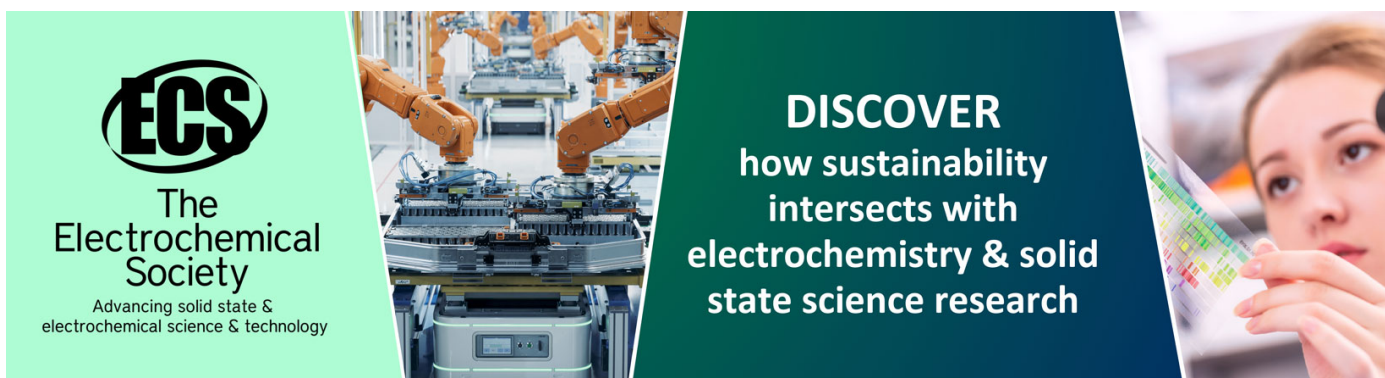
FIRST STUDY OF USING GRAVITATIONAL SIGNAL GENERATOR FOR THE MEASUREMENT OF THE GRAVITY SPEED

To cite this article: C L Fernandes *et al* 2018 *J. Phys.: Conf. Ser.* **1141** 012093

View the [article online](#) for updates and enhancements.

You may also like

- [GRAVITY Spectro-interferometric Study of the Massive Multiple Stellar System HD 93206 A](#)
J. Sanchez-Bermudez, A. Alberdi, R. Barbá et al.
- [The VLT Interferometric Measurements of Active Galactic Nuclei: Effects of Angular Momentum Distributions of Clouds in the Broad-line Region](#)
Yu-Yang Songsheng, Jian-Min Wang and Yan-Rong Li
- [The CHARA Array Interferometric Program on the Multiplicity of Classical Be Stars: New Detections and Orbits of Stripped Subdwarf Companions](#)
Robert Klement, Thomas Rivinius, Douglas R. Gies et al.



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

FIRST STUDY OF USING GRAVITATIONAL SIGNAL GENERATOR FOR THE MEASUREMENT OF THE GRAVITY SPEED

C L Fernandes¹, V B Gennari¹, C Frajuca¹

¹Instituto Federal de Educação, Ciências e Tecnologia de São Paulo
Rua Pedro Vicente, 625, Caninde – São Paulo – SP – Brazil BR
CEP: 01109-010 www.ifsp.edu.br

E-mail: cesar_fernandess@hotmail.com

Abstract. Albert Einstein's Theory of General Relativity in 1916 described a universe through its equations, where space and time are dynamic. Einstein suggested that matter and energy change the geometry of space-time through the effect of gravity. The speed of gravity is one of the great mysteries of nature and one of the most difficult to be discovered, because trying to obtain this value through observation of astrophysical objects is almost impossible to isolate the speed of gravitation and the transport of information. To simulate the effects of gravitational waves, solutions for the generation and detection of electromagnetic signals will be integrated. The development of the detection and transduction system is the focus of this research project where 3 sapphire bars will be suspended at 1 meter between them; the central sapphire will be the receiver of the electromagnetic signals, while the two "external" sapphires will transmit these electromagnetic signals by means of a PZT (piezoelectricity) system. In the calculations made in this project the sensitivity is sufficient to measure the signal. **Key words:** Detection of gravitational signals, Control and magnetic suspension, PZT-piezoceramic technology and Sapphire.

1. Introduction

Based on his theory of relativity, Albert Einstein was the first to predict the phenomenon of gravitational waves (GW) in 1916, but he believed that gravitational waves could never be directly detected. Its TRG implied that some of the most violent phenomena in the universe - such as the clash between two black holes - produce gravitational waves that expand through spacetime at the speed of light, deforming it like a stone deforming the surface of water by falling into a lake. But these bodies are so far apart that upon reaching planet Earth, the disturbances they cause would be very weak, Einstein said. It is now known that the major sources of these gravitational waves come from binary stellar systems. Next, I highlight three of these most significant binary stellar systems:

- Black Hole [1], which is characterized by being a region of space with a quantity of mass concentrated so great that nothing can escape the attraction of its force of gravity, not even light;
- Neutron star [2] is an extremely massive celestial body with extremely high rotation, with a period measured in milliseconds! These stars have a very strong magnetic field, the little radiation that escapes their surface, are radio waves and gamma rays. These neutron beams pulsate due to the rotation of the star.



