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To cite this article: Takahisa Igata *et al* 2011 *J. Phys.: Conf. Ser.* **314** 012113

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Stable Bound Orbits around Black Rings

Takahisa Igata, Hideki Ishihara and Yohsuke Takamori

Department of Mathematics and Physics,
Graduate School of Science, Osaka City University,
Osaka 558-8585, Japan

E-mail: igata@sci.osaka-cu.ac.jp

Abstract. We study stable bound orbits of a free particle around a black ring. Unlike the higher-dimensional black hole case, we find that there exist stable bound orbits in toroidal spiral shape near the ring axis and stable circular orbits on the axis. In addition, radii of stable bound orbits can be infinitely large if the ring thickness is less than a critical value.

1. Introduction

The issue with regard to higher dimensional black objects is presently under active investigation by many researchers. In particular, higher-dimensional black object solutions of the vacuum Einstein theory and various supergravity theories play crucial roles in modern physics [1]. In higher dimensions, there is a wide variety of black hole solutions in contrast to four dimensions. In the framework of the vacuum Einstein gravity, in addition to a generalization of the Kerr metric derived by Myers and Perry in [2], we have the metrics of the black object solutions with various topology of event horizon [3]. Indeed, Emparan and Reall found the five-dimensional black ring solution which has horizon topology $S^2 \times S^1$ in [4]. The existence of the black ring solution shows multiplicity of higher-dimensional black objects. Namely, the black hole uniqueness theorem does not hold in higher-dimensional spacetime as the same form in the case of four-dimensional black hole.

Among many ways of investigating black objects, the study of a free particle motion is a basic approach to understanding physical features of black objects. In recent years, free particle motion in the black ring geometry is studied by several authors [5]. In particular, Hoskisson has investigated extensively geodesics in the black ring geometry in [6]. He has shown that the separation of variable of the Hamilton-Jacobi equation for geodesics does not occur in the ring coordinates.

In this report, we then will focus on existence of stable bound orbits around a black ring. It is worth pointing out that there is no stable bound circular orbit in the higher-dimensional Schwarzschild solution. In contrast, we show that there exist stable bound orbits far from a black ring. The difference of geometry between a black hole and a black ring is clearly distinguishable by geodesic motions.

2. Black Ring Geometry

We begin by giving the geometry of the black ring. The metric is written in the form

$$ds^2 = -\frac{F(y)}{F(x)} \left(dt - CR \frac{1+y}{F(y)} d\psi \right)^2 + \frac{R^2}{(x-y)^2} F(x) \left(-\frac{G(y)}{F(y)} d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)} d\phi^2 \right), \quad (1)$$

with the range of the ring coordinates $-1 < x < 1$, $-\infty < y < -1$, where

$$F(\xi) = 1 + \lambda\xi, \quad G(\xi) = (1 - \xi^2)(1 + \nu\xi), \quad C = \sqrt{\lambda(\lambda - \nu) \frac{1 + \lambda}{1 - \lambda}}, \quad (2)$$

and R , ν , and λ are the parameters of the solution. The three parameters all have a physical interpretation: R , ν , and λ denote the ring radius, the thickness of the ring, the rotation velocity of the ring, respectively, where the parameter range is

$$R > 0, \quad 0 < \lambda \leq \nu < 1. \quad (3)$$

The parameter λ is determined by the regularity conditions at the two rotation axes of the ring in terms of ν as follows:

$$\lambda = \frac{2\nu}{1 + \nu^2}. \quad (4)$$

The black ring metrics are stationary and axisymmetric, with Killing vectors ∂_t , ∂_ϕ , and ∂_ψ . The fixed point of ψ -rotation generated by ∂_ψ , which we refer to as the ring axis, exists at $y = -1$, and the fixed point of ϕ -rotation generated by ∂_ϕ , which we refer to as the equatorial plane, exists at $x = \pm 1$. Furthermore, this metric appears to have a singularity at $y = -1/\nu, -1/\lambda, -\infty$. Evaluation of curvature invariants such as $R_{abcd}R^{abcd}$ shows that the singularity at $y = -\infty$ is a true, curvature singularity. The singularities in the metric components at $y = -1/\nu$ and $y = -1/\lambda$ are coordinate singularities, corresponding to the event horizon and the ergo surface with $S^2 \times S^1$ topology, respectively.

