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# Entropy production rate in tokamak plasmas with helical magnetic perturbations

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**Abstract.** This paper presents a variational principle based on an extremum of entropy production rate allowing the calculation of the full neoclassical equilibrium (particles, heat and momentum) in tokamaks with an arbitrary helical perturbation.

#### 1. Introduction

Determining the neoclassical equilibrium in tokamaks in presence of a helical perturbation is a question of great interest. Indeed several processes break toroidal axisymmetry of a tokamak, namely field ripple due to finite number of coils [1, 2, 3, 4, 5, 6, 7, 8], MHD instabilities, or externally applied magnetic perturbations [9, 10, 11, 12]. One important point is the calculation of toroidal and poloidal flows, since sheared flows back react on turbulence and MHD instabilities. This question has been much debated in the 90's [5, 8]).

A variational principle based on an extremum of entropy production rate is proposed here to calculate the full neoclassical equilibrium (particles, heat and momentum). This method has been applied with success to the case of ripple induced transport [13, 14]. It is extended in the present work to an arbitrary helical perturbation. This approach provides an efficient means to discriminate between various regimes, depending on collisionality and the amplitude of the helical perturbation. It appears that fluxes and damping rates are usually not negligible. Implications will be discussed, with applications to recent observations on Tore Supra [15].

The remainder of this paper is organized as follows. General properties of a variational principle based on the entropy production rate are described in section 2. The section 3 is dedicated to the application to neoclassical transport in a tokamak in presence of a helical magnetic perturbation. A conclusion follows.

#### 2. Entropy production rate in a tokamak with helical perturbation

#### 2.1. Distribution function

A general expression of the entropy production rate for a non turbulent tokamak plasma in presence of an helical perturbation can be found in ref.[14]. The analysis is restricted to the case of a simple circular concentric equilibrium. The system of coordinates is labeled  $(r, \theta, \varphi)$ , where r is the minor radius,  $\theta$  the poloidal angle, and  $\varphi$  the toroidal angle. Moreover a single ion species is considered. The equilibrium Hamiltonian is  $H = \frac{1}{2}mv_{\parallel}^2 + \mu B + e\phi$ , where m is the

mass,  $v_{\parallel}$  the parallel velocity,  $\mu$  the magnetic moment, B a reference magnetic field, and  $\phi$  the mean electric potential. The perturbed Hamiltonian reads

$$\delta H = -\mu B\epsilon \cos\theta + \mu B\delta \cos\left(N\varphi + M\theta\right) \tag{1}$$

where  $\epsilon = \frac{r}{R}$  and  $\delta(r)$  is the amplitude of the helical perturbation (it is assumed that it does not depend on the poloidal angle). We call  $M_h = M + Nq$  the helical wave number. Particles are trapped in the helical perturbation when the magnetic fields exhibit local extrema along the field lines. If  $|M_h| >> 1$ , this occurs when  $Y = \alpha |\sin \theta| < 1$ , where  $\alpha = \frac{\epsilon}{|M_h|\delta}$ . This condition defines for each minor radius r a domain in  $\theta$  for which there exists locally trapped particles. One can define an effective ripple amplitude, which is the depth of the magnetic well along the field lines between successive minima and maxima [3], namely

$$2\delta_{eff} = \frac{B_{max}}{B_{min}} - 1 = 2\delta \left(\sqrt{1 - Y^2} - Y \arccos Y\right)$$
<sup>(2)</sup>

If  $|M_h| \ll 1$ , two kinds of trapped particles exist: toroidally trapped, and helically trapped particles. The distribution function is a function of motion invariants and is close to a local Maxwellian, namely

$$F_M = \frac{\mathcal{N}}{\left(2\pi mT\right)^{3/2}} \exp\left(-\frac{H}{T}\right) \left(1 + \frac{mWv_{\parallel}}{T}\right)$$
(3)