The interior of a neutron star consists of a large nucleus formed primarily by neutrons and a small number of superconducting protons. Thus, at low temperatures, the superconducting protons, combined with the high speed of rotation of the star, produce a dynamo effect, like the one responsible for the earth's magnetic field. Around the nucleus is a mantle of neutrons, followed by a layer of iron nuclei and free electrons;

- Dwarf star [3], is characterized by being white with about 0.6 solar masses, a size sometimes larger than Earth, with a material density of 109 kg/cm³ on average, and in some cases reaching 10.000 kg/cm³, which makes white dwarfs one of the densest forms, surpassed only by neutron stars, black holes, the larger the mass of a white dwarf, the smaller its size.

Scientists around the world have focused on studies, and development of instruments that can detect GWs, through Gravitational Wave Detectors (GWD). Several GWDs were constructed, according to Figure 4, to obtain this detection of GW, a task that is not simple, because, this GW interact weakly with the matter and, in addition, the amplitude of the GW is very low, exactly what does not occur with electromagnetic waves. Among the GWD's, the LIGO (Laser Interferometer Gravitational-Wave Observatory) [4] stands out, LIGO started operating in 2002 but had its operation completed in 2010. Detectable emissions of gravitational waves are expected from binary systems - such as Black Holes, collisions of neutron stars, supernovae of massive stars, rotation of deformed neutron stars and remnants of gravitational radiation created at the beginning of the Universe. The LIGO observatory may, in theory, also observe other exotic gravitational phenomena. Physicists believe the technology is at a point where detection of gravitational waves of significant interest to astrophysics is possible. LIGO operates two observatories in synchrony, one in Livingston, Louisiana (30°29'55"N, 90°44'54"W) and one in the Hanford Nuclear Reserve, located near Richland, Washington (46°27'28"N, 119°24'35"W). The two locations are separated by 3,002 kilometers, which corresponds to 10 milliseconds at the arrival of the wave. This, in theory, will allow, by triangulation, to discover the origin of a wave in space, as can be seen on Figure 1.

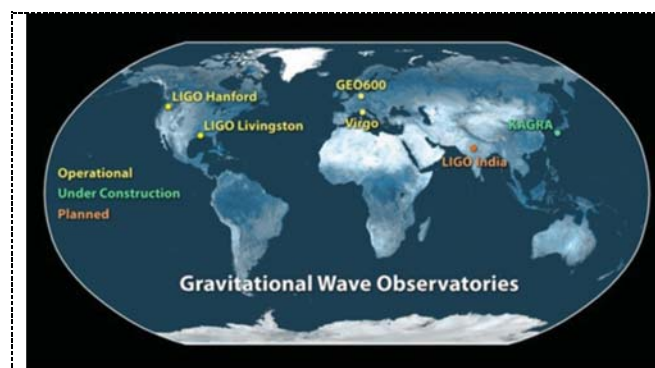


Figure 1: Main detectors and their geographical location Source : <https://www.ligo.caltech.edu/LA/>

Gravitational-Wave Observatories across the Globe, in Figure 1 the current operating facilities in the global network, can be seen including the twin LIGO detectors—in Hanford, Washington, and Livingston, Louisiana—and GEO600 in Germany. The Virgo detector in Italy and the Kamioka Gravitational Wave Detector (KAGRA) in Japan are undergoing upgrades and are expected to begin operations in 2016 and 2018, respectively. A sixth observatory is being planned in India. Having more gravitational-wave observatories around the globe helps scientists pin down the locations and sources of gravitational waves coming from space.

February 11, 2016, the LIGO project [4] announced the detection of gravitational waves from the signal found at 09:51 UTC on September 14, 2015, two black holes with about 30 solar masses in the process of fusion, the 1.2 billion light years from Earth. On October 3, 2017, the Nobel Prize in Physics was awarded

to Dr. Rainer Weiss, 85, Barry Barish, 81, and Kip Thorne, 77, "for decisive contributions to the LIGO detector and observation of gravitational waves" [4], which made the Nobel Prize jurors recognize the merits of what was considered "a discovery that shook the world," according to Göran Hansson, secretary general of the Swedish Academy of Sciences.

Brazil is also part of this international effort through the Mario Schenberg detector, however, the USP Institute of Physics (IF) donated the equipment, named after the Brazilian physicist Mario Schenberg (1914-1990), to the National Research Institute (INPE), as can be seen in Figure 2, is the result of a collaboration between the IF and INPE.



