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Electron transport in nonideal and degenerate plasmas

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Abstract. In this article, various methods of calculation for the Coulomb logarithm in the kinetic theory of electron transfer are considered. It is assumed that the interaction of electrons with ions is described by the Debye potential with a screening constant, which is determined either with or without the contribution of the ionic component. In order to take into account ionion correlations, the Ornstein–Zernike integral equation in the hypernetted chain approximation is solved numerically. The behavior of the collision cross section of electrons with ions has been studied in detail on the basis of the phase shift method.

1. Introduction

In plasma physics and astrophysics, the long-range nature of the Coulomb interaction and the gravitational one results in the appearance of various kinds of divergent integrals. To eliminate divergences in considering the kinetic equation, the Coulomb logarithm is introduced into the transport cross section for scattering of electrons by ions. The determination of this logarithm in the case of a nonideal plasma occurs with certain difficulties (see for example [1,2] and the cited literature there). In the nonideal plasma, the values of the Coulomb logarithm can be much less than unity. As shown in [3], this leads to an ultrahigh conductivity of gases in the megabar pressure range. At such pressure, the very application of the kinetic approach raises questions; therefore, a thorough study of this problem is necessary.

In this paper, we consider various methods for determination the Coulomb logarithm in the kinetic theory of electron transfer, variants for choosing a screening constant of the plasma with or without allowance for the contribution of the ionic component, and options for specifying the boundary value of the electron wave vector that determines the maximum value of the electron momentum transferred to the ions.

In the first section, the behavior of the collision cross section of electrons with ions has been studied in detail on the basis of the phase shift method. In the second section, we determine the transport cross section for scattering of electrons by ions, taking into account the ion correlation on the basis of the Ornstein–Zernike (OZ) integral equation in the hypernetted chain (HNC) approximation; the third section contains the results of the numerical calculations and the discussion of the results.

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2. The cross section for elastic scattering of electrons

Differential cross section scattering at the given angles θ and φ is defined by the formula

$$\sigma\left(\theta,\varphi\right) = \left|F\left(\theta,\varphi\right)\right|^{2},\tag{1}$$

where $F(\theta, \varphi)$ is the total scattering amplitude. The total cross section is

$$\sigma = \int_{0}^{2\pi} \int_{0}^{\pi} \sigma(\theta, \varphi) \sin \theta d\theta d\varphi.$$
 (2)

The total amplitude of the elastic scattering of electrons is determined by the asymptotic value of the phase shift of the wave function [4] $\delta_{l\infty} = \lim_{r \to \infty} \delta_l(r)$:

$$F(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} \left(2l+1\right) e^{i\delta_{l\infty}} \sin \delta_{l\infty} P_l(\cos \theta), \qquad (3)$$

where $P_l(\cos \theta)$ are the Legendre polynomials, k is the wave number of a particle with the reduced mass m: $k = mv/\hbar$, v is the particle velocity. For a collision of an electron with an ion, the reduced mass practically coincides with the electron mass m_e , while the particle velocity with reduced mass m coincides with electron velocity. From equations (2) and (3), for total scattering cross section one can obtain [4]

$$\sigma = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} \left(2l+1\right) \sin^2 \delta_{l\infty}.$$
(4)

The phase shift function is a solution of the differential equation [5-9]:

$$\frac{\mathrm{d}\delta_l\left(r\right)}{\mathrm{d}r} = -\frac{1}{k} \frac{2m_e}{\hbar^2} V\left(r\right) \left[j_l\left(kr\right)\cos\delta_l\left(r\right) - n_l\left(kr\right)\sin\delta_l\left(r\right)\right]^2 \tag{5}$$

with the initial condition

$$\delta_l \left(r = 0 \right) = 0, \tag{6}$$

where \hbar is the reduced Planck's constant, j_l , V(r) is the interaction potential, n_l are the Riccati– Bessel functions (they differ from the spherical Bessel functions in that they are additionally multiplied by an argument).