The function  $\mathcal{N}$  is defined as  $\mathcal{N} = n \exp(e\phi/T)$ . The density n, potential  $\phi$  and temperature T are functions of the toroidal kinetic momentum  $e\psi + mRv_{\parallel}$  ( $\psi$  is the poloidal magnetic flux normalised to  $2\pi$ ), while W is a function of the total energy H that ensures finite mean parallel velocity and parallel thermal flux. Once the condition  $\frac{RT}{e}\partial_{\psi}\Xi + W = V_T$  is fulfilled, where

$$\partial_{\psi}\Xi = \partial_{\psi}\ln n + \frac{e}{T}\partial_{\psi}\phi + \left(\frac{E}{T} - \frac{3}{2}\right)\partial_{\psi}\ln T \tag{4}$$

and  $V_T$  is the toroidal velocity (strictly speaking the parallel velocity), the distribution function Eq.(3) is an exact solution of the Fokker-Planck equation for the unperturbed problem. Multiplying Eq.(3) by  $v_{\parallel}^2/v_T^2$ , where  $v_T = \sqrt{T/m}$  is the thermal velocity, and integrating over the velocity space yields the conventional force balance equation in the large aspect ratio limit [13]

$$-\partial_r \phi + V_p B - V_T B_p = \frac{\partial_r p}{ne} \tag{5}$$

where  $V_p = \int d^3 \mathbf{v} F_M \frac{v_{\parallel}^2}{v_T^2} W$  is the poloidal velocity, and p the pressure. The entropy production rate can be constructed with the help of the equations derived in [13, 14]. Fluxes and thermodynamical forces can be properly defined from the resonant production rate using the following expression

$$\dot{S}_{res} = -\frac{1}{2} \int dV n \left( \frac{\Gamma}{n} \frac{\partial_r n}{n} + \frac{\mathcal{M}}{nmv_T} \frac{V_T}{v_T} + \frac{Q}{nT} \frac{\partial_r T}{T} \right)$$
(6)

where  $\Gamma$  and Q are the particle and heat fluxes, and  $\mathcal{M}$  is the rate of dissipated momentum due to toroidal collisional damping. Fluxes are therefore functional derivatives of the resonant entropy production rate.

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#### 3. Neoclassical transport tokamaks with an helical perturbation

#### 3.1. Total entropy production rate

In this section, we restrict the analysis to a single ion species. Also the analysis is restricted to the case where transit frequencies associated to  $E \times B$  and curvature drifts are smaller than collisional frequencies. Hence this analysis holds for small values of the radial electric field. The collisionality is measured with the conventional parameter for neoclassical theory  $\nu^* = \frac{\nu qR}{v_T \epsilon^{3/2}} \ll 1$ , where  $\nu$  is the collision frequency of the considered species. The collision frequency is defined here as  $\nu = \frac{4\sqrt{\pi}}{3} \frac{e^4}{(4\pi\epsilon_0)^2} \frac{n}{m_i^2 v_T^3} \ln \Lambda$ . The normalized collision frequency is  $\bar{\nu}(v) = \frac{3}{4}\sqrt{2\pi}(\Phi(v) - G(v))/v^3$ , where  $\Phi(v) = \frac{2}{\sqrt{\pi}}\int_0^v dx \exp(-x^2)$  and  $G(v) = (\Phi(v) - v\Phi'(v))/2v^2$ . All results are expressed as functions of  $\nu^*$ ,  $\delta/\epsilon$ , and wave numbers. With these conventions, the collisionality parameter related to the helical perturbation reads  $\nu_{hel}^* = \frac{\nu^*}{|M_h|} \left(\frac{\epsilon}{\delta}\right)^{3/2}$ . Also we define an effective collisionality  $\nu_{eff}^* = \max(1, M_h^2)\nu^*$ . The total resonant entropy production reads

$$\dot{S}_{res} = \dot{S}_{hel} + \dot{S}'_{tor} + \dot{S}_{tor} + \dot{S}'_{hel} \tag{7}$$

where

$$\dot{S}_{hel} = \frac{1}{2}\sqrt{\frac{\pi}{2}} \int dV n \frac{M^2}{|M_h|} \left(\frac{\delta}{\epsilon}\right)^2 \frac{qRv_D^2}{v_T} \int_0^{+\infty} du e^{-u} u^2 \\ \left(G'_0(\alpha) + G_0(\alpha) \min\left(1, \frac{4}{\pi} \mathcal{I}\nu_{hel}^* \frac{\bar{\nu}}{u^{1/2}}\right)\right) \\ \left(\partial_r \ln \mathcal{N} + \left(u - \frac{3}{2}\right) \partial_r \ln T + \frac{eB_p}{T} \frac{M_h}{M} V_T\right)^2$$

$$(8)$$

$$\dot{S}_{tor} = \frac{1}{2} \sqrt{\frac{\pi}{2}} \int dV n \frac{q R v_D^2}{v_T} \int_0^{+\infty} du e^{-u} u^{3/2} \bar{\nu}(u)$$
(9)

