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Effects of driving scale on astrophysical turbulence and a modified Chandrasekhar-Fermi method

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Abstract. Many astrophysical fluids are in turbulent state. To maintain turbulence, energy must be injected into the fluid. In turbulence simulations, it is customary to drive the fluids on a scale comparable to the size of the computational domain. In this paper, we show how some statistics of turbulence, especially turbulence statistics projected on the plane of the sky, is changed when we do not follow the conventional approach. As an example of such a statistics, we discuss in detail how the Chandrasekhar-Fermi method is affected by the small-scale driving, which is a simple and powerful technique for estimating the strength of the mean magnetic field projected on the plane of the sky.

1. Introduction

Driving is required to initiate and maintain turbulence. When hydrodynamic turbulence is driven on a scale L_f , the injected energy cascades down to small scales and, when the energy cascade reaches the dissipation scale L_d , the energy is lost through viscous damping. The energy spectrum of turbulence peaks at the driving wavenumber k_f and shows a power-law scaling in the range between k_f and the dissipation wave number k_d . The spectrum drops rapidly for wavenumbers larger than k_d .

In turbulence simulations, the driving scale L_f is usually very close to the computational box size L_{sys} . The reason for this is to maximize the inertial range. Let us assume $L_{sys} = 2\pi$ throughout the paper. In most turbulence simulations, the dissipation wavenumber k_d is determined by the numerical resolution: k_d is slightly smaller than the maximum wavenumber $N_x/2$, where N_x is the number of grid points in one direction. Therefore, in order to maximize the inertial range, it is necessary to drive turbulence near $k \sim 1$, which means $k_f \sim 1$ and $L_f \sim L_{sys} = 2\pi$.

In astronomy, observed quantities are usually integrated along the line-of-sight (LOS). Since observed quantities are two-dimensional (2D) data, obtaining three-dimensional (3D) quantities from observations is not easy. Nevertheless, there are techniques developed to derive 3D turbulence properties from 2D observations. However, those techniques have an intrinsic shortcoming: it is difficult to verify them. To overcome this difficulty, people use computer simulation data to verify their techniques.

One such example is the Chandrasekhar-Fermi (CF) method [1], which is a powerful technique for estimating the strength of the plane-of-the-sky component of the mean magnetic field, $B_{0,sky}$. The CF method makes use of polarized emission from magnetically aligned dust grains. In the



conventional CF method, we can obtain an estimate of $B_{0,sky}$ from

$$B_{0,sky} = \xi \sqrt{4\pi\bar{\rho}} \frac{\delta v_{los}}{\delta\phi}, \quad (1)$$

where ξ is a constant of order unity, $\bar{\rho}$ is average density, δv_{los} is the LOS velocity dispersion (or, typical width of an optically thin emission line), and $\delta\phi$ is the variation of the polarization angle. Researchers used numerical simulations to verify the technique and obtain the value of ξ [2, 3, 4].

In this paper, we demonstrate that we should be careful in using simulations to verify a theoretical method: if we do not take into account the driving scale, it is possible to make a mistake. We focus on the CF method. In section 2, we show that the conventional CF method is prone to such a mistake. To be specific, we show that the conventional CF method overestimates $B_{0,sky}$ by a factor of $\sqrt{N_{eddy}}$, where N_{eddy} is the number of independent eddies along the LOS. Note that $N_{eddy} \approx L_{sys}/L_f$ in simulation data. In Section 3, we introduce our modified CF method (see [5] for details). In Section 4, we provide discussions and summary.