Figure 2: Gravitational wave detector of the Institute of Physics donated to INPE - Photo: Marcos Santos / USP Images

The authors have experience with Brazilian detector of gravitational waves (GW) [5]. The central resonance frequency of this detector is around 3200 Hz with bandwidth close to 200 Hz. It contains a spherical antenna with a mass of 1150 kg, measuring 65 cm in diameter and made of a copper and aluminium alloy with 94% of Cu and 6% of Al, isolated from seismic noise by a mechanical suspension [6]. It was designed to operate with at least six electromechanical resonant transducers [7] connected to the antenna surface, arranged according to a dodecahedron half distribution, based on works by Magalhaes and collaborators [8, 9, 10]. A history of the decisions made regarding the SCHENBERG detector design can be found in [11]. Each parametric microwave transducer [12, 13, 14, 15, 16, 17] will mechanically amplify the movement of the region in which it is connected to the sphere, which occurs at the detection frequency. This amplified movement, in turn, excites the membrane of the resonant cavity in which the microwaves are pumped, to generate the electronic signal that will return carrying all the information of the GW [18, 19, 20]. In principle, the amplitude and direction of the GW can be obtained from the analysis of the output signal of these transducers.

1.1 Objective

This work aims to prove the viability of an experiment that will generate a periodic gravitational signal to be used in the measurement of the speed of gravitational interaction (gravity). This generated signal will be analysed in the continuity of this work by a set of sapphire bars cooled to 4K in the vacuum and monitored by microwaves of ultra-low noise.

The experiment is based on a system with three sapphire bars, whose choice of this material was made because it is more suitable as a resonant bar type and a sensor transducer. The sapphires will be suspended in distances of 1 meter between them, the two other sapphires are connected to the PZT (piezoelectricity) system, whose function is to press and expand these pieces; these vibrations generate the gravitational signal that will be picked up and measured by the central sapphire bar.

Gravitational waves are ripples in the curvature of space-time that propagate as waves, traveling outward from the source, carrying energy in the form of gravitational radiation, so to arrive at the result, the mathematical model of the parametric transduction system to predict the noise sources and physical properties of the sapphire. Another way would be through the detection of gravitational waves, however, this is very difficult, because until today only few waves have been detected, therefore, besides the detection of such a wave a light counterpart must be found at the same time with different instruments, the which

makes this path almost impossible; therefore, the best way is to emit a signal and detect it, which is intended in the continuation of this project.

The choice of sapphire, which is a variety of the monocrystalline form of aluminium oxide (Al_2O_3), was given by the following characteristics and properties: Material Density = $3.98 \text{ (g cm}^{-3}\text{)}$, material propagation speed = $9,4 \text{ (km s}^{-1}\text{)}$ and mechanical quality factor = $3 \cdot 10^9$.

2. Sapphire study

Sapphire is best known as a beautiful velvety blue gemstone; however, the sapphire is much more important in mechanical applications. Surprising are the mechanical properties and industrial applications the sapphire offers, its history in the synthetic crystal growth industry is already long. Sapphire is a variety of corundum mineral and this mineral is usually described using hexagonal axes.

According to Table 1, sapphire proves to be the best material for detector transducer type resonant bar, consequently, the most suitable for the development of the project.

Table 1: Physical properties of the candidate material – gravitational wave source and detection method. Source – Ribeiro, INPE 2003 -R34

Material	$\rho \text{ (g cm}^{-3}\text{)}$	$v_s \text{ (km s}^{-1}\text{)}$	Q	$\rho v_s^3 \text{ (} 10^{13} \text{ kg s}^{-3}\text{)}$	$Q \rho v_s^3 \text{ (} 10^{20} \text{ kg s}^{-3}\text{)}$
Aluminum 6061	2,7	5,1	5×10^6	36	18
Aluminum 5056	2,7	5,1	7×10^7	36	25
Niobium	8,57	3,4	2.3×10^8	34	800
Silicon	2,33	8,5	2×10^9	140	2.8×10^4
Sapphire	3,98	9,4	3×10^9	330	10^5
Lead	11,36	1,1		1,5	
Tungsten	18,8	4,3		150	
Copper(94)/Al(6)	8,0	4,6	2×10^7	77,8	155

That:

ρ : Material density;

v_s : Sound propagation velocity in material;

Q : Mechanical quality factor (thermal noise approximates Q^{-1});

ρv_s^3 : Approximation of the absorbed energy rate;

$Q \rho v_s^3$: Signal to noise ratio.

The sapphire material stands out among others by having the highest possible antenna noise in the calculation of Formula $Q \rho v_s^3$, so that the amplitude of the thermal noise of the transducer will not exceed the amplitude of the excitation signal. In addition, sapphire has the highest electrical quality factor (Q), allowing a large electromechanical coupling.