$$\min\left(1, \frac{4}{\pi} \mathcal{I}\nu^* \frac{\bar{\nu}}{u^{1/2}}\right) \left(\partial_r \ln \mathcal{N} + \left(u - \frac{3}{2}\right) \partial_r \ln T + \frac{eB_p}{T} V_T\right)^2 \tag{10}$$

where  $\mathcal{I} = 1.38$ . The entropy production rate  $\dot{S}'_{tor}$  associated to the effect of vertical drift on helically trapped particles reads when  $\nu^*_{hel} \ll 1$ 

$$\dot{S}'_{tor} = K_{hel} \int dV G_1(\alpha) n \left(\frac{Nq}{M_h}\right)^2 \left(\frac{\delta}{\epsilon}\right)^{3/2} \frac{1}{\nu^*} \frac{qRv_D^2}{v_T} \int_0^{+\infty} du e^{-u} u^{5/2} \frac{1}{\bar{\nu}(u)} \left(\partial_r \ln \mathcal{N} + \left(u - \frac{3}{2}\right) \partial_r \ln T\right)^2$$
(11)

The rate  $\dot{S}'_{tor}$  can be neglected for larger collisionality  $\nu_{hel}^* \gg 1$ . Similarly the production rate  $\dot{S}'_{hel}$  due to effect of the helical perturbation on toroidally trapped particles is small when  $\nu^* \gg 1$ . If  $\nu^* \ll 1$ , the expression of  $\dot{S}'_{hel}$  depends on the effective collision frequency. In the low effective collisionality regime  $\nu_{eff}^* \ll 1$ 

$$\dot{S}_{hel}' = K_{hel}' \int dV n (Nq)^2 \left(\frac{\delta}{\epsilon}\right)^2 \frac{1}{\nu^*} \frac{qRv_D^2}{v_T} \int_0^{+\infty} du e^{-u} u^{5/2} \frac{1}{\bar{\nu}(u)} \left(\partial_r \ln \mathcal{N} + \left(u - \frac{3}{2}\right)\partial_r \ln T\right)^2$$
(12)

while for larger collisionality  $\nu_{eff}^* \gg 1$  one has

$$\dot{S}_{hel}' = \frac{1}{2}\sqrt{\frac{\pi}{2}} \int dV n \frac{N^2 q^2}{|M_h|} \left(\frac{\delta}{\epsilon}\right)^2 \frac{qRv_D^2}{v_T}$$

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$$\int_{0}^{+\infty} du e^{-u} u^2 \left( \partial_r \ln \mathcal{N} + \left( u - \frac{3}{2} \right) \partial_r \ln T \right)^2 \tag{13}$$

where  $dV = 4\pi^2 Rr dr$ ,  $v_D = \frac{T}{eBR}$  is the thermal curvature drift velocity,  $K_{hel} = \left(\frac{2}{\pi}\right)^{3/2} \min\left(\frac{8}{9}, |M_h|\right)$ , and  $K'_{hel} = \left(\frac{2}{\pi}\right)^{3/2} \min\left(\frac{8}{9}, \frac{1}{|M_h|^3}\right)$ . When  $\alpha \gg 1$ , the form factors  $G_0$ ,  $G'_0$ ,  $G_1$  are given by the relations  $G_0(\alpha) = \frac{1}{2\pi} \int_{Y<1} d\theta$ ,  $G'_0(\alpha) = \frac{1}{2\pi} \int_{Y>1} d\theta$ , and  $G_1(\alpha) = \frac{1}{\pi} \int_{Y<1} d\theta \sin^2 \theta$ . In the opposite limit one gets  $G_0 = G_1 = 1$  and  $G'_0 = 0$ . The integration domains that correspond to the form factors  $G_0$ ,  $G'_0$ ,  $G_1$  do not span the whole phase space. The reason is that each contribution to the entropy production rate is associated to a resonant surface in the phase space where the entropy production rate is maximum. Indeed the production rates come from collisional boundary layers near transitions from trapped to passing particles.

The strategy to determine the neoclassical equilibrium is rather straightforward. First the extremum of the entropy production rate with respect to density variations yields the particle flux. It appears readily that the electron flux is smaller than the ion flux by a factor  $\sqrt{m_e/m_i} \ll 1$ . Thus the ambipolarity constraint reduces to a condition of vanishing ion flux,  $\Gamma = 0$ . This constraint gives a relation between the radial electric field, toroidal velocity and gradients of density and temperature (the poloidal velocity can be eliminated using the force balance equation). Second the extremum with respect to toroidal velocity provides the damping rate in the toroidal direction and therefore another constraint. Finally, the extremum with respect to the temperature gradient yields the thermal diffusivity. Several situations may occur, depending on whether local trapping occurs or not, and also depending on collisionality. We will call "weak perturbation regime" the situation where local trapping does not occur, and "strong perturbation regime" the case with local trapping. These two limits are detailed in the following sections.

