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A simple description of turbulent transport in a stratified shear flow devoted to the simulation of thermohydrodinamics of inland waters

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Abstract. The paper presents three major models used to describe the thermohydrodynamics of inland water objects: the standard one-dimensional E- ε (k- ε) model, the EFB-model by S. S. Zilitinkevich, and RANS-type model of turbulent transfer by L. O. Ostrovsky and Yu. I. Troitskaya. For models that take into account the two-sided transformation of the kinetic and potential energies of turbulent pulsations, the dependences of the turbulent Prandtl number on the gradient Richardson number are provided. The obtained dependences were used in the E-E model to parameterize the coefficient of heat transfer eddy diffusivity in order to take into account stratification when calculating the thermohydrodynamic regimes of inland waters. The results of verification of the modified E- ε model based on experimental data are presented.

1. Introduction

By now, the role of inland water bodies in the processes of general circulation of the atmosphere, the ocean and land, their thermohydrodynamic and biological properties in weather forecasts, and also in climate change has been established [1-6]. The basis of physico-mathematical simulation of thermodynamics and hydrodynamics of inland waters is RANS (Reynolds-averaged Navier-Stokes equations) [7]. Since this system contains unknown quantities (turbulent flows), it is necessary to involve additional hypotheses for its closure. Practically significant is the so-called E- ε (k- ε) model based on the equations for the kinetic energy of turbulence E and its dissipation rate ε . The ratio of the coefficient of momentum eddy diffusivity to the coefficient of heat transfer eddy diffusivity (the turbulent Prandtl number, PrT) is assumed to be constant. This, in particular, limits the description of the known effect [8-9] associated with the existence of turbulence for large gradient Richardson numbers Ri. At the same time, both laboratory and field measurements demonstrate the dependence of PrT on Ri. In this connection, special attention in the simulation of the thermohydrodynamic regime of inland water bodies is paid to stratification and, in particular, to its role in the processes of turbulent mixing, thermocline dynamics, etc. Recently, modernized approaches to the description of geophysical

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turbulence are actively developing, taking into account stable and unstable stratification, as well as internal waves interacting with turbulence and even amplifying it.

Authors propose geophysical turbulence models that take into account the two-sided transformation of the kinetic and potential energies of turbulent pulsations. This is the so-called EFB model [10] and the closed Reynolds system of equations (RANS) [11] obtained on the basis of the sequential kinetic approach. These models allowed, in particular, to explain the case of maintaining turbulence observed in laboratory and field conditions by weak velocity shears, as well as to calculate the dependence of PrT on Ri. In the present work, a parameterization of the coefficient of heat transfer eddy diffusivity for the E- ε model is proposed in order to take into account stratification when calculating the thermohydrodynamic regimes of inland waters. Verification of the proposed parameterization is presented.

2. E- ϵ (k- ϵ) model for the description of thermohydrodynamics of inland waters

In this paper, the model is used in the form proposed in [12]:

$$\frac{\partial E}{\partial t} = \frac{\alpha_E}{h^2} \frac{\partial}{\partial \xi} k \frac{\partial E}{\partial \xi} + \frac{\xi}{h} \frac{dh}{dt} \frac{\partial E}{\partial \xi} + P - \varepsilon, \qquad (2.1)$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{\alpha_{\varepsilon}}{h^2} \frac{\partial}{\partial \xi} k \frac{\partial \varepsilon}{\partial \xi} + \frac{\xi}{h} \frac{dh}{dt} \frac{\partial \varepsilon}{\partial \xi} + C_1 \frac{\varepsilon}{E} (P - \varepsilon),$$
(2.2)

where a_E and a_c are dimensionless constants, P is total generation of turbulence energy through the shear of velocity and effect of density stratification, C_1 is the function of Reynolds number. The beginning of the vertical downward z-coordinate is supposed to be coupled with water surface z=h(t), so we can turn from z to the new independent variable $\xi = \frac{z}{h}$, where h=h(t) is a depth of water body, and t is a time.

The coefficient of momentum eddy diffusivity is as follows:

$$k = C_e \frac{E^2}{\varepsilon},$$

where C_e is dimensionless constant.