The electron–ion interaction is described by the Debye potential

$$V(r) = -\frac{e^2 z_i}{r} \mathrm{e}^{-k_s r},\tag{7}$$

where k_s is the screening constant, ez_i is the ion charge.

2.1. Approximate solution for the Debye potential

In order to find the approximate solution of equation (5) for small values of the phase function, we rewrite (5) in the following form:

$$\frac{\mathrm{d}\tan\delta_l\left(r\right)}{\mathrm{d}r} = -\frac{1}{k} \frac{2m_e}{\hbar^2} V\left(r\right) \left[j_l\left(kr\right) - n_l\left(kr\right)\tan\delta_l\left(r\right)\right]^2. \tag{8}$$

Neglecting the phase tangent in square brackets, from (8) we find the first approximation:

$$\tan \delta_l(r) = -\frac{1}{k} \frac{2m_e}{\hbar^2} \int_0^r V(r) j_l^2(kr) \, \mathrm{d}r = -\frac{\alpha}{k} \int_0^r \frac{\mathrm{e}^{-k_s r}}{r} j_l^2(kr) \, \mathrm{d}r,\tag{9}$$

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where α is the quantity with the dimension of the reciprocal length and is determined by the relation

$$\alpha = \frac{2m_e e^2 z_i}{\hbar^2}.$$

For the phase shift, using the value of the improper integral [10]

$$\int_{0}^{\infty} \left[J_{l+1/2} \left(kr \right) \right]^{2} \mathrm{e}^{-k_{s}r} \mathrm{d}r = \frac{1}{\pi k} Q_{l} \left(1 + \frac{k_{s}^{2}}{2k^{2}} \right), \tag{10}$$

from (9) we get

$$\tan \delta_l\left(\infty\right) = -\frac{\alpha}{2k} Q_l\left(1 + \frac{k_s^2}{2k^2}\right) \equiv -\frac{e^2 z_i k}{2\varepsilon_e} Q_l\left(1 + \frac{k_s^2}{2k^2}\right). \tag{11}$$

Here Q_k is the Legendre function of the second kind, ε_e is the electron energy: $\varepsilon_e = \hbar^2 k^2 / 2m_e$. Now taking into account the sum [11]

$$\sum_{k=0}^{\infty} \left(2k+1\right) \left[Q_k\left(x\right)\right]^2 = \frac{1}{x^2-1},\tag{12}$$

for the elastic collision cross section of an electron with an ion, provided that the phase shift is small from (11) we get

$$\sigma = \frac{\pi e^4 z_i^2}{\varepsilon_e^2} \frac{k^2}{k_s^2} \frac{4k^2}{4k^2 + k_s^2} \equiv \frac{4\pi\alpha^2}{k_s^2 (4k^2 + k_s^2)}.$$
(13)

2.2. The Born approximation

In the Born approximation, the elastic scattering amplitude is given by [4]

$$f_B = -\frac{m}{2\pi\hbar^2} \int U(r)e^{-\mathbf{i}\mathbf{q}\mathbf{r}} \mathrm{d}\mathbf{r},\tag{14}$$

where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, $q = 2k \sin \frac{\theta}{2}$, θ is the angle between the vectors \mathbf{k} and \mathbf{k}' , which is a scattering angle, \mathbf{k} , \mathbf{k}' are the wave vectors of the initial and scattered electrons: $k = p/\hbar$, U(r) is the interaction potential. In order to apply the Born approximation, at least one of the following two conditions must be fulfilled:

$$|U| \ll \frac{\hbar^2}{ma^2} \text{ or } |U| \ll \frac{\hbar v}{a},\tag{15}$$

where a is the characteristic radius of U(r), |U| is the order of its magnitude in the main area of its action. When the first condition is met, the Born approximation is applicable at all velocities, and the second one only at sufficiently high velocities.

For the Debye potential (7) for the elastic scattering amplitude from (14) we get

$$f_B = -\frac{\alpha}{k_s^2 + 2k^2 \left(1 - \cos\theta\right)}.$$
(16)

The differential scattering cross section in the Born approximation in the case of the Debye potential is obtained by the formula

$$\sigma\left(\theta\right) = \frac{\alpha^2}{\left[k_s^2 + 2k^2\left(1 - \cos\theta\right)\right]^2}.$$
(17)

By integrating (2) with (17), for the total cross section, we arrive at (13). Consequently, the first approximation (9) corresponds to the Born approximation.

