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Research on Characteristics of Morphological Filters

Fang Xie[1,2,3,4,5], Kangkang Liang[1,2,3,4,5,*] ,Wenming Wu[1,2,3,4,5] ,Wenjie Hong[1,2,3,4,5] and Wei Hu[6]

[1]School of Electrical Engineering and Automation, Anhui University, Hefei, China
[2]National Engineering Laboratory of Energy-Saving Motor & Control Technology, Anhui University, Hefei, China
[3]Engineering Research Center of Power Quality, Ministry of Education, Anhui University, Hefei, China
[4]Anhui Province Laboratory of Industrial Energy-Saving and Safety, Anhui University, Hefei, China
[5]Collaborative Innovation Center of Industrial Energy-Saving and Power Quality Control, Anhui University, Hefei, China
[6]Hefei Hengda Jianghai Pump Co., Ltd

*Corresponding author e-mail: kangkang9986@163.com

Abstract. As a kind of non-linear filter, morphology filter is widely applied in sampled power system single processing and motor control with its effective denoising performance. If the structural elements are selected properly, the morphological filter can do better in maintaining the original characteristics of the signal. However, in practical applications, it is a major difficulty to grasp the factors that affect the cut-off frequency of morphological filters. This paper will solve these problems. In this paper, the frequency response characteristic of the combined morphological filter is presented by excitation signal inputted into the given morphological filter. The research indicates when the sampling frequency is fixed, the cut-off frequency increases with the increase of the length of the structural elements. The cut-off frequency gets smaller as the width of structure element grows, and the relationship between them is nonlinear. When the width of the structure element is fixed, as the sampling frequency changed, the cut-off frequency varies directly with the sampling frequency. Experiments show the corresponding rule of morphological filter frequency.

1. Introduction

Mathematical Morphology (MM) [1,2] was first proposed in Doctor J.serra’s paper, and it was applied in many field in the past, such as signal processing, image recognition and image processing[3]. Nowadays, MM has been gradually applied to other fields. Ander Landstrom used MM to deal with incomplete data [4]. Pascal Dieu Seul ASSALA used it to detect overvoltage in power system [5], and Zhao Jing used it to detect power quality disturbances [6]. Moreover, the application of MM in other fields is becoming more and more mature.

In power system, the morphological filter can be applied in denosing, because it has the advantage of fast calculation speed, easy to implement with hardware and good real-time performance. After years of development, mathematical morphology has made great achievements in theoretical research and application, and has become a theory with complete mathematical foundation, which contains very rich contents. With the in-depth of research and the interdisciplinary integration with related disciplines, mathematical morphology will become more and more perfect. However, how to use morphological filters has still many problems.

It is a key to select the appropriate structure elements in thinning algorithm which based on the MM. Paper [7,8,9] offer different selection principle of structural elements, and paper [7,8] points out the importance of structural elements. The choice of structural elements and the width of structural elements both have a great influence on the response characteristics of MM filters. In order to convenience for study, the linear structure element is only chosen in this paper. Meanwhile, the width of structural elements can determine the cut-off frequency of morphological filter.

The other key for morphological filters is to select the appropriate sampling frequency. When the type of structural element and the width of structural are fixed, the cut-off frequency of the morphological filter will also change as the sampling frequency of the signal changed. This paper uses the linear structure element, the characteristics of the reaction were analyzed by the curves at different sampling frequencies.

This paper presents morphological filter with different structure element's width and different sampling frequency. Section 1 introduces the implication for the research. Section 2 describes the basic theories about MM. Section 3, a signal model is proposed, and the experiment results prove the correction of the proposed method. Finally, the conclusion is summarized in section 4.

2. BASIC THEORIES OF MM

MM is an effective tool in many fields. Erosion and dilation are the basic operations of MM. Based on these two operations other operations are called as open operator and close operator.

Set the original signal $z(n)$ as discrete function distributed on the Z=(0,1,2,⋯N-1), and "$x(n)$" is structure element distributed on the X=(0,1,2,⋯,M-1), N≧M.