Vertical heat exchange, expressed as the coefficient of heat transfer eddy diffusivity λ , is defined as:

(2.3)

 $\lambda = c\rho k.$

It is used in the calculations of the temperature distribution (see below) and the parameter P in equations (2.1-2.2)

The numerical model for calculating the temperature distribution is based on solving a onedimensional heat equation:

$$c\rho\frac{\partial T}{\partial t} = \frac{1}{h^2}\frac{\partial}{\partial\xi}\left(\lambda\frac{\partial T}{\partial\xi}\right) + c\rho\frac{dh}{dt}\frac{\xi}{h}\frac{\partial T}{\partial\xi} - c\rho\frac{1}{h}B_w\frac{\partial T}{\partial\xi} - \frac{1}{h}\frac{\partial S}{\partial\xi},$$

where c is heat capacity of the water, ρ – its density, λ – the coefficient of heat transfer eddy diffusivity, T – temperature, $B_w = dh_0/dt$ – water balance at free surface, S – solar radiation flux penetrating the water body.

As for the water object dynamics, in [12] the zonal u and the meridional v components of the flow velocity used to compute the parameter P and the gradient Richardson number Ri in our study, are found using the following equations:

$$\frac{\partial u}{\partial t} = \frac{1}{h^2} \frac{\partial}{\partial \xi} k \frac{\partial u}{\partial \xi} + \frac{\xi}{h} \frac{dh}{dt} \frac{\partial u}{\partial \xi} - fv,$$
$$\frac{\partial v}{\partial t} = \frac{1}{h^2} \frac{\partial}{\partial \xi} k \frac{\partial v}{\partial \xi} + \frac{\xi}{h} \frac{dh}{dt} \frac{\partial v}{\partial \xi} + fu,$$

where f is the Coriolis frequency. Components of friction stress are used to calculate the boundary conditions for u and v according to the Monin-Obukhov similarity theory at the upper boundary (water – atmosphere) and according to the Chezi formulas [13] taking into account the surface roughness at the lower boundary (water – soil).

3. EFB-model and parameterization of turbulent Prandtl number

This model, presented in [9-10], is based on the balance equations for the kinetic energy of turbulence E_K , potential energy E_P , turbulent fluxes of momentum τ_{i3} and potential temperature F_z , as well as on the relaxation equation for the turbulent time scale t_T :

$$\frac{DE_K}{Dt} - \frac{\partial}{\partial z} K_E \frac{\partial E_K}{\partial z} = -\tau_{i3} \frac{\partial U_i}{\partial z} + \beta F_z - \frac{E_K}{t_T},$$
(3.1)

$$\frac{DE_P}{Dt} - \frac{\partial}{\partial z} K_E \frac{\partial E_P}{\partial z} = -\beta F_z - \frac{E_P}{C_P t_T},$$

$$D\tau_{i2} = \partial \frac{\partial}{\partial z} \tau_{i2} = -\beta F_z - \frac{\partial}{\partial L_i} \tau_{i2}$$
(3.2)

$$\frac{D\tau_{i3}}{Dt} - \frac{\partial}{\partial z} K_{FM} \frac{\partial\tau_{i3}}{\partial z} = -2E_z \frac{\partial U_i}{\partial z} - \frac{\tau_{i3}}{C_\tau t_T} \quad (i = 1, 2),$$
(3.3)

$$\frac{DF_z}{Dt} - \frac{\partial}{\partial z} K_{FH} \frac{\partial F_z}{\partial z} = -2(E_z - 2C_\theta E_P) \frac{\partial \Theta}{\partial z} - \frac{F_z}{C_F t_T},$$
(3.4)

$$\frac{Dt_T}{Dt} - \frac{\partial}{\partial z} K_T \frac{\partial t_T}{\partial z} = -C_R \left(\frac{t_T}{t_{TE}} - 1\right).$$
(3.5)

Here D/Dt is the substantial derivative, K_{E} , K_{FM} , K_{FH} , K_{T} are the coefficients of turbulent transport, β is the buoyancy parameter, E_z is the vertical component of the kinetic energy, C_P , $C\tau$, C_{θ} , C_F , C_R are constants, Θ is the average potential temperature, t_{TE} is the equilibrium relaxation time.