2.3. The semiclassical approximation

For large l, the motion is quasiclassical, therefore, the phase of the wave function is determined by the difference in the phase incursions when moving with and without the field U(r) [4]:

$$\delta_l = \int_{kr_1}^{\infty} \sqrt{1 - \frac{(l+1/2)^2}{x^2}} - \frac{2m_e U(x/k)}{k^2 \hbar^2} dx - \int_{kr_2}^{\infty} \sqrt{1 - \frac{(l+1/2)^2}{x^2}} dx,$$
(18)

where x = kr, r_1 , r_2 are the roots of the radicands (area $r > r_{1(2)}$ is the classically accessible motion area):

$$k^{2} - \frac{\left(l+1/2\right)^{2}}{r_{1}^{2}} - \frac{2m_{e}U\left(r_{1}\right)}{\hbar^{2}} = 0, \qquad r_{2} = \frac{l+1/2}{k}.$$
(19)

In the case of the Debye potential

$$\delta_l = \int_{kr_1}^{\infty} \sqrt{1 - \frac{(l+1/2)^2}{x^2} + \frac{\alpha e^{-\beta x}}{kx}} dx - \int_{kr_2}^{\infty} \sqrt{1 - \frac{(l+1/2)^2}{x^2}} dx$$
(20)

and r_1 is defined by the expression

$$k^2 r_1^2 + \alpha r_1 e^{-k_s r_1} - (l+1/2)^2 = 0.$$

For large l, the values of r_1 and r_2 are also large, therefore, U(r) is small in the entire domain of integration and $r_1 \approx r_2$ with high accuracy. In this case, from (18) one can get an approximate expression [4]:

$$\delta_l = \frac{m_e}{\hbar^2 k^2} \int_{(l+1/2)}^{\infty} \frac{U(x/k)}{\sqrt{x^2 - (l+1/2)^2}} x dx = -\frac{\alpha}{2k} \int_{(l+1/2)}^{\infty} \frac{e^{-\beta x}}{\sqrt{x^2 - (l+1/2)^2}} dx.$$
 (21)

2.4. Numerical calculations of the cross section

Figure 1 shows the dependence of the phase function on the distance for different values of the orbital angular momentum l with energy $\varepsilon_e = 1$ eV. Number densities of electrons and ions $n_{e0} = n_{i0} = 10^{16}$ cm⁻³ ($z_i = 1$), temperature T = 300 K. As a screening constant, we used the complete Debye constant:

$$k_s = k_D \equiv \left[\frac{4\pi e^2 n_{e0} \left(1 + z_i\right)}{T}\right]^{1/2}.$$

Figure 2 demonstrates partial cross sections as functions of the orbital angular momentum at $n_{e0} = n_{i0} = 10^{16} \text{ cm}^{-3} (z_i = 1), T = 300 \text{ K}$. One can see that the total cross sections calculated by the method of phase functions and in the Born approximation are in good accordance.

Table 1 compares the total cross sections for elastic scattering at different electron energies calculated by the different methods described above. It is seen that at high energies $\varepsilon_e \gg T$ all four methods give practically identical cross sections, but as the energy decreases, the discrepancies grow. We draw attention to the fact that a quasiclassical method with allowance for the mismatch of the turning points of an electron moving in a force field and without it yields the closest cross sections to the exact values obtained by the phase–function method (18). From table 1 it is also seen that the accuracy of the Born approximation decreases with decreasing the electron energy due to the violation of the conditions of its applicability.



Figure 1. Phase functions of electron elastic scattering by ions with the Debye potential for different orbital numbers l as a function of the reduced radial coordinate at the electron energy $\varepsilon_e = 1 \text{ eV}$, electron and ion number densities $n_{e0} = n_{i0} = 10^{16} \text{ cm}^{-3}$ ($z_i = 1$) and the temperature T = 300 K.