#### 3.2. Weak perturbation

For small amplitudes of the helical perturbation Y > 1, no local trapping occurs, so that only banana particles matter. We restrict the analysis here to the banana regime  $\nu^* \ll 1$ . In that case two regimes are possible: banana-drift and ripple-plateau (by analogy with the particular case of ripple M = 0). These names follow the terminology proposed by Yushmanov [3], and corresponds respectively to the " $1/\nu$ " regime and superbanana-plateau regime of Shaing [12]. The transition regime, called " $\sqrt{\nu}$  regime", is not addressed here.

3.2.1. Banana-drift regime Two contributions remain in the entropy production rate: the collisional friction of banana particles on passing particles Eq.(10), and a second related to the effect of ripple on banana particles Eq.(12). The ambipolarity condition reads

$$(1+k_1)\partial_r \ln \mathcal{N} + (3.37 - 0.17k_1)\partial_r \ln T + k_1 \frac{eB_p V_T}{T} = 0$$
(14)

The collisionality parameter  $k_1$  is similar to the one defined in [8]

$$k_{1} = \frac{c_{\nu}}{c_{bd}} \frac{1}{\min\left(\frac{8}{9}, \frac{1}{|M_{h}|^{3}}\right) N^{2} q^{2}} \left(\frac{\epsilon}{\delta} \nu^{*}\right)^{2}$$
(15)

where  $c_{\nu} = \mathcal{I}\sqrt{\frac{2}{\pi}} \int_{0}^{+\infty} du e^{-u} u^{3/2} \bar{\nu}(u) \approx 1.1$  and  $c_{bd} = \left(\frac{2}{\pi}\right)^{3/2} \int_{0}^{+\infty} du e^{-u} u^{5/2} \frac{1}{\bar{\nu}(u)} \approx 7.4$ . Hence the high collisionality regime is defined by the condition  $\nu^* \gg Nq \left[\min\left(\frac{8}{9}, \frac{1}{|M_h|^3}\right)\right]^{1/2} \frac{\delta}{\epsilon}$ . Note that this condition is consistent with the banana-drift condition  $\nu_{eff}^* \ll 1$  if  $\frac{\delta}{\epsilon} \ll 1$ .  $\frac{1}{Nq}\min\left(1,\frac{1}{|M_h|^{1/2}}\right)$ . The extremalisation with respect to the toroidal velocity yields the following evolution equation

$$\partial_t V_T = -c_\nu \frac{1}{1+k_1} \sqrt{\epsilon} \nu \left( V_T - 3.54 \frac{\partial_r T}{eB_p} \right) \tag{16}$$

When the toroidal velocity has relaxed, i.e.  $V_T = 3.54 \frac{\partial_r T}{eB_p}$ , the poloidal velocity is found by combining this expression Eq.(14) with the force balance equation. It turns out that, whatever the collisionality, it is equal to the conventional neoclassical poloidal velocity

$$V_p = 1.17 \frac{\partial_r T}{eB} \tag{17}$$

The force balance equation (or equivalently Eq.(14)) yields the radial electric field

$$\frac{eE_r}{T} = \partial_r \ln n + 3.37 \partial_r \ln T \tag{18}$$

Finally the extremalisation with respect to  $\partial_r \ln T$  yields the heat flux  $Q = -\chi_i n \partial_r T$  where

$$\chi_i = 36.94 \min\left(\frac{8}{9}, \frac{1}{|M_h|^3}\right) N^2 q^2 \frac{\delta^2}{\epsilon^{1/2}} \frac{v_D^2}{\nu} \left(1 + 0.246k_1\right)$$
(19)

These results are consistent with the values given by Kovrizhnykh [8].