It should be noted that this model is rather difficult for numerical simulation, but it allows to calculate various characteristics of turbulence in the conditions from neutral stratification to extremely stable one. The model of gradient transfer under the condition of equilibrium versions of balance equations for turbulent fluxes of momentum and potential temperature follows from the system (3.1-3.5). In this case:

$$F_z = -K_H \frac{\partial \theta}{\partial z}, \qquad \tau_{i3} = -K_M \frac{\partial U_i}{\partial z}, \tag{3.6}$$

$$K_H = 2C_F E_z t_T \left(1 - C_\theta \frac{E_P}{E_z} \right), \qquad K_M = 2C_\tau E_z t_T,$$
(3.7)

 K_H is the coefficient of heat transfer eddy diffusivity (is equivalent to λ), K_M is the coefficient of momentum eddy diffusivity (is equivalent to k), $E_z = A_z E_K$, $A_z = A_z(\Pi)$, $\Pi = \frac{E_P}{E_K}$.

Let us consider the following simple task of the evolution of homogeneous turbulence in a stationary shear flow with a given velocity profile $\vec{u} = U_0(z)\vec{x}$, where $\partial U_0/\partial z = const$. Shear flow is stably stratified and with a constant Brunt–Väisälä frequency N: $N^2 = \beta \partial \Theta/\partial z$. The turbulent regime of such a fluid will be described by a system of equations of the first order in time:

$$\frac{DE_K}{Dt} = -\tau_{13} \frac{\partial U_1}{\partial z} + \beta F_z - \frac{E_K}{t_T},$$

$$\frac{DE_P}{Dt} = -\beta F_z - \frac{E_P}{C_P t_T},$$
(3.8)

The system has two equilibrium states: an unstable one $E_P = E_K = 0$ and stable one with a non-zero value of the energy parameter of the stratification $\Pi = E_P/E_K$, defined by the relation:

$$\Pi(Ri) = \frac{C_P \frac{C_F}{C_r} \left(1 - C_{\theta} \frac{\Pi(Ri)}{A_z(\Pi)}\right) Ri}{1 - \frac{C_F}{C_r} \left(1 - C_{\theta} \frac{\Pi(Ri)}{A_z(\Pi)}\right) Ri}, A_z(\Pi) = \frac{\Pi_{\infty} \left(C_r - 3\frac{\Pi}{C_P}\right) (C_P + \Pi) - 2C_r C_0 (C_P + \Pi_{\infty})\Pi}{3\Pi_{\infty} (1 + C_r) (C_P + \Pi) - 2C_r (1 + C_0) (C_P + \Pi_{\infty})\Pi'}$$
(3.9)

where Π_{∞} is the limiting value of the parameter equal to 0.287. So, from the definition of the turbulent Prandtl number and (3.7–3.9) we get:

$$PrT(Ri) = \frac{C_P Ri \left(1 + \frac{\Pi(Ri)}{C_p}\right)}{\Pi(Ri)}.$$
(3.10)

In Fig. 1 (see below) the dependence of the turbulent Prandtl number on the gradient Richardson number is presented. You can see that PrT(Ri) is an unboundedly increasing function, and for large $Ri \gg 1$, the Prandtl number grows linearly.

4. Turbulent transfer model by L. Ostrovsky and Yu. Troitskaya

On the basis of the kinetic approach, the expressions for turbulent fluxes and average values of the corresponding hydrodynamic quantities were obtained in [11]. This allowed us to close the RANS equations:

$$\frac{\partial \langle u_i \rangle}{\partial t} + \langle u_j \rangle \frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{1}{\rho_0} \frac{\partial \langle \rho \rangle}{\partial x_i} + g_i \frac{\langle \rho \rangle - \rho_0}{\rho_0} = \frac{\partial}{\partial x_j} \left(L \sqrt{b} \left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) \right), \tag{4.1}$$

$$\frac{\partial \langle \rho \rangle}{\partial t} + \langle u_i \rangle \frac{\partial \langle \rho \rangle}{\partial x_i} = 2 \frac{\partial}{\partial x_i} L \sqrt{b} \left(\frac{\partial \langle \rho \rangle}{\partial x_i} + \frac{3}{2b\rho_0} (g_i \langle \rho'^2 \rangle + g\beta_i) \right), \tag{4.2}$$

$$\frac{\partial b}{\partial t} + \langle u_i \rangle \frac{\partial b}{\partial x_i} - L\sqrt{b} \left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right)^2 - \frac{g}{\rho_0} L\sqrt{b} \times \left(\frac{\partial \langle \rho \rangle}{\partial z} + \frac{3}{2b\rho_0} (g\langle \rho'^2 \rangle + g\beta_z) \right) -$$
(4.3)