Figure 2. Partial cross sections as a function of the orbital number for $n_{e0} = n_{i0} = 10^{16} \text{ cm}^{-3}$ $(z_i = 1), T = 300 \text{ K}$ at $\varepsilon_e = 10^{-2}$ (1), 0.1 (2) and 1 eV (3). The solid curves have been obtained by the phase shift method, the symbols with thin lines have been calculated within the Born approximation.

3. Transport cross section for elastic scattering of electrons

To describe the transfer of light particles to a heavy particle, the Lorentz approximation [12] can be used. In this case, the integral of the electron–ion collisions takes the simple form of [12]

Table 1. The total cross sections σ (in cm²) of electron elastic scattering by ions with taking into account the partial cross sections for orbital numbers up to $l_{\text{max}} = 60$ at $n_{e0} = n_{i0} = 10^{16}$ cm⁻³ ($z_i = 1$) and T = 300 K.

$\varepsilon_e, \mathrm{eV}$	10^{-3}	10^{-2}	10^{-1}	1
The phase-function method (4) The Born approximation (13) The semiclassical approach (18) The semiclassical approach (21)	$\begin{array}{l} 1.91\times 10^{-10}\\ 1.46\times 10^{-10}\\ 1.87\times 10^{-10}\\ 1.22\times 10^{-10}\end{array}$	$\begin{array}{l} 9.36\times 10^{-11}\\ 7.76\times 10^{-11}\\ 9.52\times 10^{-11}\\ 7.86\times 10^{-11}\end{array}$	$\begin{array}{l} 4.80\times10^{-11}\\ 4.71\times10^{-11}\\ 4.80\times10^{-11}\\ 4.72\times10^{-11}\end{array}$	$\begin{array}{c} 2.23\times 10^{-11}\\ 2.23\times 10^{-11}\\ 2.23\times 10^{-11}\\ 2.23\times 10^{-11}\\ \end{array}$

equation $\text{St}f = -\nu_{\text{ei}}\delta f$, where ν_{ei} is the transport frequency: $\nu_{\text{ei}} = n_i v \sigma_{\text{tr}}$; σ_{tr} is the transport cross section, δf is the deviation of the distribution function from the equilibrium, v is the electron velocity. The transport cross section is defined by the formula

$$\sigma_{\rm tr} = \int_{0}^{2\pi} \int_{0}^{\pi} (1 - \cos\theta) \,\sigma\left(\theta\right) \sin\theta \mathrm{d}\varphi \mathrm{d}\theta \tag{22}$$

and in the Born approximation we have

$$\sigma_{\rm tr} = \frac{m^2}{2\pi\hbar^4} \int_0^{\pi} (1 - \cos\theta) \left| \int U(r) \, e^{-i\mathbf{q}\mathbf{r}} d\mathbf{r} \right|^2 \sin\theta d\theta.$$
(23)

Taking into account the ion correlations, the interaction potential of electrons with N_i ions in the volume V is determined as follows:

$$U(\mathbf{r}) = \sum_{i} U_{\rm ei} \left(\mathbf{r} - \mathbf{R}_{i} \right), \tag{24}$$

where the summation is performed for all ions with coordinates R_i , U_{ei} is the electron–ion interaction potential. For the Fourier transform of the potential (24) per ion, we find

$$U_{\mathbf{k},\mathbf{k}'} = U_{\mathbf{q}ei} \frac{1}{N_i} \sum_{i} e^{-i\mathbf{q}\mathbf{R}_i},\tag{25}$$

where

$$U_{\mathbf{q}ei} = \int U_{\mathrm{ei}}\left(\mathbf{r}\right) \mathrm{e}^{-\mathrm{i}\mathbf{q}\mathbf{r}} \mathrm{d}\mathbf{r}.$$