3. PZT operations in MHz [7]

PZT is the ability of some crystals to generate electrical voltage by response to mechanical pressure. PZT is also called piezoelectricity, this term meaning tightening / pressing, comes from the Greek word piezein. The active element of most ultrasonic devices and transducers, which is what is proposed in this work, is piezoelectric. The PZT may belong to one of the following groups: quartz crystals, water soluble crystals, monocrystals, piezoelectric semiconductors, piezoelectric ceramics, polymers and piezoelectric composites.

Here we detail the principle of piezoelectric, perovskite and polarization. The piezoelectricity or so-called direct piezo effect (Figure 3) is the ability of some crystals to generate an electric charge when mechanically

charged with pressure or voltage; (Figure 4) are crystals that undergo a controlled deformation when exposed to an electric field, this study is based on the following concept: The polarity of the charge depends on the orientation of the crystal relative to the direction of pressure.

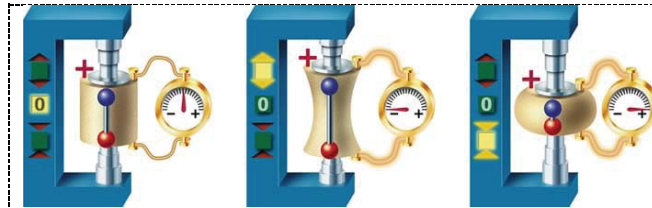


Figure 3: Direct piezoelectric effect

https://www.ceramtec.com.br/_img/content/werkstoffe.piezokeramik.grundlagen.direkter.piezoefekt.jpg

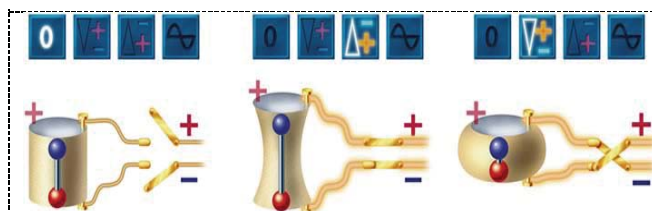


Figure 4: Reverse piezoelectric effect

https://www.ceramtec.com.br/_img/content/werkstoffe.piezokeramik.grundlagen.direkter.piezoefekt.jpg

4. Transducer of Sapphire - Simulation of the gravitational signal, resonant bars

This system is based on SQUID (Superconducting Quantum Interface Device) amplifiers that amplify the signals of an "inductive superconductive" transducer for gravitational signals with resonant bars using coupled parametric transducers, as can be seen on Figure 5.

The geometry of the sapphire transducer will be in the form of a cylindrical rod as massive as possible, and regarding the manufacture of the sapphire, some care must be taken to make the Q_c and Q_m the highest possible values.

Adaptation of the complete gravitational wave detector model to the receiver element for the development of the project, the connection of the receiver element to the low-noise oscillator went straight with the Sapphire and needed some adjustments. The Sapphire transducer geometry will be in the form of a cylindrical bar as massive as possible, and in relation to the manufacture of sapphire,

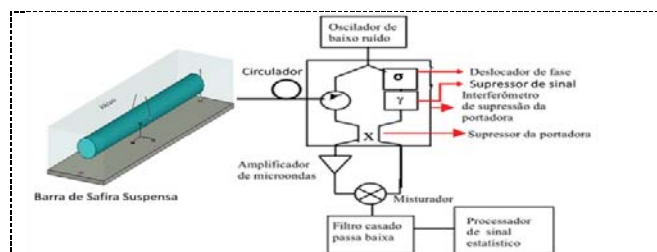


Figure 5: Complete model of the gravitational wave detector adapted to the design - Low Noise Oscillator connected to the Project receiving Sapphire – Unprecedented

5 Project development

This work aims to develop a system with the objective of generating a periodic gravitational signal to be used to measure the speed of gravitational interaction (gravity).

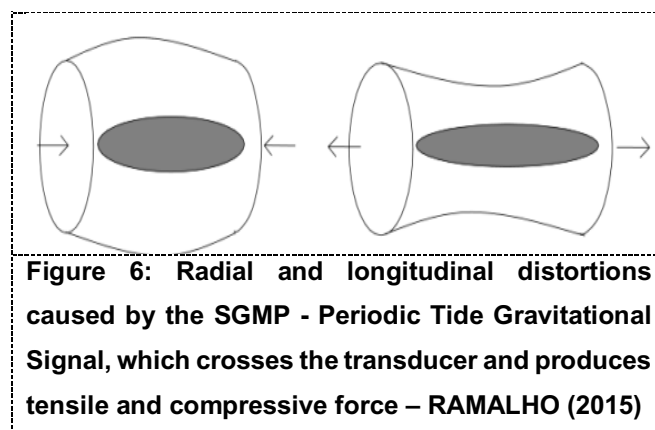
5.1 Sapphire Transduction Mechanism

The study of the acoustic resonance frequency of the sapphire cylindrical will be developed by the calculations of the forces that will act on both the generating elements and the receiving element. Based on the study developed by Carlos Frajuca in 2014, and calculations revisited with a differentiated proposal of frequency emitted and received by 3 pieces of Sapphire and how these elements behave like a mechanical resonator. We will prove that the Sapphira element resounds its dimensions, and how they change and act forces internally. Thus, we will observe the two mechanisms of transduction: (a) the change in the dimensions in the crystal due to the oscillations change their dimensions, and (b) such changes induce changes in the dielectric permittivity altering the electric modes.