3. Stable Bound Orbits in the Black ring geometry

We turn to a discussion of geodesic motion in the black ring geometry. Let p_μ be the 5-momentum of a particle with rest mass m , we define the Hamiltonian of geodesic motion as

$$H = \frac{1}{2} \left[\frac{(x-y)^2}{R^2} \frac{G(x)}{F(x)} p_x^2 - \frac{(x-y)^2}{R^2} \frac{G(y)}{F(x)} p_y^2 + E^2 \left(U_{\text{eff}} + \frac{m^2}{E^2} \right) \right], \quad (5)$$

where

$$U_{\text{eff}} = -\frac{F(x)}{F(y)} - \frac{C^2(x-y)^2(y+1)^2}{G(y)F(x)F(y)} + \frac{(x-y)^2}{R^2 G(x)} \ell_\phi^2 - \frac{F(y)(x-y)^2}{R^2 G(y)F(x)} \ell_\psi^2 + 2 \frac{C(x-y)^2(y+1)}{RG(y)F(x)} \ell_\psi \quad (6)$$

is the effective potential of the geodesic motion, and $E = -p_t$, $\ell_\phi = p_\phi/E$, and $\ell_\psi = p_\psi/E$, respectively. The three components, p_t , p_ϕ , and p_ψ , yield constants of motion since the black ring metric admits three commutable Killing vectors.

In what follows, in order to give intuitive picture of particle motion, we use ζ - ρ coordinates which are defined as

$$\zeta = R \frac{\sqrt{y^2 - 1}}{x - y}, \quad \rho = R \frac{1 - x^2}{x - y}. \quad (7)$$

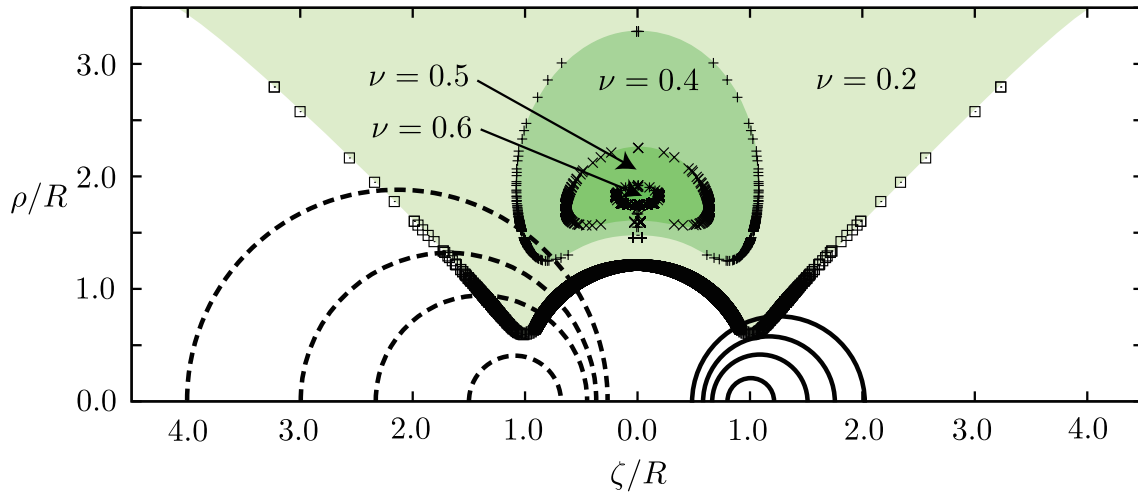


Figure 1. The shaded region shows the domain of stable bound orbits in the black ring spacetime for each ν . The dashed and solid semicircles show the ergo surfaces and the event horizon, respectively, where $\nu = 0.2, 0.4, 0.5$, and 0.6 from the small one to the large one.

In the flat limit, the black ring metric reduces to the flat metric in the form $ds^2 = -dt^2 + d\zeta^2 + \zeta^2 d\psi^2 + d\rho^2 + \rho^2 d\phi^2$.