For the square of the Fourier transform of the potential, we have

$$\left|U_{\mathbf{k},\mathbf{k}'}\right|^{2} = U_{\mathbf{q}ei}^{2} \left|\frac{1}{N_{i}}\sum_{i} e^{-i\mathbf{q}\mathbf{R}_{i}}\right|^{2} = U_{\mathbf{q}ei}^{2}S_{i}\left(q\right),\tag{26}$$

where $S_i(q)$ is the static structure factor (SSF)

$$S_{i}(q) = \left| \frac{1}{N_{i}} \sum_{i} e^{-i\mathbf{q}\mathbf{R}_{i}} \right|^{2} = 1 + \frac{N_{i}}{V} \int \left[g\left(\mathbf{r}\right) - 1 \right] \frac{\sin\left(qr\right)}{qr} 4\pi r^{2} \mathrm{d}r.$$
(27)

In the case of the Debye potential (7) we get

$$U_{\mathbf{q}ei} = \frac{4\pi e^2 z_i}{q^2 + k_s^2},$$

so as a consequence

$$\left|U_{\mathbf{k},\mathbf{k}'}\right|^2 = \frac{16\pi^2 e^4 z_i^2}{\left(q^2 + k_s^2\right)^2} S_i\left(q\right).$$
⁽²⁸⁾

By substituting of (28) to (23), we get

$$\sigma_{\rm tr} = \frac{4\pi m^2 e^4 z_i^2}{\hbar^4 k^4} \int_0^{2\kappa} \frac{S_i(y)}{\left(y^2 + k_s^2\right)^2} y^3 \mathrm{d}y.$$
(29)

Let us consider the case when the ion correlations can be neglected: $S_i = 1$. In this case from (29) we obtain (see [13, 14])

$$\sigma_{\rm tr} = \frac{2\pi m^2 e^4 z_i^2}{\hbar^4 k^4} \left[\ln\left(1 + \frac{4k^2}{k_s^2}\right) - \frac{4k^2}{4k^2 + k_s^2} \right]. \tag{30}$$

From equation (30) we see that the transport cross section in the Born approximation has no singularities, which is well known. But, as noted above, the Born approximation has limited applicability, so equation (30) needs to be clarified.

In works [15–19], Ziman introduced and actively used the concept of the "atom" as an ion with a shielding electron cloud to describe electron transport properties of metals. In this case, the electron scattering occurs on such an "atom" with a pseudopotential equal to the screened Debye one, and the interaction of "atoms" with each other is described by the same potential. In [15–19], it was assumed that only electrons participate in the screening, the randomness or ordering of the arrangement of "atoms" or simply ions is taken into account by the static structure factor. This theory was very successful in describing the transport properties of electrons in various metals and their alloys, so in this paper we use the same approach.

The electron screening in a plasma is generally determined by the expression [12, 14]:

$$k_{De}^{2} = \frac{8\pi e^{2}}{T} \frac{1}{\lambda_{e}^{3}} \mathcal{F}_{-1/2}(\eta_{e}), \qquad (31)$$

where T is the electron temperature, $\eta_e = \mu_e/T$, μ_e is the chemical potential of the electron gas, $\mathcal{F}_{-1/2}(\eta_e)$ is the Fermi–Dirac integral defined by the relation [20]:

$$\mathcal{F}_k(\eta_e) = \frac{1}{\Gamma(k+1)} \int_0^\infty \frac{x^k \mathrm{d}x}{e^{x-\eta_e} + 1},\tag{32}$$

 $\Gamma(x)$ is the gamma function. The number density of electrons is

$$n_e = \frac{2}{\lambda_e^3} \mathcal{F}_{1/2}\left(\eta_e\right). \tag{33}$$

By combining (31) and (33), we get

$$k_{De}^{2} = k_{De,0}^{2} \frac{\mathcal{F}_{-1/2}(\eta_{e})}{\mathcal{F}_{1/2}(\eta_{e})},\tag{34}$$

where $k_{De,0}$ is an electronic screening constant in the nondegenerate case:

$$k_{De,0}^2 = \frac{4\pi e^2 n_e}{T}.$$
(35)