Dilation is defined as,

$$(z \oplus x)(n) = \max\{z(n-m) + x(m)\} \quad (1)$$
$$m \in 0,1,\cdots M-1$$

Erosion is defined as,

$$(z \Theta x)(n) = \min\{z(n+m) - x(m)\} \quad (2)$$
$$m \in 0,1,\cdots M-1$$

Where $\oplus$ and Θ denote the dilation and erosion operators.

Two basic operations can make up many complex operations, the most used are open operator and close operator.

Open operator is defined as,

$$(z \circ x)(n) = (z \Theta x \oplus x)(n) \quad (3)$$

Close operator is defined as,

$$(z \bullet x)(n) = (z \oplus x \Theta x)(n) \quad (4)$$

Where $\circ$ and $\bullet$ denoted the open and close operator.

The open operator can inhibit the positive pulse signal, while the close operator can inhibit the negative pulse signal. Open operator, close operator and their free combination can all form different morphological filters. In order to eliminate both positive and negative pulses in the signal at the same time, open operator and close operator are used simultaneously. Set $z(n)$ as a one-dimension input signal, and both of two structure elements are "x", the generalized close-open and open-close filter are defined as follow.

Generalized open-close filter is defined as,

$$G_{oc}(z(n)) = (z \circ x \bullet x)(n) \quad (5)$$

Generalized close-open filter is defined as,

$$G_{co}(z(n)) = (z \bullet x \circ x)(n) \quad (6)$$

The contractibility of open-close filter makes the output smaller, and the expansibility of close-open filter makes the output bigger. Therefore there will be bias. The Equation (7) can be used to avoid it.

$$y(n) = \frac{1}{2}[G_{co}(z(n)) + G_{oc}(z(n))] \tag{7}$$

We use MM filter as pretreatment tools, it has much advantage on the choice of structure element to cover the insufficient of normal MM filter.

3. ANALYSIS AND SIMULATION

The mathematical morphology filter, as a high performance and nonlinear time-domain low pass filter, is the precondition and theoretical basis of its application, which can accurately determine its frequency response characteristics.

3.1. Research on the low pass filtering of MM filer

This paper reference analysis method of nonlinear filter frequency response characteristics [10, 11] and method of median-mean, and input different frequency signals to determined MM filter, analysis the changes of the response, get the MM filter response.

Set the number of analysis points to N, the Equation of single frequency sinusoidal signal is shown by (8).

$$y = \sin(2\pi f \frac{n}{f_s} + 2\pi f \frac{j}{10}) \tag{8}$$
$$n = 0,1\ldots, N-1$$
$$j = 0,1\ldots,9$$

Where f_s is the sampling frequency, f is the frequency of sinusoidal signal.

Set the sine signal defined by Equation (8) is used as the excitation of Equation (7) morphological combined filter, and the range of frequency f is calculated by (9).

$$f = \frac{f_s}{N}, \frac{2f_s}{N}, \cdots, (\frac{N}{2}-1)\frac{f_s}{N} \tag{9}$$

The frequency range of input single frequency sinusoidal signal is $0 < f < f_s / 2$, and $2^{\pi \frac{f}{f_s} \frac{j}{N}}$ is the initial phase of a sinusoidal signal.

To avoid the influence of initial phase change, the methods of phase averaging and median-mean method based on the characteristics of nonlinear median filter is referenced.

One way is phase averaging. The initial phase of sine signal changes 10 times at each frequency f and the amplitudes of the filter response are obtained at each phase. The average value of the ten amplitudes is taken as the result of the filter's response. With the change of signal frequency, a series of mean values will be obtained. Set these values be ordinate and frequency value be abscissa, the amplitude frequency characteristic can be obtained. That's the method of phase averaging.

Another way is median-mean. The initial phase of sine signal changes 10 times at each frequency f and the amplitudes of the filter response are obtained at each phase. After removing the maximum value and the minimum value, the average value of remaining eight amplitudes is taken as the result. With the change of signal frequency, a series of values will be obtained. Set these values be ordinate and frequency value be abscissa, the amplitude frequency characteristic can be obtained. That's the method of median-mean.

3.2. The relationship between structural elements with different widths and cut-off frequency

To explore the relationship between different structural elements widths and cutoff frequency, different width structure elements are chosen to get the corresponding response curve.

