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# Some topological properties of the Inverse Lens Mapping

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**Abstract**. Away from critical curves, lens mapping can be seen as a linear invertible transformation of the plane even for regions (cells) of relatively large size. However, close to critical curves the departures from linearity can be very strong. We discuss the topological problems induced by the mapping of regions of the image plane that include critical curves (critical cells).

## 1. Inverse lens mapping as a linear transformation of the plane.

Lens mapping can be seen locally as a linear invertible transformation of the plane that transforms parallelograms into parallelograms [1]. These transformations are the composition of rotation, dilation (including reflection) and shear (see figures 1,2 and 3). Lens mapping is symmetric too.

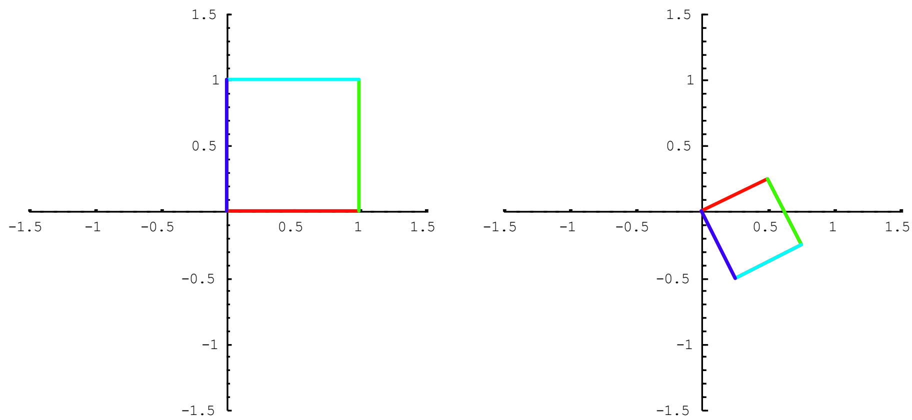


**Figure 1.** Dilation + rotation.

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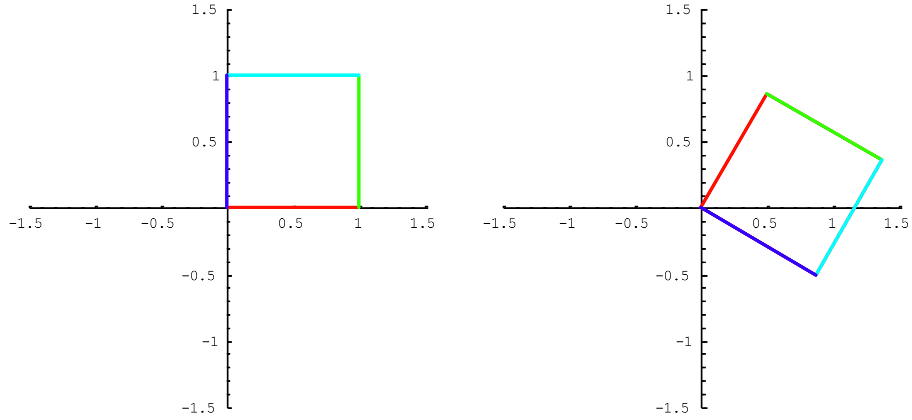