It is important to highlight that at room temperature and $n_e \leq 10^{18} \text{ cm}^{-3}$ the electron screening constant is practically the same as (35) (figure 3), but in the strongly degenerate case

$$k_{De}^2 = \frac{m_e e^2}{\pi^2 \hbar^2} \left(\frac{3n_e}{\pi}\right)^{1/3}.$$
(36)

In calculating, the reduced chemical potential η_e was found from (33) using the given electron number density n_e , then from (31) or (34) the screening constant k_{De} was determined. The Fermi-Dirac integrals were calculated according to [20]. Here we also give an expression for the mean kinetic energy of electrons

$$\langle \varepsilon_e \rangle = \frac{3}{2} T \frac{\mathcal{F}_{3/2}(\eta_e)}{\mathcal{F}_{1/2}(\eta_e)}.$$
(37)

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In the nondegenerate plasma $\langle \varepsilon_e \rangle = \frac{3}{2}T$, and in the strongly degenerate plasma $\langle \varepsilon_e \rangle = \frac{3}{5}\varepsilon_F$, where ε_F is the Fermi energy:

$$\varepsilon_F = \left(3\pi^2 n_e\right)^{2/3} \frac{\hbar^2}{2m_e}.\tag{38}$$

The transport cross section for electron scattering by ions with allowance for the ion–ion correlation is defined by the expression [21]

$$Q_{\rm ei}\left(\varepsilon_e\right) = \frac{\pi z_i^2 e^4}{\varepsilon_e^2} \Lambda_{\rm ei},\tag{39}$$

where ε_e is the electron energy, Λ_{ei} is the Coulomb logarithm

$$\Lambda_{ei} = \int_{0}^{q_m} \frac{k^3}{(k^2 + k_s^2)^2} S_i(k) \,\mathrm{d}k,\tag{40}$$

 q_m is the maximum value of the wave vector (determined by the maximum value of the electron momentum transferred to the ion in the collision). In [15], it was assumed that $q_m = 2k_F$ (for an electron on the Fermi surface, the maximum value of the momentum transfer is $2\hbar k_F = 2\sqrt{2m_e\varepsilon_F}$), where k_F is the wave number of an electron on the Fermi surface:

$$k_F = \left(3\pi^2 n_e\right)^{1/3}.$$
(41)

In [3] it was assumed that

$$q_m = \min\left\{E_T/z_i e^2, 2/\lambda_e\right\},\tag{42}$$

where it was supposed that E_T was equal to the temperature (in energy units) in the nondegenerate case and to the Fermi energy in the degenerate case; the de Broglie wavelength in these cases was also determined using the Fermi energy or temperature.

In a rarefied plasma, when the ion–ion correlations can be neglected, $S_i \approx 1$. In this case, we find from (40) (see [13,14])

$$\Lambda_{ei} = \frac{1}{2} \left[\ln \left(1 + \chi_i \right) - \frac{\chi_i}{1 + \chi_i} \right],\tag{43}$$

where $\chi_i = (q_m/k_s)^2$.

In this paper, in order to determine the structure factor of ions, the OZ equation in the HNC approximation [22–24] is solved numerically:

$$h(\mathbf{r}) = C(\mathbf{r}) + n_i \int h(\mathbf{r}_1) C(|\mathbf{r} - \mathbf{r}_1|) \,\mathrm{d}\mathbf{r}_1,$$
(44)

where g(r) = 1 + h(r) is the pair correlation function, C(r) is the direct correlation function, n_i is the ion number density. In order to close the OZ equation, we use the HNC approximation [25]:

$$C(\mathbf{r}) = \exp\left[-\frac{U_{\mathrm{ii}}(\mathbf{r})}{T} + \gamma(\mathbf{r})\right] - \gamma(\mathbf{r}) - 1, \qquad (45)$$

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where $U_{ii}(r)$ is the interaction potential of the particles under consideration, $\gamma(r) = h(r) - C(r)$. In our case, the Debye potential (7) is used to describe the ion interaction. The HNC approximation turns out to be sufficiently accurate to describe the Coulomb systems and a system of particle interacting by the Debye potential (see, for example, [26]).