Besides the crystal is part of the detection system of the SGMP, it is also used in the transduction system, since when its dimensions are changed its dielectric properties change, in this way this effect is used as a parameter in a resonant circuit. When the dielectric permittivity changes, the circuit capacitance changes by changing its frequency response, this mechanism performs the parametric transduction of the SGMP.

The excitation forces under the transducer are from the SGMP, which causes longitudinal distortions in the sapphire bar. The tidal gravitational signal mechanically excites the detector (the sapphire bar) causing traction and compression forces that oscillate at the resonant frequency of the bar, exciting its acoustic modes, causing it to resonate by modifying its dimensions. Consequently, the oscillations caused due to the acoustic resonant effect cause the longitudinal distortions that alter the dielectric permittivity of the sapphire bar. This effect is cancelled using of active transduction.

To perform the active transduction the microwave signal generated by a microwave signal generator is used. This signal is then injected and collected, in which case the longitudinal distortions in the transducer cause a change in the dielectric constant of the sapphire, thus, a signal of microwaves applied in the crystal when being collected of it undergoes a change of amplitude and phase due to the changes in the dielectric constant. This effect is called parametric transduction, where an electric parameter, in this case the dielectric constant, is changed causing a modulation in the capacitance of the system. This is only possible, because the mechanical distortions caused by the interaction of the SGMP with the transducer cause changes in the modes of electromagnetic resonance of the transducer, this phenomenon is illustrated Figure 6.



5.2 Gravitational signal generation and detection

The study was initiated by the system proposed by Frajuca and Ruiz (2014) where we have the representation of two bodies of mass ' M ' rotating in a specific radius ' a ' around an axis displaced in a distance ' r ' [21]. This system as we see in figure 7 has become the basis for the evolution of the project.

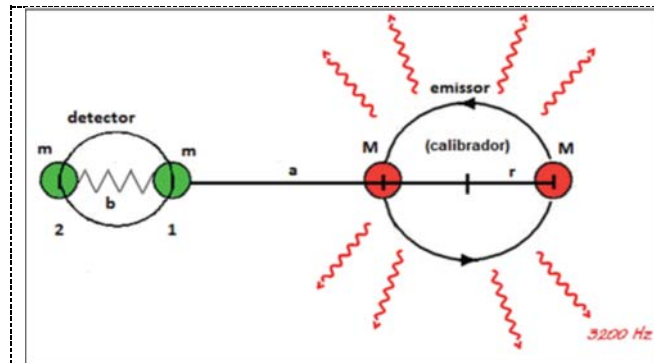


Figure 7: Simplified model of the detector and the emitter of periodic tidal gravitational signals.

Source - Frajuca and Ruiz (2014)

With this initial idea, the evolution of the project occurred with the study of Sapphire as the most suitable material for the emission and detection of signals, by the vibration of suspended pieces of 20 cm, and weight of 100 grams in each of them; connected to a PZT system at 10 m, under vacuum at a temperature of 4 K. These values were taken as initial study parameters for the definition of the calculation formulas, as seen on Figure 8.

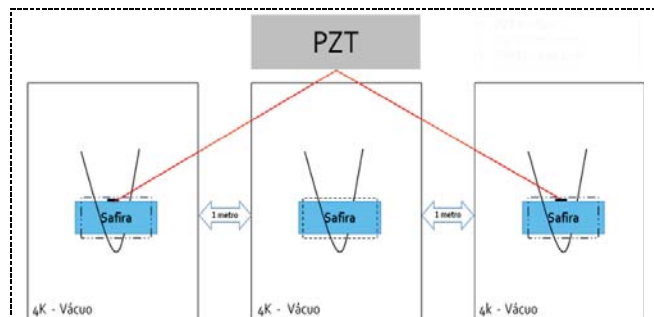
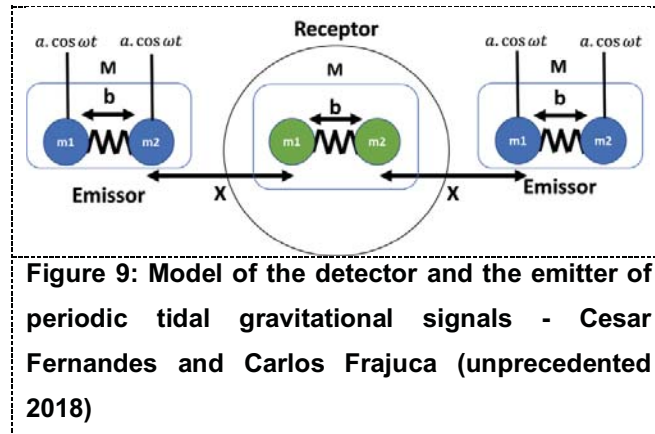