2. The conventional Chandrasekhar-Fermi (CF) method

2.1. Derivation of the conventional CF method

Consider a turbulent medium with a mean magnetic field ($\mathbf{B}_0^{3D}$) and a fluctuating magnetic field ($\mathbf{b}^{3D}$). For simplicity, let us assume that the mean magnetic field is along the x direction and the LOS is along the z direction. In this setup, the strength of the plane-of-the-sky mean magnetic field ($B_{0,sky}^{3D}$) is just B_0^{3D} :

$$B_{0,sky}^{3D} = B_0^{3D}, \quad \text{if } \mathbf{B}_0^{3D} \perp \text{LOS}. \quad (2)$$

Let us consider a fluid filled with Alfvén waves or Alfvénic turbulence. In Alfvénic disturbances, the r.m.s. fluctuation of magnetic field (δb^{3D}) and the r.m.s. velocity (δv^{3D}) are related by

$$\frac{\delta b^{3D}}{\sqrt{4\pi\bar{\rho}}} \sim \delta v^{3D} \quad \text{or} \quad 1 \sim \sqrt{4\pi\bar{\rho}} \frac{\delta v^{3D}}{\delta b^{3D}}, \quad (3)$$

where $\bar{\rho}$ is average density. Since we are dealing with Alfvén waves, we assume the density is constant. If we multiply both sides by the mean plane-of-the-sky magnetic field $B_{0,sky}^{3D}$, we obtain

$$B_{0,sky}^{3D} \sim \sqrt{4\pi\bar{\rho}} \frac{\delta v^{3D}}{\left(\delta b^{3D}/B_{0,sky}^{3D}\right)}. \quad (4)$$

It is relatively easy to estimate the numerator on the right-hand side (i.e. δv^{3D}) from observations. If we observe the turbulent medium using an optically thin molecular emission line, then the width of the emission line profile ($\equiv \delta v_{los}$) is approximately $\sim \delta v^{3D}$:

$$\delta v^{3D} \sim \delta v_{los}. \quad (5)$$

Note that δv^{3D} roughly corresponds to the typical length of the velocity vectors in the 3D space, which is $\sim \sqrt{3}$ times larger the typical length of the LOS velocity vectors in the 3D space (see red arrows in Figure 1(a)).

Estimation of the denominator on the right-hand side of Equation (4) is rather tricky. Suppose that we have coherent Alfvén waves (see Figure 1(b)). In this case, it is trivial to show that

$$\frac{b_y^{2D}}{B_{0,sky}^{2D}} = \frac{\int b_y^{3D} dz}{\int B_{0,sky}^{3D} dz} = \frac{b_y^{3D}}{B_{0,sky}^{3D}}, \quad (6)$$

where $B_{0,sky}^{2D}$ is the plane-of-the-sky mean magnetic field, which is proportional to $B_{0,sky}^{3D}$ integrated along the LOS, and b_y^{2D} is the y component of the plane-of-the-sky magnetic field, which is proportional to b_y^{3D} integrated along the LOS. Note that, if we ignore the fluctuating magnetic field in the x direction, $B_{0,sky}^{2D}$ and b_y^{2D} are proportional to the x and y components of the plane-of-the-sky magnetic field, respectively, measured by a distant observer located on the z axis. Therefore, the ratio $b_y^{2D}/B_{0,sky}^{2D}$ determines the direction of the observed magnetic field on the plane of the sky. From (6), we can write

$$\frac{\delta b_y^{3D}}{B_{0,sky}^{3D}} = \delta \left(\frac{b_y^{3D}}{B_{0,sky}^{3D}} \right) = \delta \left(\frac{b_y^{2D}}{B_{0,sky}^{2D}} \right) = \frac{\delta b_y^{2D}}{B_{0,sky}^{2D}} = \delta (\tan \phi) \sim \delta \phi, \quad (7)$$

where ϕ is the angle between the plane-of-the-sky magnetic field and the mean plane-of-the-sky magnetic field ($\propto \mathbf{B}_{0,sky}^{2D}$) and $\delta \phi$ is its variation.

Since the polarized emission from magnetically aligned dust grains is perpendicular to the direction of the observed magnetic field on the plane of the sky, the angle ϕ in Equation (7) is virtually identical to the polarization angle with respect to the average polarization direction. Therefore, if we observe the variation of polarization angle (i.e. $\delta \phi$), we can obtain an estimate for the denominator in Equation (4).