The OZ equation is solved by the iterative method [27,28], and the values of the SSF in the Debye approximation are used as the initial solution. We used the procedure proposed in [29] to speed up the convergence.

4. Numerical calculations and discussion of the results

In this paper, we consider two variants for determining the Debye screening constant taking into account,

$$k_s = k_D \equiv \sqrt{k_{\rm De}^2 + k_{\rm Di}^2},$$

and without taking into account the contribution of the ionic component, $k_s = k_{\text{De}}$. The value of the boundary wave number q_m is specified either as in [3]

$$q_m = q_{\rm mS} \equiv \min\left(r_0^{-1}\sqrt{1 + (\varepsilon_F/T)^2}, 2k_E\right),\tag{46}$$

or as the inverse Landau radius in a nondegenerate plasma with a transition to $2k_F$ in a degenerate plasma (as in [15]):

$$q_m = q_{\rm mZ} \equiv \sqrt{r_0^{-2} + 4k_F^2}.$$
(47)

Here k_E is the wave number determined from the mean kinetic energy of electrons (37)

$$k_E = \left(\frac{10}{3} \frac{\langle \varepsilon_e \rangle m_e}{\hbar^2}\right)^{1/2},\tag{48}$$

which in a strongly degenerate case becomes the wave number of an electron with the Fermi energy. As a result, in this paper, calculations have been made for four variants of the choice of the maximum value of the wave number and the choice of the screening constant: for variant 1 we set $k_s = k_{De}$ and $q_m = q_{mZ}$; for 2— $k_s = k_D$ and $q_m = q_{mZ}$; for 3— $k_s = k_{De}$ and $q_m = q_{mS}$; for 4— $k_s = k_D$, $q_m = q_{mS}$.

Consideration of the different variants for choosing the constant screening and the boundary value of the wave number is due to the following fact. As noted above, in a number of papers the electron screening constant is used as a screening constant. Such a choice is suitable for metals in which the ions are located at the sites of the crystal lattice, but in the case of a plasma with not too high values of the nonideality parameter (figure 4, which shows an increase of the order of the ion distribution in the growth of Γ), the ionic component also takes part in the screening of the electric field of the ions. Also, the choice of the boundary value of the wave number equal to $2k_F$ for metals is clear, in which the transfer is carried out mainly by electrons with the energy near the Fermi surface, whereas this choice is open to question in the case of plasma.

It is seen from figure 3 that the value $q_{\rm mS}$, coinciding at small electron number densities with the classical inverse Landau radius, at the number density of the order 10^{19} cm³ tends to a value equal to the inverse Landau radius determined with the Fermi energy as the temperature, and then tends to $2k_F$ for the number density of the order of 10^{25} cm⁻³. We note that the behavior of $q_{\rm mS}$ is very close to the behavior of the inverse Landau radius calculated using the mean kinetic energy $\langle \varepsilon_e \rangle$ instead of the temperature (see curve 7 in figure 3). The value of $q_{\rm mZ}$ coincides with r_0^{-1} for small number densities, and tends to $2k_F$ as the number density increases.



Figure 3. The dependencies on the electron number density of the main plasma parameters with the inverse length dimension, which define the boundary wave number q_m at T = 300 K and $z_i = 1$: the inverse Landau length $r_0^{-1} = T/(e^2 z_i)$ (1), the inverse Landau length with the Fermi energy ε_F instead of the temperature: $r_0^{-1} = \varepsilon_F/(e^2 z_i)$ (2), $q_m = 2/\lambda_e$ (3), $q_m = 2k_F$ (4), k_{De} (5), k_{Di} (6), the inverse Landau length with the mean electron energy $\langle \varepsilon_e \rangle$: $r_0^{-1} = \langle \varepsilon_e \rangle/(e^2 z_i)$ (7), $q_m = q_{\text{mS}}$ (46) (red solid line with magenta triangles), and $q_m = q_{\text{mZ}}$ (47) (black solid line with cyan circles).

