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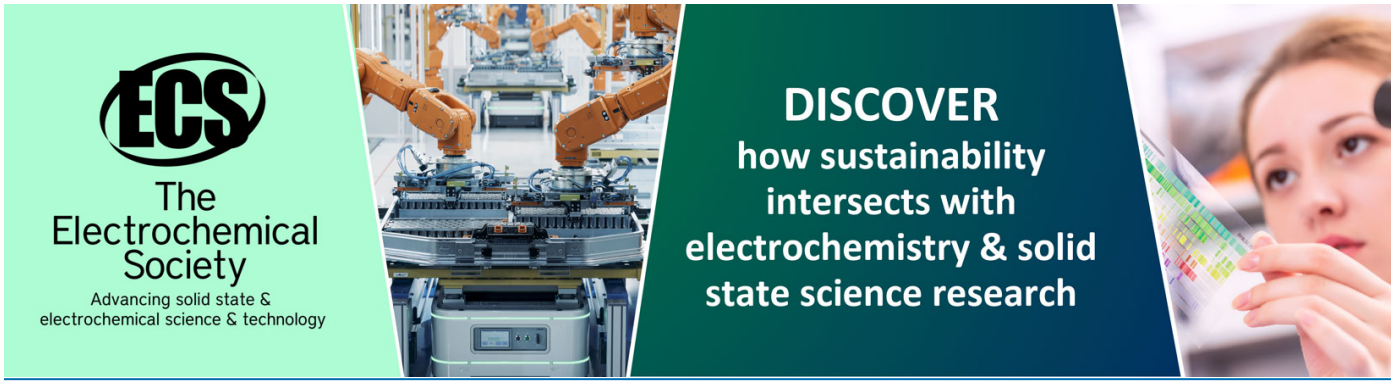

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Alpha Decay Width of ^{212}Po from a quartetting wave function approach

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Abstract. An effective α particle equation is derived for cases where an α particle is bound to a doubly magic nucleus. As an example, we consider ^{212}Po with the α on top of the ^{208}Pb core. The fully quantal solution of the problem is inspired by the THSR (Tohsaki-Horiuchi-Schuck-Röpke) wave function concept that has been successfully applied to light nuclei. Shell model calculations are improved by including four-particle (α-like) correlations that are of relevance when the matter density becomes low. In the region where the α-like cluster penetrates the core nucleus, the intrinsic bound state wave function transforms at a critical density into an unbound four-nucleon shell model state. We present a microscopic calculation of both α-cluster preformation probability and decay width in typical α-emitter ^{212}Po. Using actually measured density distribution of the ^{208} Pb core, the calculated alpha decay width of ^{212}Po agrees fairly well with the measured one. Applications to other nuclei (^{20}Ne) are discussed.

1. Introduction

The radioactive α decay is known for a long time in nuclear physics. In contrast to extensive and precise experimental data, see [1] and refs. given there, the theoretical description needs some semiempirical assumptions at present. Whereas the tunneling of an α particle across the Coulomb barrier is well described in quantum physics, the problem in understanding the α decay within a microscopic approach is the preformation of the α cluster in the decaying nucleus.

Recently many attempts are performed to investigate α-like correlations in nuclear systems. For nuclear matter, it has been shown that α-like correlations are possible in the low-density region, but are dissolved because of the Pauli blocking around nucleon density $n_B = 0.03$ fm^{-3} [2]. Such low-density nuclear systems occur, for instance, in excited selfconjugate light nuclei

(^{8}Be, ^{12}C, ^{16}O, ^{20}Ne). Four-nucleon correlations have been identified successfully within the THSR approach [3], see also [4]. Recently, α-like correlations have been investigated also in the outer tails of the density of a nucleus where the nucleon density is below 0.03 fm^{-3} [5, 6].

2. Four nucleons moving in a mean field

We consider a quantum many-particle approach. The propagation of a quartet $(n_\uparrow, n_\downarrow, p_\uparrow, p_\downarrow)$ is described by a four-particle Green function which leads to the in-medium Schrödinger equation

$$[E_4 - \hat{h}_1 - \hat{h}_2 - \hat{h}_3 - \hat{h}_4]\Psi_4(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\mathbf{r}_4) = \int d^3\mathbf{r}'_1\, d^3\mathbf{r}'_2 \langle \mathbf{r}_1\mathbf{r}_2|B\; V_{N-N}|\mathbf{r}'_1\mathbf{r}'_2\rangle \Psi_4(\mathbf{r}'_1\mathbf{r}'_2\mathbf{r}_3\mathbf{r}_4)$$
$$+ \int d^3\mathbf{r}'_1\; d^3\mathbf{r}'_3 \langle \mathbf{r}_1\mathbf{r}_3|B\; V_{N-N}|\mathbf{r}'_1\mathbf{r}'_3\rangle \Psi_4(\mathbf{r}'_1\mathbf{r}_2\mathbf{r}'_3\mathbf{r}_4) + \text{four further permutations.} \quad (1)$$