Figure 8: Proposed design of the project - PZT in phase / sapphire with 20cm / Distance of 10m between emitters and signal receivers (unprecedented 2018)

From this theory, the simulation of the transduction is made, taking into account the calculation of Force F_n and F_a as considered in the initial study, nevertheless changed the design of the project, taking into consideration the complete system, as we see in the Figure 8. Calculations will be made with Newtonian signal analysis generated from the ' M ' masses of the sapphires, which vibrate and transmit the signal to the detector.

The proposed model, part of a system formed by three sets of masses m_1 e ' m_2 ', as seen in Figure 9, displaced together by the distance ' b ' coupled by a spring, as seen first at the beginning of these studies. The results of the calculations will be the basis for the projection of the suspended sapphire system.



5.2.1 Calculations for Demonstrating Theory

The calculations will be based on Figure 9 where we will measure the forces F_{1x} and F_{2x} , in relation to the masses ' m_1 ' e ' m_2 ', the distance ' b ' coupled by a spring, representing the specific mass ' M ' and the coefficient of rigidity of the materials of detectors and receivers plus distance ' X '. This relation will give the subsidies to the more adequate definition of the elements that will be used in the project. Therefore, the analysis of the calculations is based on the following concepts and equations:

5.2.1.1 Analysis of the Newtonian signal

Newton's constant G value adopted is $6,67408 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$. The analysis of the Newtonian signal generated from the ' m_1 ' e ' m_2 ' masses of two emitting elements that vibrate and interact with the matter of the detector, which is at the center, receives these vibrations in GHz.

Therefore, we can obtain tidal interaction forces that will be caused by ' m_1 ' and ' m_2 ' with different intensities due to the distance ' b ' plus the distance ' X ', with this system formed by ' m_1 ' e ' m_2 ' in all the elements of the design, detectors and receivers, we have the mechanical harmonic oscillator. From an external excitation in PZT, whose frequency is the fundamental frequency of this system, the excitation signal will be amplified, due to the resonant characteristic of the material, thus, the excitations caused by the vibrations of the emitting elements with masses ' m_1 ' e ' m_2 ', we have the forces of interaction Tidal force on the masses ' m_1 ' e ' m_2 ' of the receiver, shown by the following equations F_{1x} and F_{2x} .

$$F_{1x} = \frac{MmG}{(b+X+a.\cos \omega t)^2} + \frac{MmG}{(2b+X-a.\cos \omega t)^2} \quad (1)$$

$$F_{2x} = M_{ef}mG \left(\frac{1}{(X+a.\cos \omega t)^2} + \frac{1}{(b+X-a.\cos \omega t)^2} \right) \quad (2)$$

That:

- G : Newton constant;
- M_{ef} : Total mass of the emitting and receiving elements;
- m : Effective mass of emitters and detectors;
- ω : Speed of mass vibration;
- a : Length / distance of sapphire piece vibrating;
- X : Distance between the emitters and the detector;
- b : Effective length of transmitters and receiver;
- t : Time.

5.2.1.2 Calculation of tidal interaction forces

$$a = a \cos wt \quad \text{and} \quad a \ll b \ll X$$

$$F_{1x} - F_{2x} = \frac{M_{ef}mG}{x^2} \left(\frac{1}{\left(1+\frac{b+a}{x}\right)^2} + \frac{1}{\left(1+\frac{2b-a}{x}\right)^2} \right) - \left(\frac{1}{\left(1+\frac{a}{x}\right)^2} + \frac{1}{\left(1+\frac{b-a}{x}\right)^2} \right) \quad (3)$$

$$\frac{GmM_{ef}}{x^2} \left(-\frac{4b}{x} + \frac{12b^2}{x^2} - \frac{8(b(3a^2-3ab+4b^2))}{x^3} \right) \quad (4)$$

using a first order approximation around a small device compared to the distance:

$$F_{1x} - F_{2x} = MmG \cdot \frac{24ab^2}{x^5}, \quad (5)$$

using the expression for an harmonic oscillator, the vibration amplitude in b is given b:

$$\Delta b = QGM \left(\frac{24ab^2}{x^5 \cdot w^2} \right) \cdot \sin wt \quad (6)$$

$$\Delta b = 3 \cdot 10^{-18} m \quad (7)$$

calculated with the characteristics described in next chapter.

6. Quantum limit, equipment sensitivity and thermal noise

From the calculations that will be demonstrated in this chapter, it will be proven that the sapphire can measure oscillations above the quantum limit.