**Figure 2.** Reflection + rotation.

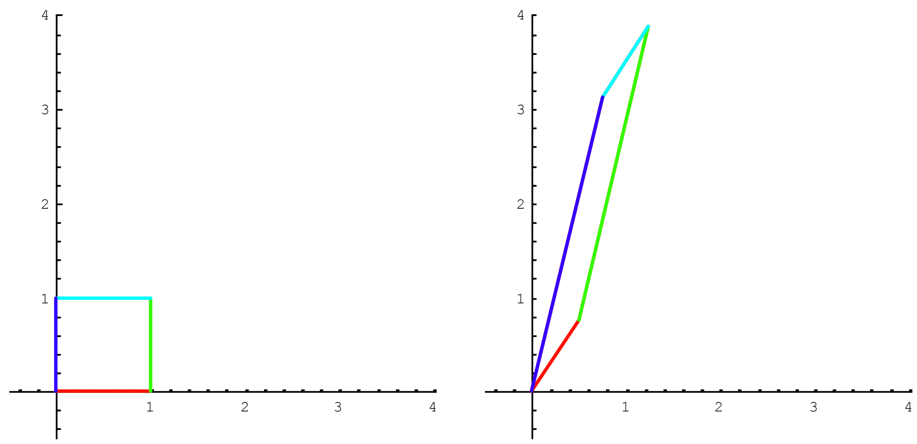


**Figure 3.** Rotation + shear.

## 2. Inverse lens mapping close to critical curves.

Close to critical curves the departure from linearity can be strong. Let us consider a small cell located in the neighborhoods of a critical curve. Close to a fold the inverse lens mapping collapse the cell in the direction perpendicular to the caustic (see figure 4). In addition to the collapse the cell is also rotated and suffers a strong shear. Close to a cusp (see figure 5), the inverse lens transformation is clearly non-linear; straight lines blend and parallelism is not preserved. Nevertheless, also in this case the cell collapse in the direction perpendicular to the caustics that join at the cusp.

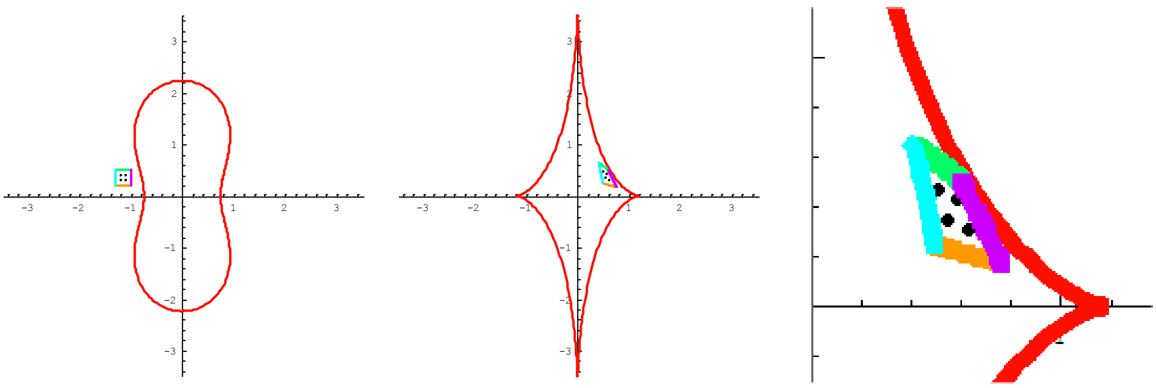

**Figure 4.** Non-critical cell close to a fold

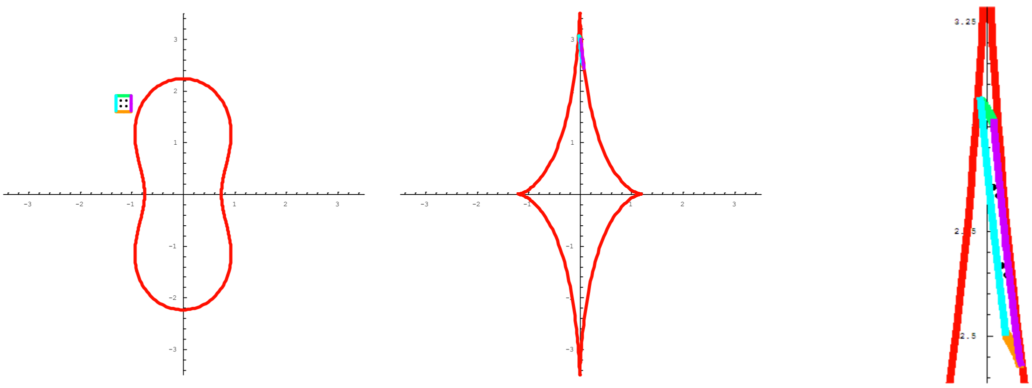

**Figure 5.** Non-critical cell close to a cusp

## 3. Inverse lens mapping of critical cells.

If the simply connected region is crossed by a critical curve the transformed region will be not simply connected. This can have very significant consequences. The boundary of the critical cell will be not transformed in the boundary of the transformed region (see figure 6). Another consequence is auto-overlapping (see figure 7). Finally, a someway different example of topological pathology is the transformation of a simply connected region including a point where f is discontinuous (see figure 8).