In summary, the conventional CF method can be written by

$$B_{0,sky}^{3D} \sim \xi \sqrt{4\pi\rho} \frac{\delta v_{los}}{\delta \phi}, \quad (\leftarrow \text{for coherent waves}) \quad (8)$$

where ξ is a constant of order unity. Note that we have assumed that Alfvén waves are coherent (see Figure 1(b)) to obtain this relation.

2.2. A problem with the conventional CF method

In the previous subsection, we have assumed that Alfvén waves are all in phase. In this case, the conventional CF method should work fine. But, if either Alfvén waves have different phases or they form turbulence, then the conventional CF method needs modification.

Suppose that we have Alfvénic turbulence. Then, several large-scale eddies can exist along the LOS (see Figure 1(c)). It is natural to assume that physical quantities do not change much inside each large-scale eddy. For example, we may assume that the value of b_y^{3D} is roughly constant in each large-scale eddy. However, the direction of b_y^{3D} may change from eddy to eddy. Therefore, if we integrate b_y^{3D} along the LOS, we have

$$b_y^{2D} \propto \int b_y^{3D} dz \sim \delta b_y^{3D} L_f \sqrt{N_{eddy}}, \quad (9)$$

where N_{eddy} ($\approx L_{sys}/L_f$) is the number of large-scale eddies along the LOS and L_f is the driving scale of turbulence. Note that the typical size of the large-scale eddies is similar to the driving scale L_f .

Since the direction of the 3D mean magnetic field does not change along the LOS, integration of $B_{0,sky}^{3D}$ is different from that of b_y^{3D} :

$$B_{0,sky}^{2D} \propto \int B_{0,sky}^{3D} dz = B_{0,sky}^{3D} L_{sys} = B_{0,sky}^{3D} L_f N_{eddy}. \quad (10)$$

From Equations (9) and (10), we have

$$\frac{b_y^{2D}}{B_{0,sky}^{2D}} \sim \sqrt{N_{eddy}} \frac{b_y^{3D}}{B_{0,sky}^{3D}}, \quad (11)$$