$$-\frac{C B^{\prime/2}}{L} = \frac{5}{3} \frac{\partial}{\partial x_i} \left(L \sqrt{b} \frac{\partial B}{\partial x_i} \right),$$

$$\frac{\partial \langle \rho^{\prime 2} \rangle}{\partial t} + \langle u_i \rangle \frac{\partial \langle \rho^{\prime 2} \rangle}{\partial x_i} - 2 \frac{\partial \langle \rho \rangle}{\partial x_i} L \sqrt{b} \times \left(\frac{\partial \langle \rho \rangle}{\partial x_i} + (g_i \langle \rho^{\prime 2} \rangle - g \beta_i) \frac{3}{2b\rho_0} \right) = \frac{\partial}{\partial x_i} L \sqrt{b} \frac{\partial \langle \rho^{\prime 2} \rangle}{\partial x_i}, \quad (4.4)$$

The equation (4.3) is equivalent to (3.1), (4.4) – to (3.2), the kinetic energy of turbulence is denoted by the symbol b, and the potential energy is considered through density fluctuations.

In the framework of this approach, it is possible to calculate, in particular, the dependence of the turbulent Prandtl number on the gradient Richardson number:

$$PrT(Ri)^{-1} = \frac{(4-3R) + \frac{1}{Ri} - \left(\left((4-3R) + \frac{1}{Ri}\right)^2 - \frac{4}{Ri}\right)^{1/2}}{2},$$
(4.5)

where *R* is a parameter characterizing the anisotropy of small-scale turbulence and depending on the ratio of the vertical and horizontal scales of correlation field of density fluctuations. Values of *R* equal to 0.3 and 0.5 were considered, PrT(Ri) is presented in Fig. 1.



Figure 1. The dependence of the turbulent Prandtl number *PrT* on the gradient Richardson number *Ri* in different models.

5. Results of calculation

In the previous work of the group of authors [14] fairly accurate results for calculating the thermodynamics of inland water bodies were obtained with the standard E- ε model. They were used to verify the results of the current study. The verification was based on experimental data obtained at the Gorky reservoir experimental site in the period 2014-2017.

In the present work, the parameterization of the coefficient of heat transfer eddy diffusivity for the E- ε model is proposed in order to take into account stratification when calculating the thermohydrodynamic regimes of inland water bodies. Formula (2.3) turns into the following form:

$$\lambda = c\rho k \cdot PrT^{-1},\tag{5.1}$$

where (3.10) and (4.5) are used as the turbulent Prandtl number.

Within the framework of verification, the temperature distribution profiles obtained as a result of simulation using the standard model and the ones with parameterizations were compared. Several cases were considered, two examples are presented in the Fig. 2.



Figure 2. Comparison of the temperature distribution profiles obtained as an experimental data and as a result of simulation with the standard E- ε model and the models with parameterizations.

It is demonstrated that the use of parameterizations made it possible to obtain no less accurate results, but, as the analysis of the standard deviation (see Table 1) showed, even more accurate ones.

Table 3. Standard deviation (in $^{\circ}$ C) obtained with the standard E- ϵ model and with the parameterizations

	Standard	EFB	Ostrovsky & Troitskaya, R=0.3
Case 1 (Fig. 1a)	0.23	0.19	0.20
Case 2 (Fig. 1b)	0.63	0.60	0.60

6. Conclusion

In this paper, we consider three main models of turbulence, which are used to describe the thermohydrodynamics of both inland waters, and the ocean and atmosphere. For models that take into account the two-sided transformation of the kinetic and potential energies of turbulent pulsations, the

dependences of the turbulent Prandtl number on the gradient Richardson number are provided. The obtained dependences were used in the E- ε model to parameterize the coefficient of heat transfer eddy diffusivity in order to take into account stratification when calculating the thermohydrodynamic regimes of inland waters. Based on the previously obtained results [14], it was found that the use of parameterizations allows reducing the standard deviation from the real values

Due to the fact that such parameterization allows to correctly describe the processes of turbulence in inland waters, in the future they are planned to be used for the conditions of the ocean and atmosphere, including for large values of the gradient Richardson number.

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