Set $N = 512$, $f_s = 1Hz$, use linear type structural element with a width of L . Set $L = 2,4,6,8$. Get the frequency response of the filter, and a horizontal curve with the amplitude of 0.707 is drawn to get the cut-off frequency, as shown in figure 2 and figure 3.

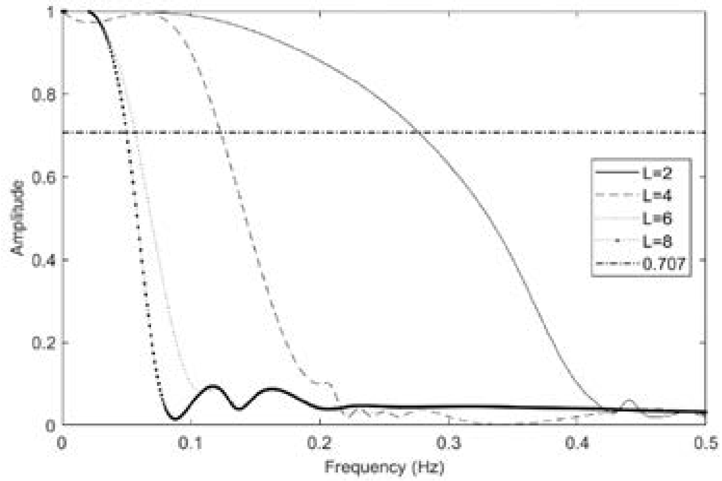

Figure 1. Amplitude frequency characteristics of morphological filters with different structural element widths based on average phase method

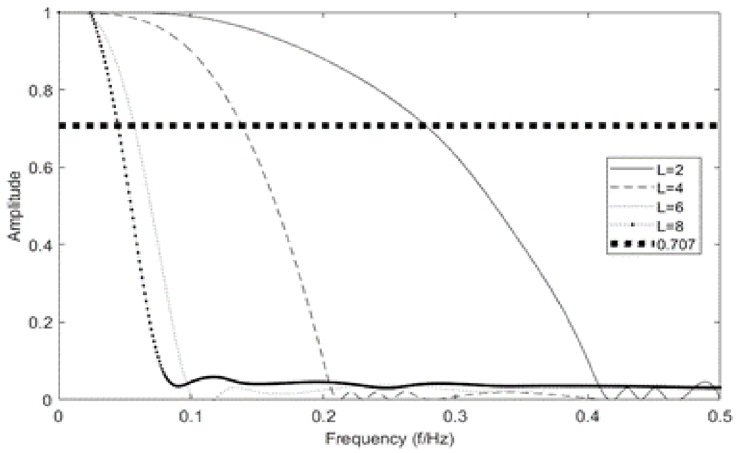

Figure 2. Amplitude frequency characteristics of morphological filters with different structural element widths based on median-mean method

Both Figure 1 and Figure 2 show good low pass characteristics. And the variation of the bandpass in both Figure1 and Figure 2 are almost the same. As the width of the structural element increases, the width of the bandpass becomes narrowing. With the increasing of structural element width, the cut-off frequency nonlinear is decreased. When the shape of the structural element is fixed, change the analysis points in the experimental.

The results show that the number of analysis points only affects the resolution of the frequency, and has no effect on the frequency response of the morphological filter. When different methods are taken to avoid the influence of initial phase change, the results are the same. It ensures the effectiveness of the experiment.

3.3. The relationship between different sampling frequency and cut-off frequency

In order to analyse the effect of sampling frequency change on the response characteristic of the filter, the sampling frequency in Equation (9) is change into 1Hz, 2Hz, 4Hz and 8Hz. In order to ensure the highest sampling frequency, it still has better frequency resolution, set N=2048, the frequency of the signal is 0 Hz<f<0.5 Hz, the increment of the frequency is $\frac{1}{256}$.

A linear structural element with a width of L=10 is selected to get the frequency response characteristic of the morphological filter at different sampling frequencies, as is shown in Figure 4 and figure 5.