The single-nucleon operator $\hat{h}_1$ contains the kinetic energy and the potential energy of an external mean field, produced by the other nucleons which are not under consideration. The six nucleon-nucleon interaction terms contain besides the nucleon-nucleon potential V_{N-N} also the blocking operator B which can be given in quasi-particle state representation. For the first term on the r.h.s. of Eq. (1), the expression $B(1,2) = [1 - f_1(\hat{h}_1) - f_2(\hat{h}_2)]$ results. The phase space occupation $f_\nu(\hat{h}) = \sum_n^{\text{occ.}} |n,\nu\rangle\langle n,\nu|$ indicates the phase space which is occupied and therefore according to the Pauli principle not available for an interaction process. In Ref. [5], the Tamm-Dancoff expression $[1 - f_1(\hat{h}_1)][1 - f_2(\hat{h}_2)]$ is used which neglects pairing.

It is advantageous to introduce new variables, the c.o.m. position $\mathbf{R} = \sum_i^4 \mathbf{r}_i/4$ and the relative coordinates $\mathbf{s}_j$, $j = 1\ldots3$, in particular Jacobian coordinates. It has been shown [5] that we can subdivide the wave function $\Psi(\mathbf{R}, \mathbf{s}_j)$ into two parts in an unique way,

$$\Psi(\mathbf{R}, \mathbf{s}_j) = \varphi^{\text{intr}}(\mathbf{s}_j, \mathbf{R})\, \Phi(\mathbf{R}). \quad (2)$$

A coupled system of wave equations is obtained describing the c.o.m. motion

$$-\frac{\hbar^2}{8m}\nabla_R^2\Phi(\mathbf{R}) - \frac{\hbar^2}{4m}\int ds_j \varphi^{\text{intr},*}(\mathbf{s}_j,\mathbf{R})[\nabla_R\varphi^{\text{intr}}(\mathbf{s}_j,\mathbf{R})][\nabla_R\Phi(\mathbf{R})]$$
$$-\frac{\hbar^2}{8m}\int ds_j \varphi^{\text{intr},*}(\mathbf{s}_j,\mathbf{R})[\nabla_R^2\varphi^{\text{intr}}(\mathbf{s}_j,\mathbf{R})]\Phi(\mathbf{R}) + \int dR'\, W(\mathbf{R},\mathbf{R}')\Phi(\mathbf{R}') = E\,\Phi(\mathbf{R}) \quad (3)$$

and a similar equation for the intrinsic motion.

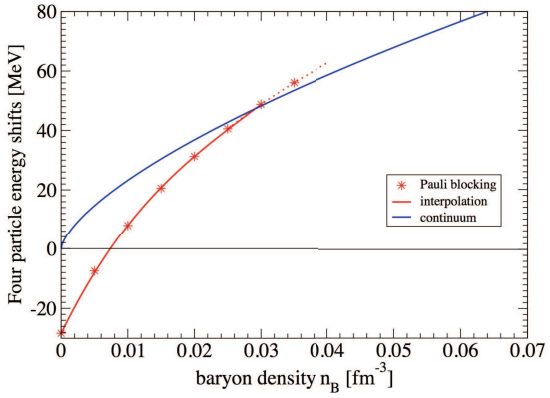

Figure 1. Pauli blocking: four-nucleon continuum edge (full line) and α-like bound state (stars) as function of n_B (homogeneous nuclear matter). The bound state energy of the α particle is shifted with increasing density n_B of the surrounding matter and merges with the four-nucleon continuum $\mu_4 = 4E_F = 2\hbar^2(3\pi^2 n_B/2)^{2/3}/m$ at the critical density $n_B^{\text{cr}} \approx 0.03$ fm^{-3}.