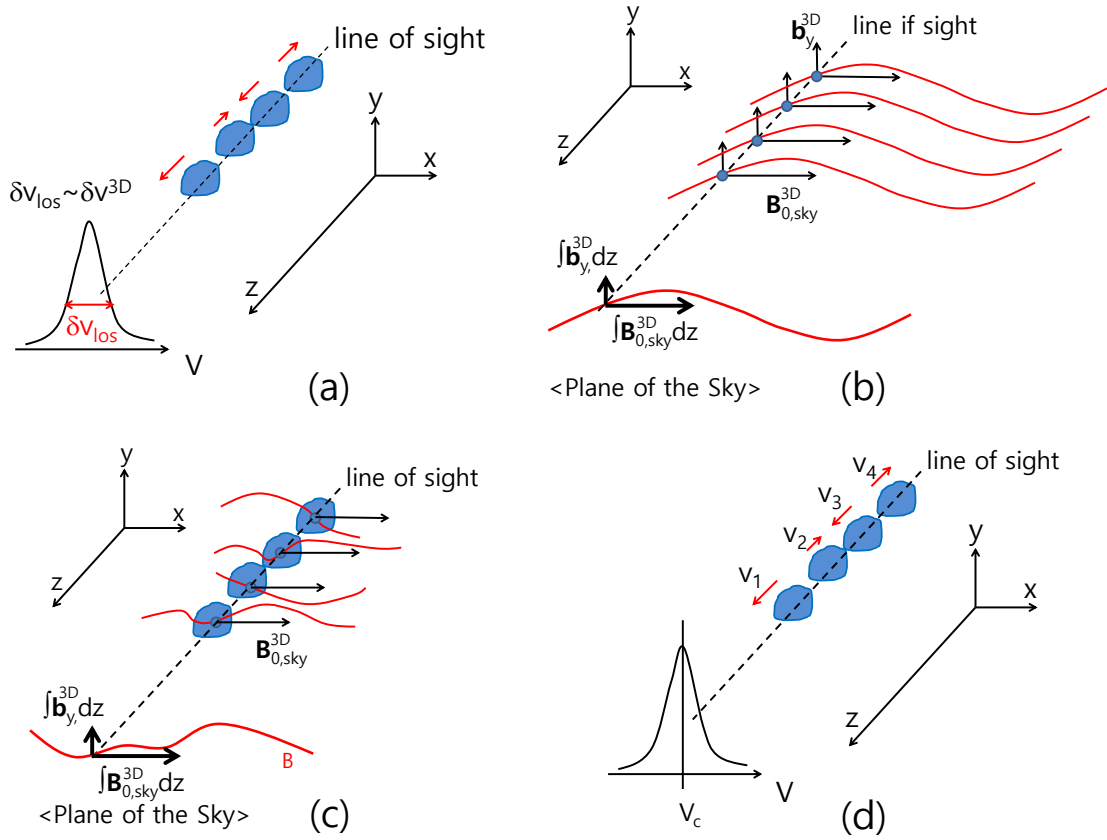


Figure 1. Observations of velocity and magnetic field. The line of sight (LOS) is along the z direction and the sky plane is parallel to the xy plane. For simplicity, we assume density is constant. (a) The width of an optically thin line is equal to the LOS velocity dispersion δv_{los} . (b) When Alfvén waves are all in phase, we have $b_y^{2D}/B_{0,sky}^{2D} = b_y^{3D}/B_{0,sky}^{3D}$, where b_y^{2D} and $B_{0,sky}^{2D}$ are magnetic fields integrated along the LOS. (c) When Alfvén waves are out of phase, we have $b_y^{2D}/B_{0,sky}^{2D} < b_y^{3D}/B_{0,sky}^{3D}$ due to random-walk-like behavior of b_y^{3D} . (d) The centroid velocity V_c is an average velocity. The area of a line profile on the left-hand side of V_c and that on the right-hand side of V_c are same.

which is obviously different from the relation for coherent waves in Equation (6). If we insert this relation into Equation (4), we get

$$\sqrt{N_{eddy}} B_{0,sky}^{3D} \sim \sqrt{4\pi\rho} \frac{\delta v_{los}}{\delta\phi}, \quad (\leftarrow \text{for turbulence}) \quad (12)$$

where we have replaced δv^{3D} with δv_{los} and used relations in Equation (7). Equation (12) implies that the conventional CF technique (see the right-hand side of the equation) overestimates $B_{0,sky}^{3D}$ by a factor of $\sqrt{N_{eddy}}$.

Another way to look at this problem is that, when there are many independent eddies along the LOS, $\delta\phi$ becomes reduced due to averaging effects, which makes the conventional CF method to overestimate $B_{0,sky}^{3D}$. This averaging effect, in fact, has been discussed by earlier researchers (see, for example, [6, 7, 8]).

2.3. Numerical tests of the conventional CF method

We use direct numerical simulations to test the conventional CF method (see [5] for details). We solve the following compressible MHD equations in a periodic box of size 2π using an Essentially Non-Oscillatory scheme (see [9]):

$$\partial\rho/\partial t + \nabla \cdot (\rho\mathbf{v}) = 0, \quad (13)$$

$$\partial\mathbf{v}/\partial t + \mathbf{v} \cdot \nabla\mathbf{v} + \rho^{-1}\nabla(C_s^2\rho) - (\nabla \times \mathbf{B}) \times \mathbf{B}/4\pi\rho = \mathbf{f}, \quad (14)$$

$$\partial\mathbf{B}/\partial t - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \quad (15)$$

with $\nabla \cdot \mathbf{B} = 0$ and an isothermal equation of state $P = C_s^2\rho$, where C_s is the sound speed and ρ is density. Here $\mathbf{v}$ is the velocity, $\mathbf{B}$ is the magnetic field, and $\mathbf{f}$ is the driving force. We use 512^3 grid points. In our simulations, $C_s = 0.1$, $\bar{\rho} = 1$, and $B_0/\sqrt{4\pi\bar{\rho}} = 1$. In all simulations, the r.m.s. velocity v_{rms} is between ~ 0.7 and ~ 0.8 , and the sonic Mach number is $M_s \equiv v_{rms}/C_s \sim 7$. Since the Alfvén speed of the mean field ($V_A = B_0/\sqrt{4\pi\bar{\rho}}$) is 1, the Alfvén Mach number is $M_A \equiv v_{rms}/V_A \sim 0.7$, which means that turbulence considered here is sub-Alfvénic.