It is also seen from figure 3 that at room temperature the degeneracy begins to affect the electron Debye radius at $n_e > 10^{18} \text{ cm}^{-3}$, and the electron screening constant becomes noticeably smaller than the ion one.

Figure 4 shows the dependencies of the two-particle correlation function and the ion-ion static structure factor with the ionization multiplicity $z_i = 1$ for T = 300 K on r/a and ka for different values of the nonideality parameter Γ , which is defined by the expression

$$\Gamma = \frac{e^2 z_i^2}{aT}, \qquad a = \left(\frac{3}{4\pi n_i}\right)^{1/3}.$$
 (49)

The Coulomb logarithm (40) is calculated by using similar dependencies of the static structure factor. Usually, the inequality $k_{\text{max}} > q_m$ is fulfilled, but if it is violated, then $S_i = 1$ is assumed in the range $k_{\text{max}} < k \leq q_m$, where k_{max} is the maximum value of the wave number in the calculations of the static structure factor.

Figure 5 shows the dependencies of the Coulomb logarithm on the electron number density for vatiant 1. We see that Λ_{ei} decreases with increasing n_e and passes through the minimum at $n_e \sim 10^{19}$ cm⁻³. Curve 2 calculated for $S_i = 1$ behaves similarly, so we can conclude that the appearance of a minimum is not connected with the variations of SSF with a change in the ion number density. From the comparison of curves 1 and 2 we can conclude that taking into account the ion correlation contributes to the Coulomb logarithm already at $n_e \sim 10^{12}$ cm⁻³, but this contribution becomes especially noticeable at $n_e > 10^{20}$ cm⁻³, when the allowance for the correlation of the ions is necessary and leads to a noticeable decrease in the electron scattering cross section.



Figure 4. The radial distribution functions (a) and the static structure factors (b) at T = 300 K and $z_i = 1$ for different values of the nonideality parameter: $\Gamma = 4.2 \times 10^{-3}$ (1), 0.416 (2), 89.7 (3), 193 (4), 416 (5), and 897 (6).

In figure 6 numerical calculation results by four above mentioned variants are compared. It can be seen that in the calculations within the variant $q_m = q_{mS}$ the values of the Coulomb logarithm are significantly less than in calculations within the variant $q_m = q_{mZ}$. The values of Λ_{ei} also turn out to be smaller in calculations within the variant $k_s = k_D$ than within the variant $k_s = k_{De}$. As can be seen in figure 6, allowance for the ion correlation is important only in calculations without allowance for the screening by ions. A similar picture is observed in the calculations for T = 300, 1500 and 3000 K.



Figure 5. The Coulomb logarithm as a function of the electron number density for $z_i = 1$ in variant 1 with $q_m = q_{mZ}$ and $k_s = k_{De}$ at T = 300 (1), 1000 (2) and 3000 K (3). The solid curves are obtained by numerical integration of (40) with the SSF from the HNC OZ equation, dash curves are calculated from equation (43).



Figure 6. The Coulomb logarithm as a function of n_e for $z_i = 1$ and T = 5000 K for variant 1 with $q_m = q_{mZ}$ and $k_s = k_{De}$ (1), variant 2 with $q_m = q_{mZ}$ and $k_s = k_D$ (2), variant 3 with $q_m = q_{mS}$ and $k_s = k_{De}$ (3), and variant 4 with $q_m = q_{mS}$ and $k_s = k_D$ (4). The solid curves are from (40) with the SSF from the HNC OZ equation, dash curves are from equation (43).

5. Conclusion

In this paper, the different methods of calculation of the differential cross section of electrons scattering by ions have been considered, that is the method of phase functions, the Born and

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semiclassical approximations, as well as various methods for determining the Coulomb logarithm in the kinetic theory of electron transport, the different choice of constant plasma screening with or without the contribution of ionic components and the different choice of boundary values of the wave vector of electrons. The Ornstein–Zernike integral equation in the hypernetted chain approximation has been solved numerically to take into account the ion–ion correlations. It has been shown that there are values of the Coulomb logarithm much less than one that requires a more careful study of the definition of the transport section of electron scattering in strongly coupled plasmas.

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