The available technology is sufficient to measure the Standard Quantum Limit (SQL) of a low-loss acoustic oscillator with reading based on a parametric microwave transducer. The experiment uses the low electrical and acoustic losses in the monocrystalline sapphire and low noise microwave technology. The crystal acts as an electric vibration sensor and an acoustic oscillator in a monolithic structure. In the structure of the sapphire bar dielectric transducer, we found that, with a double-phase noise suppression system of 40 to 60 dB double cavity, the SQL can be measured using the sapphire bar. We show that the SQL of this structure can be measured with a standard parametric reading. The operating principle is demonstrated by some simple experiments of ambient temperature, with all results verified using finite element analysis. We have, therefore, been able to measure SQL; analyzing the properties of a microwave displacement measuring system based on a parametric high Q_m transducer and a dual frequency oscillator. The objective of detecting gravitational radiation, has necessarily forced the measurement technologies, at increasingly sensitive levels. Currently, the measurement of mass displacements, of the order of 10^{-19} m, is achievable in state-of-the-art detectors around the world. In theory, a macroscopic mass will exhibit quantum mechanical properties if its displacement can be measured with sufficient precision. In this case, the Displacement uncertainty will be governed by the Heisenberg uncertainty principle, with the limit for the measurement, known as the standard quantum boundary (SQL).

The existence of SQL for a macroscopic object is possible for the following reasons:

- (1) sapphire with ultra-low and acoustic dielectric [22], demonstrated losses are available;
- (2) demonstration of a low noise parametric transducer reading [22], based on a low-noise phase oscillator, which can now reach levels as low as -185 dBc/Hz at 1 kHz off set;
- (3) ultra-sensitive sapphire dielectric transducer [22];

(4) the new transducer configuration, which combines the good electromagnetic and acoustic properties of the sapphire into a single monolithic structure, because the SQL can never be overcome with a displacement measurement.

Following are the values set for the Quantum Limit, Equipment Sensitivity Limit and Thermal Noise Limit calculations:

$$\begin{aligned} M_{ef} &= 1 \text{ kg}; \\ \text{Distance between the masses} &= 1 \text{ m}; \\ Q &= 10^9; \\ G &= 6,67 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}; \\ A &= 10^{-4} \text{ (Vibration of the bars)}; \\ B &= 0,2 \text{ m (size of the bars)}; \\ \hbar &= 6,626 \ 069 \cdot 10^{-34} \text{ J} \cdot \text{s} \\ \text{Frequency bandwidth BW} &= 10 \text{ Hz}; \\ \text{Temperature} &= 4 \text{ K} \end{aligned}$$

6.1 Quantum Limit

The experiment is based on the vibration of the suspended sapphires. And for vibrating bodies the energy packets are called Phonons, the minimum energy that can be measured for the quantum boundary is 1 Phonon which is represented by the formula:

$$E = \hbar \omega \quad (8)$$

That corresponds to the minimum limit, because the smallest number of Phonons is 1; therefore,

$$\hbar \omega = \frac{1}{2} \cdot A^2 \omega^2 m \Rightarrow \hbar \omega = \frac{A^2 \omega^2 M_{ef}}{2} \quad (9)$$

$$\Delta b_{QL} = A = \sqrt{\frac{2\hbar}{\omega M_{ef}}} \quad (10)$$

$$\Delta b_{QL} = A = \sqrt{\frac{2\hbar}{M_{ef} 2\pi}} = \sqrt{\frac{2 \cdot 10^{-34}}{2\pi \cdot 5 \cdot 10^5}} = 2,5 \cdot 10^{-20} \text{ m} \quad (11)$$

$$\text{Quantum Limit: } \Delta b_{QL} = 2,5 \cdot 10^{-20} \text{ m} \quad (12)$$

6.2 Equipment Sensitivity Limit

The sensitivity of the displacement of a parametric transducer can be characterized by the frequency shift of the electrical resonance in relation to the displacement (df/dx). For the sapphire transducer, this value is (2 MHz)/(μ m.) For the displacement (df/dx) and phase noise of the pump oscillators, the limit for spectral performance of the transducer can be calculated by:

$$S_x(f) = \left(\frac{df}{dx}\right)^{-2} S_\phi(f) f^2 \quad (13)$$

$$S_x = \sqrt{10^{-39,5}} = 5 \cdot 10^{-19} \frac{\text{m}}{\sqrt{\text{Hz}}} \quad (14)$$

Using *bandwidth* $BW = 10$ Hz: The $S_{\varphi} = -185$ dBc at 1000 Hz of the carrier (TOBAR, 1996)

$$\text{Equipment Sensitivity Limit: } S_x = 1,6 \cdot 10^{-19} m \quad (15)$$

6.3 Thermal Noise Limit

Variations in detector length due to thermal effects (Δ_{xt}) should be reduced below quantum limits. And the variations in detector length, with the addition of thermal energy, cause a noise known as Nyquist thermal noise; the study of these length variations has as reference.