Table 1. Simulations

Runs	Resolution	Sonic Mach number(M_s)	$B_{0,sky}^{3D}/\sqrt{4\pi\bar{\rho}}$	k_f	N_{eddy} ($= L_{sys}/L_f$)
KF3	512^3	~ 7	1	3	~ 3
KF5	512^3	~ 8	1	5	~ 5
KF10	512^3	~ 7	1	10	~ 10
KF20	512^3	~ 7	1	20	~ 20

We perform 4 different simulations with different driving wavenumbers (see Table 1). The driving wavenumbers are $k_f=3, 5, 10$, and 20 , respectively and the corresponding driving scales are $L_f=L_{sys}/3, L_{sys}/5, L_{sys}/10$, and $L_{sys}/20$, respectively. Note that the number of independent eddies along the LOS is $\sim 1/k_f$.

Except the driving scale, other conditions are virtually same (see Table 1). Using the MHD turbulence data and numerical method in Ref. [10] (see also [4]), we obtain synthetic polarization maps arising from magnetically-aligned dust grains at a far-infrared/sub-mm wavelength. Using the polarization maps, we calculate $\delta\phi$. We also calculate δv_{los} using the turbulence data. After getting $\delta\phi$ and δv_{los} , we calculate the quantity

$$\sqrt{4\pi\bar{\rho}} \frac{\delta v_{los}}{\delta\phi}, \quad (16)$$

which is proportional to $B_{0,sky}^{3D}$ in the conventional CF method. Since $B_0/\sqrt{4\pi\bar{\rho}} = 1$ in all our simulations, the above expression should be constant if the conventional CF method is correct. We plot the results in the left panel of Figure 2. As we can see in the figure, the conventional CF method overestimates $B_{0,sky}^{3D}$ as N_{eddy} ($\sim 1/k_f$) increases.

3. A Modified CF Method

3.1. Estimation of N_{eddy} and our modified CF method

Since the conventional CF method overestimates $B_{0,sky}^{3D}$ by a factor of $\sqrt{N_{eddy}}$, we need to know N_{eddy} to correct it. In this subsection, we show that variation of centroid velocity δV_c is