Let us consider stationary particle motion determined by

$$U_{\text{eff}}(\zeta_{\text{st}}, \rho_{\text{st}}) + \frac{m^2}{E^2} = 0, \quad \frac{\partial U_{\text{eff}}}{\partial \zeta}(\zeta_{\text{st}}, \rho_{\text{st}}) = \frac{\partial U_{\text{eff}}}{\partial \rho}(\zeta_{\text{st}}, \rho_{\text{st}}) = 0, \quad (8)$$

i.e., $(\zeta_{\text{st}}, \rho_{\text{st}})$ are positions of the extrema of U_{eff} . Restricting attention to stable bound orbits, we impose in addition to equation (8) the requirement that

$$\det \mathcal{H}(\zeta_s, \rho_s) > 0, \quad \frac{\partial^2 U_{\text{eff}}}{\partial \rho^2}(\zeta_s, \rho_s) > 0, \quad (9)$$

where

$$\mathcal{H}(\zeta, \rho) = \begin{pmatrix} \frac{\partial^2 U_{\text{eff}}}{\partial \rho^2}(\zeta, \rho) & \frac{\partial^2 U_{\text{eff}}}{\partial \rho \partial \zeta}(\zeta, \rho) \\ \frac{\partial^2 U_{\text{eff}}}{\partial \zeta \partial \rho}(\zeta, \rho) & \frac{\partial^2 U_{\text{eff}}}{\partial \zeta^2}(\zeta, \rho) \end{pmatrix}, \quad (10)$$

so that (ζ_s, ρ_s) denote positions of local minima of U_{eff} .

In Figure 1, the domains of (ζ_s, ρ_s) are plotted by solving the conditions (8) and (9) numerically. The figure shows that there are stable bound orbits on and near the ring axis of the black ring geometry while there is no stable bound orbit on the equatorial plane. The orbits on the time slice of the Killing time have toroidal spiral shape near the ring axis and circular shape on the ring axis, which is generated by the two axial Killing vectors.

We discuss the asymptotic form of U_{eff} near the ring axis to confirm the existence of the stable bound orbit near infinity in analytical way. If $\ell_\psi = 0$, the form of U_{eff} expanded in ρ near asymptotic infinity on the ring axis is

$$U_\infty \equiv U_{\text{eff}}(\zeta = 0, \rho) \simeq -1 - \frac{4\nu R^2 - (1 - \nu)\ell_\phi^2}{(1 - \nu)^2} \frac{1}{\rho^2} + \frac{2\nu R^2[2(R^2 + \zeta^2) - \ell_\phi^2]}{(1 - \nu)^2} \frac{1}{\rho^4}. \quad (11)$$

By solving the equation $\partial U_\infty / \partial \rho = 0$ and imposing the two inequalities $4\nu R^2 - (1 - \nu)\ell_\phi^2 > 0$ and $2R^2 - \ell_\phi^2 > 0$, we obtain the local minimum point on the ring axis in the form

$$\rho_s = \sqrt{\frac{4\nu R^2(2R^2 - \ell_\phi^2)}{4\nu R^2 - (1 - \nu)\ell_\phi^2}}, \quad (12)$$

where conditions (9) hold

$$\det \mathcal{H}(\zeta_s = 0, \rho_s) = \frac{128\nu^2 R^4(2R^2 - \ell_\phi^2)}{(1 - \nu)^4 \rho_s^{10}} > 0, \quad (13)$$

$$\frac{\partial^2 U_\infty}{\partial \rho^2}(\zeta_s = 0, \rho_s) = \frac{[4\nu R^2 - (1 - \nu)\ell_\phi^2]^3}{4\nu^2 R^4(1 - \nu)^2(2R^2 - \ell_\phi^2)^2} > 0. \quad (14)$$

If ℓ_ϕ^2 approaches to $4R^2\nu/(1 - \nu)$ under the above conditions, the radius of the stable circular orbit ρ_s approaches infinity only in the cases $0 < \nu < \nu_\infty := 1/3$. We note that the angular momentum $L_\phi = \ell_\phi E$ for the stable bound circular orbit is finite even if its radius is infinite.

4. Summary

In this short paper, we have discussed stable bound orbits of a free particle in the black ring geometry. By using the way of an effective potential of a free particle motion, we show the existence of stable bound orbits in the black ring spacetime which is a characteristic property of the black ring geometry unlike the black hole case. Furthermore, in the case of $0 < \nu < \nu_\infty = 1/3$, we find that the radii of the stable circular orbits can be infinity. The basic mechanism for the appearance of stable bound orbits is different from the case of a four-dimensional black hole. They occur by a balance of Newton's gravitational term and subleading higher multipole term. Detailed discussion is found in our paper [7]. In addition, interesting issue of dynamical particle motion in the black ring geometry is discussed in [8].

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