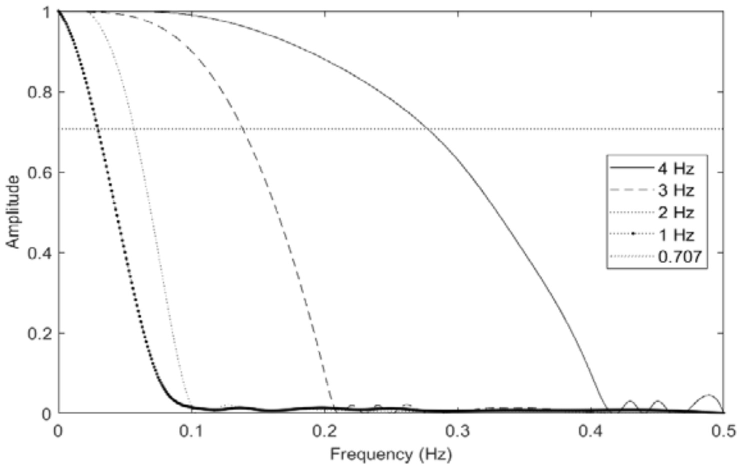

Figure 3. Amplitude frequency characteristics of morphological filter with different sampling frequency based on average phase method

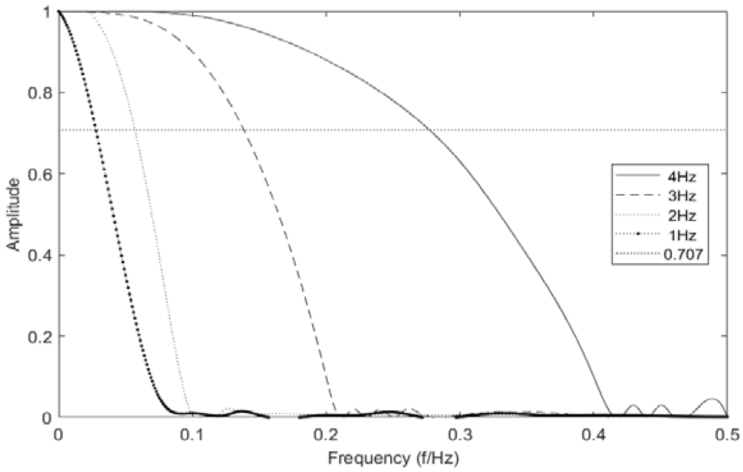

Figure 4. Amplitude frequency characteristics of morphological filter with different sampling frequency based on median-mean method

The width and shape of the structural elements are fixed in Figure 3. The higher the sampling frequency is, the higher the width of the bandpass is. The corresponding cut-off frequencies in Figure 3 are 0.03144 Hz, 0.06287 Hz, 0.12568 Hz and 0.25143 Hz at the sampling frequency of 1 Hz, 2 Hz, 3 Hz and 4 Hz. The ratio of the cut-off frequency is about 1:2:3:4. In Figure 4, the corresponding cut-off frequencies are 0.03153 Hz, 0.06301 Hz, 0.12598 Hz and 0.25257 Hz at the sampling frequency of 1 Hz, 2Hz, 3 Hz and 4 Hz. The ratio of the cut-off frequency is also about 1:2:3:4. The cut-off frequency is proportional to the sampling frequency. In fact, we only need to know the cut-off frequency corresponding to the different widths of the structural elements at the sampling frequency of 1 Hz. Then it multiplies by the actual sampling frequency, the cut-off frequency was gotten accurately.

When using other structural elements, such as semicircular, triangle, sine or cosine structure element, the analysis results are almost the same as the linear structural elements. Therefore, the conclusion is universal.

4. Conclusion

In this paper, two key factors are discussed that affect the morphological filter response of a linear structural element. As the width of the structure element increases, the cut-off frequency decreases when the sampling frequency is fixed. The cut-off frequency gets smaller as the width of structure element growing bigger. And the relationship between them is nonlinear. When the width of the structure element is fixed, the cut-off frequency and sampling frequency of the morphological filter are proportional to the sampling frequency. This kind of algorithm regulates the width of the structure element and makes the continuous morphological transform of signal, having well kept the main characteristic of original signal.

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