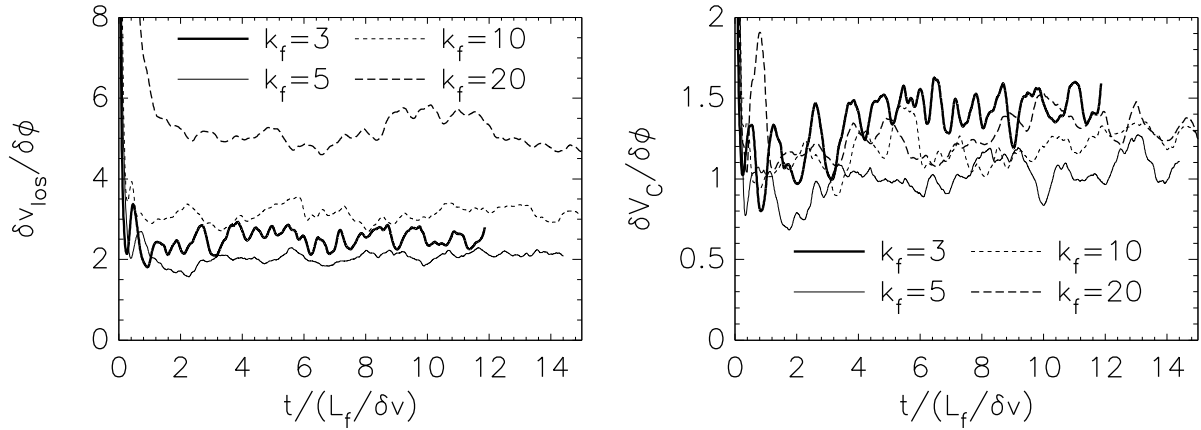


Figure 2. Estimates of $B_{0,sky}^{3D}/\sqrt{4\pi\rho}$, which is 1 in our simulations, from the conventional CF method (left panel) and our modified CF method (right panel). (Left) The conventional CF method overestimates $B_{0,sky}^{3D}$ if k_f is large. Note that, if turbulence is driven near k_f , there are $\sim k_f$ independent eddies along the LOS. (Right) Our modified CF method returns better estimates of $B_{0,sky}^{3D}$ even if k_f is large. From [5].

proportional to $\sqrt{N_{eddy}}$ (see [5] for details). The centroid velocity is a kind of average velocity. When we have an emission line with intensity profile $I(v)$, the centroid velocity is defined by

$$V_c = \int v_{los} I(v_{los}) dv_{los} / \int I(v_{los}) dv_{los}. \quad (17)$$

Suppose that there are N_{eddy} independent eddies along a LOS (see Figure 1(d)). Each eddy has typical LOS velocity $v_{los,i}$, which we denote as v_i for simplicity. Although v_i 's are not directly observable, we can consider the following sum for the LOS:

$$v_1 + v_2 + \dots + v_{N_{eddy}}. \quad (18)$$

Now, let us consider similar summations for many *different* LOS's. Since we have now many summations, we can calculate the standard deviation of the summations. Note that v_i 's show a random-walk-like behavior and the summation in Equation (18) corresponds to the net displacement of a one-dimensional random walk. Therefore, the standard deviation of the summations follows

$$\text{Standard deviation of } (v_1 + v_2 + \dots + v_{N_{eddy}}) \sim \sqrt{N_{eddy}} \delta v_{los}, \quad (19)$$

which becomes

$$\text{Standard deviation of } (v_1 + v_2 + \dots + v_{N_{eddy}})/N_{eddy} \sim \frac{\delta v_{los}}{\sqrt{N_{eddy}}}. \quad (20)$$

The left-hand side is equal to the standard deviation of the (arithmetic) average velocity. Centroid velocity is also a kind of average velocity. Therefore, we expect that the standard deviation of centroid velocity also follows a similar scaling:

$$\delta V_c \sim \frac{\delta v_{los}}{\sqrt{N_{eddy}}}. \quad (21)$$

From Equations (12) and (21), we have

$$B_{0,sky}^{3D} \sim \sqrt{4\pi\rho} \frac{\delta v_{los}}{\delta\phi} \frac{1}{\sqrt{N_{eddy}}} \sim \sqrt{4\pi\rho} \frac{\delta v_{los}}{\delta\phi} \frac{\delta V_c}{\delta v_{los}} \sim \sqrt{4\pi\rho} \frac{\delta V_c}{\delta\phi}. \quad (22)$$

Therefore, our modified CF method becomes

$$B_{0,sky}^{3D} = \xi' \sqrt{4\pi\rho} \frac{\delta V_c}{\delta\phi}, \quad (23)$$

where ξ' is a constant of order unity that can be determined by numerical simulations.

3.2. Numerical tests of the modified CF method

The right panel of Figure 2 shows our main result, in which we plot estimates of $B_{0,sky}^{3D}/\sqrt{4\pi\rho}$ from our modified CF method:

$$\frac{\delta V_c}{\delta\phi}. \quad (24)$$

In the panel, we can see that the estimates do not depend on the driving scale strongly and are fluctuating between ~ 1.0 and ~ 1.5 . Therefore, since $B_{0,sky}^{3D}/\sqrt{4\pi\rho} = 1$ in our simulations, the constant ξ' in Equation (23) is between ~ 0.7 and ~ 1 .