3.2.2. Ripple-plateau regime Again two contributions remain in the entropy production rate: banana Eq.(10), and ripple-plateau Eq.(13). The ambipolarity condition reads

$$(1+k_2)\partial_r \ln \mathcal{N} + (1.5 - 0.17k_2)\partial_r \ln T + k_2 \frac{eB_p V_T}{T} = 0$$
(20)

where the collisionality parameter  $k_2$  is defined as

$$k_2 = \sqrt{\frac{2}{\pi}} c_\nu \frac{|M_h|}{N^2 q^2} \left(\frac{\epsilon}{\delta}\right)^2 \nu^* \tag{21}$$

Note that the parameter  $k_2$  corresponds to  $k_2^{-1}$  in [8]. The weak collisional regime is defined as  $\nu^* \ll \frac{N^2 q^2}{|M_h|} \left(\frac{\delta}{\epsilon}\right)^2$ , which is consistent with the ripple-plateau condition  $\nu_{eff}^* > 1$  if  $\frac{\delta}{\epsilon} \gg \frac{1}{Nq} \min\left(|M_h|^{1/2}, \frac{1}{|M_h|^{1/2}}\right)$ . The extremalisation with respect to the toroidal velocity yields the following evolution equation

$$\partial_t V_T = -c_\nu \frac{1}{1+k_2} \sqrt{\epsilon} \nu \left( V_T - 1.67 \frac{\partial_r T}{eB_p} \right)$$
(22)

The poloidal velocity is found by combining the relaxed toroidal velocity  $V_T = 1.67 \frac{\partial_r T}{eB_p}$  with the force balance equation and the ambipolarity condition Eq.(20). Again it is found to be equal to the conventional neoclassical poloidal velocity  $V_p = 1.17 \frac{\partial_r T}{eB}$  for any collisionality parameter  $k_2$ . The force balance equation yields the radial electric field

$$\frac{eE_r}{T} = \partial_r \ln n + 1.5\partial_r \ln T \tag{23}$$

Finally the extremalisation with respect to  $\partial_r \ln T$  yields the heat flux  $Q = -\chi_i n \partial_r T$  where

$$\chi_{i} = 3\sqrt{\frac{\pi}{2}} \frac{N^{2} q^{2}}{|M_{h}|} \left(\frac{\delta}{\epsilon}\right)^{2} \frac{q R v_{D}^{2}}{v_{T}} \left(1 + 0.41 k_{2}\right)$$
(24)

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#### 3.3. Strong perturbation

In that regime, all 4 contributions must be kept in the entropy production rate. It is assumed that  $\nu^* \ll 1$ . Several situations must be considered. Let us assume first that helically trapped particles are in the plateau regime  $\nu_{hel}^* = \frac{\nu^*}{|M_h|} \left(\frac{\epsilon}{\delta}\right)^{3/2} \gg 1$ . In that case,  $\dot{S'}_{tor}$  is negligible. Depending whether  $|M_h|$  is of order one, or larger than one, the parameter  $\nu_{eff}^* = \max(1, M_h^2)\nu^*$  can be smaller or larger than one. If  $\nu_{eff}^* \gg 1$ , toroidally trapped particles (bananas) are in the ripple plateau regime. In this particular case,  $\dot{S}_{hel}$  and  $\dot{S'}_{hel}$  are of the same amplitude, if  $M \sim Nq$ . In that case, the radial electric field satisfies the relation

$$\partial_r \ln \mathcal{N} + \frac{3}{2} \partial_r \ln T + \frac{MM_h}{M^2 + N^2 q^2} \frac{eB_p V_T}{T} = 0$$
<sup>(25)</sup>

and the toroidal friction rate reads

$$\partial_t V_T = -\sqrt{\frac{\pi}{2}} |M_h| \frac{N^2 q^2}{M^2 + N^2 q^2} \frac{v_T}{qR} \delta^2 V_T$$
(26)

If  $\nu_{eff}^* \ll 1$ , toroidally trapped particles are in the banana-drift regime so that  $\dot{S}'_{hel} \gg \dot{S}_{hel}$ . The radial electric field is given by Eq.(18). The toroidal friction rate reads

$$\partial_t V_T = -\sqrt{\frac{\pi}{2}} \left| M_h \right| \frac{v_T}{qR} \delta^2 \left( V_T - 1.87 \frac{M}{M_h} \frac{\partial_r T}{eB_p} \right)$$
(27)