For thermal noise to be minimized, the relationship between the standard quantum boundary and Nyquist thermal noise should be maximized.

The thermal noise must be small enough that the limits imposed by the project are related to the quantum limit. This ensures the highest accuracy of the system.

$$\Delta_{xth} = \sqrt{\frac{KT}{2M_{eff}\omega_m Q_m B_{wD}}} \quad (16)$$

$$\text{Thermal Noise Limit: } \Delta_{xth} = 6,4 \times 10^{-20} m \quad (17)$$

7. Conclusion

The measurement is limited by the following limits:

- ✓ Signal strength:
 - $\Delta b = 3 \times 10^{-18} m$
- ✓ Quantum limit:
 - $\Delta b_{QL} = 2,5 \times 10^{-20} m$
- ✓ Equipment sensitivity limit:
 - $S_x = 1,6 \times 10^{-19} m$
- ✓ Thermal noise limit:
 - $\Delta_{xth} = 6,4 \times 10^{-20} m$

The signal caused by the vibration is greater than the quantum limit, that the sensitivity of the equipment and that the thermal noise, it is concluded, therefore, that with the adopted parameters for the measurement of the speed of gravity is possible by the proposed experiment, being the experiment possible.

8. References:

- [1] Black Hole: InfoEscola - <https://www.infoescola.com/astrologia/buraco-negro/>, *Wikipedia* - https://pt.wikipedia.org/wiki/Buraco_negro

- [2] Neutron star: https://pt.wikipedia.org/wiki/Estrela_de_n%C3%A9utrons, *Misterios do Mundo* - <https://misteriosdomundo.org/o-que-e-uma-estrela-de-neutrons/>
- [3] Dwarf star: https://pt.wikipedia.org/wiki/An%C3%A3_branca Site Astronomia - <http://www.siteastronomia.com/estrela-ana-branca>
- [4] LIGO <https://pt.wikipedia.org/wiki/LIGO>
- [5] The Gravitational Waves International Committee Roadmap (GWIC). The future of gravitational wave astronomy – A global plan. June 2010. Glasgow: University of Glasgow - Department of Physics and Astronomy - Kelvin Building (G12 8QQ), 117p.
- [6] Bortoli F S, Frajuca C, De Sousa S T, de Waard A, Magalhaes N S 2016 *Brazilian Journal of Physics* **46** 308
- [7] Frajuca C, Ribeiro K L, L. A. Andrade, W. F. Velloso Jr, J. L. Melo, O. D. Aguiar and N. S. Magalhaes 2002 *Class. Quantum Grav.* **19** 1961
- [8] N. S. Magalhaes, O.D. Aguiar, W.W. Johnson and C. Frajuca 1997 *Gen. Relat. Grav.* **29** 1511
- [9] N. S. Magalhaes, W. W. Johnson, C. Frajuca and O. D. Aguiar 1995 *MNRAS* **274** 670
- [10] N. S. Magalhaes, W. W. Johnson, C. Frajuca and O. D. Aguiar 1997 *Astroph. J.* **475** 462
- [11] O. D. Aguiar et al. 2002 *Brazilian Journal of Physics* **32** 866
- [12] C. Frajuca, F. S. Bortoli, N. S. Magalhães. 2005 *Brazilian Journal of Physics* **35** 1201
- [13] C. Frajuca, K.L.Ribeiro, O.D.Aguiar, L. A Andrade, P.J.Castro, N.S.Magalhaes and R. M. Marinho Jr., 2004 *Class. Quantum Grav.* **21** 1107
- [14] C. Frajuca, N. S. Magalhaes, A. M. Horiguti. 2008 *Journal of Physics: Conference Series (Online)* **122** 012029.
- [15] C. Frajuca, F. S. Bortoli N. S. 2006 *Journal of Physics: Conference Series (Online)* **32**, 319
- [16] C. Frajuca, F.S. Bortoli; N. S. Magalhães 2005 *Brazilian Journal of Physics*, **35** 1201
- [17] F. S. Bortoli , C Frajuca , N Magalhaes and E N Duarte 2010 *Journal of Physics: Conference Series* **228** 012011.
- [18] C. R. Locke, M.E. Tobar, E. N. 2000 *Review of Scientific Instruments* **71** 2737
- [19] Ribeiro K L et al. 2004 *Class. Quantum Grav.* **21** 1225
- [20] Andrade L A et al. 2004 *Class. Quantum Grav.* **21** 1225
- [21] Ramalho W. C. S. – 2015 *Transdutor de Safira para medição da velocidade da interação gravitacional*

[22] Tobar M.E., Ivanov E.N. Oi D.K.L., Cuthbertson, B.D., Blair D.G. Sapphire test-masses for measuring the standard quantum limit and achieving quantum non-demolition - Department of Physics, The University of Western Australia, Nedlands, **6907**, WA, Australia