4. Discussions and Summary

In this paper, we have considered the effects of driving scale on the estimates of the mean plane-of-the-sky magnetic field $B_{0,sky}^{3D}$ from the Chandrasekhar-Fermi (CF) method. We have shown that the conventional CF method tends to overestimate $B_{0,sky}^{3D}$ if the driving scale is smaller than the system size and proposed a modified CF method that can correct the effect. The method we propose in Equation (23) with $0.7 < \xi' < 1.0$ does not require new observations [5]. That is, the method is readily applicable for present observational data. Apart from numerical constants, the only difference between our method and the conventional CF method is that our method requires the standard deviation of velocity centroids δV_c , while the conventional method requires average width of the emission line profiles δv_{los} . The standard deviation of velocity centroids δV_c can be easily obtained from existing optically-thin emission line profiles. If such emission line profiles ($I(v_{los})$'s) are available for n_{obs} lines of sight, then we need the following two steps to obtain δV_c :

- (i) We calculate the centroid velocity V_c (see Equation (17)) for each line of sight. Let $V_{c,i}$ be the centroid velocity for line of sight i :

$$V_{c,i} = \int v_{los} I_i(v_{los}) dv_{los} / \int I_i(v_{los}) dv_{los}, \quad (25)$$

where $I_i(v_{los})$ is the optically-thin emission line profile for the line of sight.

- (ii) We calculate δV_c from the formula

$$\delta V_c^2 \equiv \frac{1}{n_{obs}} \sum_{i=1}^{n_{obs}} V_{c,i}^2 - \left(\frac{1}{n_{obs}} \sum_{i=1}^{n_{obs}} V_{c,i} \right)^2. \quad (26)$$

In this paper we have demonstrated that the conventional CF method indeed overestimates the mean plane-of-the-sky magnetic field $B_{0,sky}^{3D}$ by a factor of $\sqrt{N_{eddy}}$, where N_{eddy} is the number of independent eddies along the line of sight. We have found that the standard deviation of centroid velocities divided by the average line-of-sight velocity dispersion ($\delta V_c/\delta v_{los}$) is proportional to $1/\sqrt{N_{eddy}}$ (Equation (21)). Therefore Equation (23) with $\xi' = 0.7 \sim 1$ provides a better estimate for $B_{0,sky}^{3D}$.

References

- [1] Chandrasekhar S and Fermi E 1953 *Astrophys. J.* **118** 113
- [2] Ostriker E C, Stone J M and Gammie C F 2001 *Astrophys. J.* **546** 980–1005 (*Preprint astro-ph/0008454*)
- [3] Padoan P, Goodman A, Draine B T, Juvela M, Nordlund Å and Rognvaldsson Ö E 2001 *Astrophys. J.* **559** 1005–1018 (*Preprint astro-ph/0104231*)
- [4] Heitsch F, Zweibel E G, Mac Low M M, Li P and Norman M L 2001 *Astrophys. J.* **561** 800–814 (*Preprint astro-ph/0103286*)
- [5] Cho J and Yoo H 2016 *Astrophys. J.* **821** 21 (*Preprint 1603.08537*)
- [6] Myers P C and Goodman A A 1991 *Astrophys. J.* **373** 509–524
- [7] Zweibel E G 1996 *Polarimetry of the Interstellar Medium (Astronomical Society of the Pacific Conference Series vol 97)* ed Roberge W G and Whittet D C B p 486
- [8] Houde M, Vaillancourt J E, Hildebrand R H, Chitsazzadeh S and Kirby L 2009 *Astrophys. J.* **706** 1504–1516 (*Preprint 0909.5227*)
- [9] Cho J and Lazarian A 2002 *Physical Review Letters* **88** 245001 (*Preprint astro-ph/0205282*)
- [10] Fiege J D and Pudritz R E 2000 *Astrophys. J.* **544** 830–837 (*Preprint astro-ph/0005363*)