We now investigate the other extreme case where all particles are in the helically trapped regime, i.e.  $\nu_{hel}^* \ll 1$ ,  $G'_0 = 0$ , and  $G_0 = G_1 = 1$ . The entropy production rate is then dominated by the two contributions proportional to the inverse of the collision frequency, namely  $\dot{S'}_{tor}$  and  $\dot{S'}_{hel}$ . Moreover  $\dot{S'}_{tor}$  is larger than  $\dot{S'}_{hel}$  provided that  $\delta < \epsilon/M_h^4$  if toroidally trapped particles are in the banana-drift regime  $\nu_{eff}^* \ll 1$ , and provided that  $\nu^* \ll (\epsilon/\delta)^{1/2} / |M_h|$  if they are in the ripple-plateau regime  $\nu_{eff}^* \gg 1$ . These conditions are usually met. The extremum of Eq.(11) with respect to  $\partial_r \ln \mathcal{N}$  and  $\partial_r \ln T$  then the particle and heat fluxes

$$\begin{pmatrix} \Gamma \\ Q \end{pmatrix} = -nD_{lt} \begin{pmatrix} \partial_r \ln \mathcal{N} + 3.37 \partial_r \ln T \\ 5 \partial_r T \end{pmatrix}$$
(28)

where  $D_{lt} = 6.58 \min\left(1, \frac{9}{8} | M_h |\right) \frac{N^2 q^2}{M_h^2} \delta^{3/2} \frac{v_D^2}{\nu}$ . This expression is a generalization of the expression given by Connor et al. [2] for ripple induced fluxes in tokamaks. Since electron ripple losses are small, the ambipolarity constraint  $\Gamma = 0$  imposes the value of the radial electric field, which is the Connor value Eq.(18). At this point, one has still to find the ordering between  $\dot{S}_{hel}$  and  $\dot{S}_{tor}$ . One finds easily that  $\dot{S}_{hel}/\dot{S}_{tor}$  is of the order of  $\left(\frac{M}{M_h}\right)^2 \left(\frac{\delta}{\epsilon}\right)^{1/2}$ . If  $\frac{M}{M_h} \ll \left(\frac{\epsilon}{\delta}\right)^{1/4}$ , the banana term  $\dot{S}_{tor}$  is dominant. The extremum with respect to the toroidal velocity yields the value  $V_T = 3.54 \frac{\partial_r T}{eB_p}$  and a damping rate  $\nu_{\varphi} = c_{\nu}\sqrt{\epsilon\nu}$ . The toroidal velocity can be combined with the radial electric field in the force balance equation to find the poloidal velocity  $V_p = 1.17 \frac{\partial_r T}{eB}$ . If  $\frac{M}{M_h} \gg \left(\frac{\epsilon}{\delta}\right)^{1/4}$ , the term  $\dot{S}_{hel}$  is dominant. One finds  $V_T = 3.54 \frac{M}{M_h} \frac{\partial_r T}{eB_p}$  with the damping rate  $\nu_{\varphi} = c_{\nu}\sqrt{\delta\nu}$ . The toroidal electric field in the force balance equation to find the radial electric field in the force balance equation to find the radial electric field in the force balance equation to find the radial electric field in the force balance equation to find the radial electric field in the force balance equation to find the radial electric field in the force balance equation to find the radial electric field in the force balance equation to find the radial electric field in the force balance with the radial electric field in the force balance equation to find the radial electric field in the force balance equation to find the poloidal velocity  $V_p \approx \left(3.54 \frac{M}{M_h} - 2.54\right) \frac{\partial_r T}{eB}$ .

#### 4. Discussion and conclusion

In conclusion, a variational principle based on a principle of extremum of entropy production rate has been developed. This method allows the calculation of the neoclassical equilibrium in

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tokamaks with arbitrary helical perturbation. In tokamaks, helical perturbations usually come from MHD perturbations or external coils (RMPs). Since the amplitude is small, the relevant regime is banana-drift in the nearly collisionless regime [9, 10, 11, 12]. The regime that is usually retained corresponds to  $k_1 \gg 1$  (so-called " $1/\nu$ " regime). Nevertheless, it turns out that the weakly collisional regime  $k_1 \ll 1$  is not that difficult to reach. In that case the toroidal friction rate is of the order of  $\nu\sqrt{\epsilon}$ . More interestingly, the diffusion coefficient is not that small in this regime, and exceeds the banana diffusion coefficient. However it remains smaller than a turbulent diffusion coefficient. In ohmic plasmas in the Tore Supra tokamak, the level of density fluctuations, and therefore turbulent transport, has been found to be very small (~  $10^{-3}$ ) inside the q = 1 surface. It appears that the measured diffusion coefficient of electrons is larger than the neoclassical banana value. However the impurity flux does agree with the neoclassical value when assuming axisymmetric geometry [15]. This paradox can be resolved if one accounts for the presence of a (1,1) internal kink mode with a reasonable amplitude. Electron collisional diffusion increases due to the helical perturbation, while the impurity flux is barely affected. Hence ripple induced transport matters for toroidal momentum transport, but also in some cases for particle transport.

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