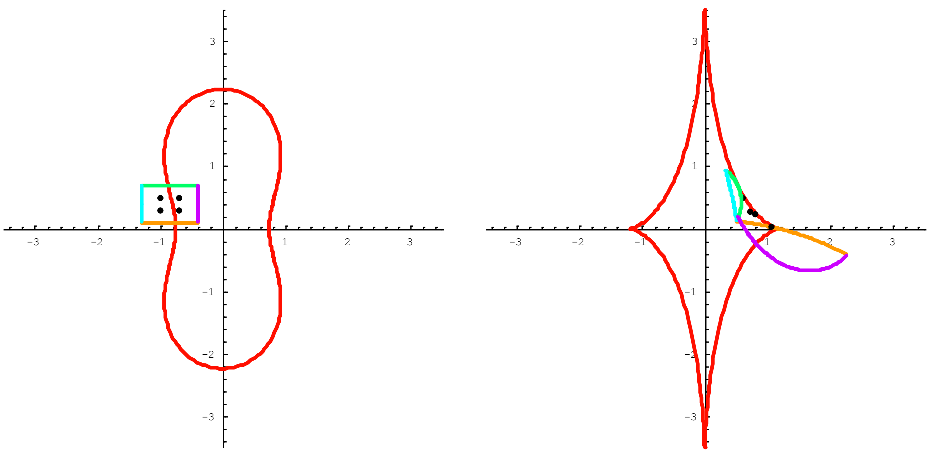

**Figure 6.** The boundary of the critical curve will be not transformed in the boundary of the transformed region.

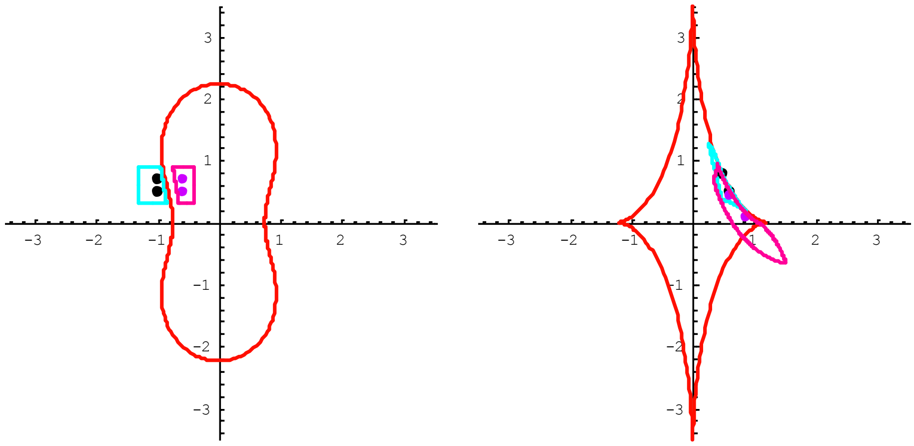


**Figure 7.** Autooverlaping.

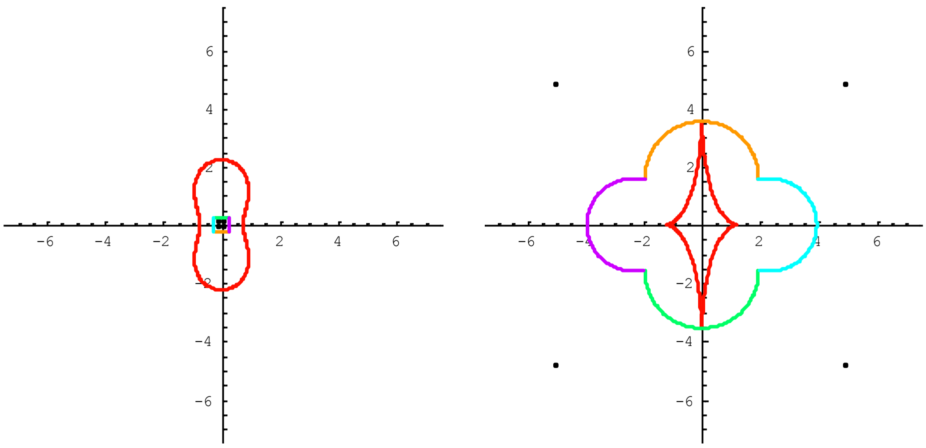


**Figure 8.** Transformation of a cell including a point where f  is discontinuous.

## 4. Conclusions.

Inverse lens mapping induces strong topological pathologies in the transformation of critical cells. To avoid this problem in the computation of magnification maps, a partition of the image plane in polygonal cells not including critical points (except perhaps at the cells boundaries) should be considered.

## References

[1] Mediavilla E, Muñoz J, Lopez P, Mediavilla T, Abajas C, Gonzalez- Morcillo C and Gil-Merino 2006 *ApJ* **635** 942-953