The c.o.m. potential

$$W(\mathbf{R},\mathbf{R}') = \int ds_j\, ds'_j\, \varphi^{\text{intr},*}(\mathbf{s}_j,\mathbf{R})\left[T[\nabla_{s_j}]\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{s}_j-\mathbf{s}'_j) + BV(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)\right]\varphi^{\text{intr}}(\mathbf{s}'_j,\mathbf{R}')$$

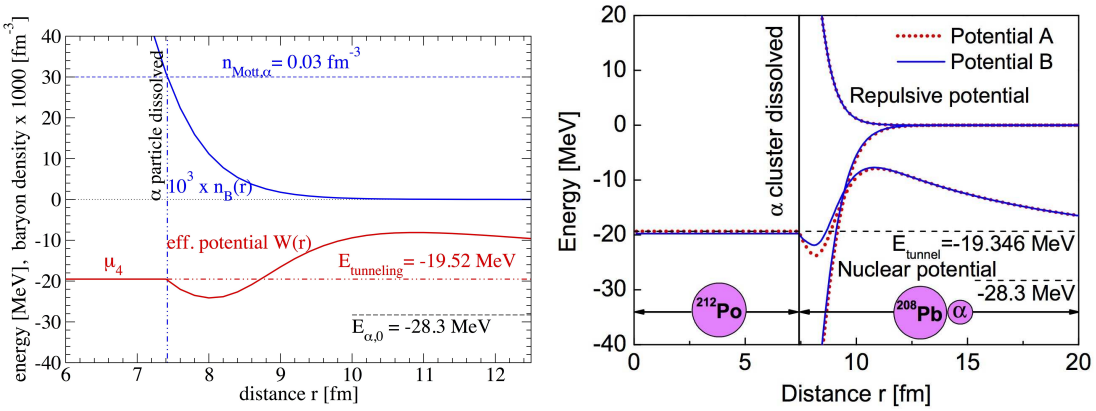

Figure 2. Potential for the c.o.m. motion of the four-nucleon system on top of the Pb core. Left: core nucleus density and $r_{\rm cr}$, Thomas-Fermi condition (Potential A). Right: different contributions to the c.o.m. potential (A and B).

contains the external (mean-field) contribution $V_\tau^{\rm mf}(r)$ and the intrinsic energy of the α-like cluster. Neglecting gradient terms in (3), the main problem to solve the intrinsic energy is the Pauli blocking which is in general non-local.

Within a local-density approximation, results for homogeneous nuclear matter can be used; see Fig. 1. The intrinsic wave function with lowest energy changes from an α-like bound state to a product of quasi-free single nucleon states at the critical density $n_B^{\rm cr} \approx 0.03$ fm^{-3}.

3. Results for ^{212}Po

Following [5, 6], we consider ^{212}Po which decays into a ^{208}Pb core and an α particle, half-life 0.299 μs and decay energy $Q_\alpha = 8954.13$ keV. The core nucleons are distributed with the baryon density $n_B(r)$ [7] and produce a mean field $V_\tau^{\rm mf}(r)$ acting on the two neutrons ($\tau = n$) and two protons ($\tau = p$) moving on top of the lead core. In addition, $n_B(r)$ is responsible for the Pauli blocking shift shown in Fig. 1. The critical density $n_B = 0.03$ fm^{-3} occurs at $r_{\rm cr} = 7.4383$ fm. This means that α-like clusters can exist only at distances $r > r_{\rm cr}$, for smaller values of r the intrinsic wave function is characterized by the uncorrelated motion. The mean-field potential of the ^{208}Pb core is obtained as the M3Y double-folding potential, see [8], in addition to the Coulomb interaction.

An important issue is that we select four nucleons which are described microscopically and can form an α like bound state. The interaction with the ^{208}Pb core is taken as mean field so that correlations between the nucleons of the core and the four quartet nucleons are neglected. Furthermore, we will not give a microscopic description of the core nucleons (e.g., Thomas-Fermi or shell model calculations) but consider both $n_B(r)$ and $V_\tau^{\rm mf}(r)$ as phenomenological input.

The local c.o.m. potential $W(r)$ is shown in Fig. 2. To derive the the M3Y double-folding potential, a parameterized nucleon-nucleon effective interaction $v(s) = c\exp(-4s)/(4s) - d\exp(-2.5s)/(2.5s)$ has been used, describing a short-range repulsion (c) and a long-range attraction (d); s denotes the nucleon-nucleon distance. At $r = r_{\rm cr}$, the four nucleons are at the Fermi energy μ_4, see Fig. 1. Within the Thomas-Fermi model, the chemical potential μ_4 for the free nucleon gas inside the core nucleus is constant, not depending on r.

Solving a Schrödinger equation describing the c.o.m. motion with the potential $W(r)$, the energy eigenvalue E_4 of the quartet is obtained. The c.o.m. wave function is found, the preformation factor is calculated from the probability to find the quartet c.o.m. position at $r > r_{\rm cr}$. In that region, α particles are formed. The tunneling rate and the half-life are obtained

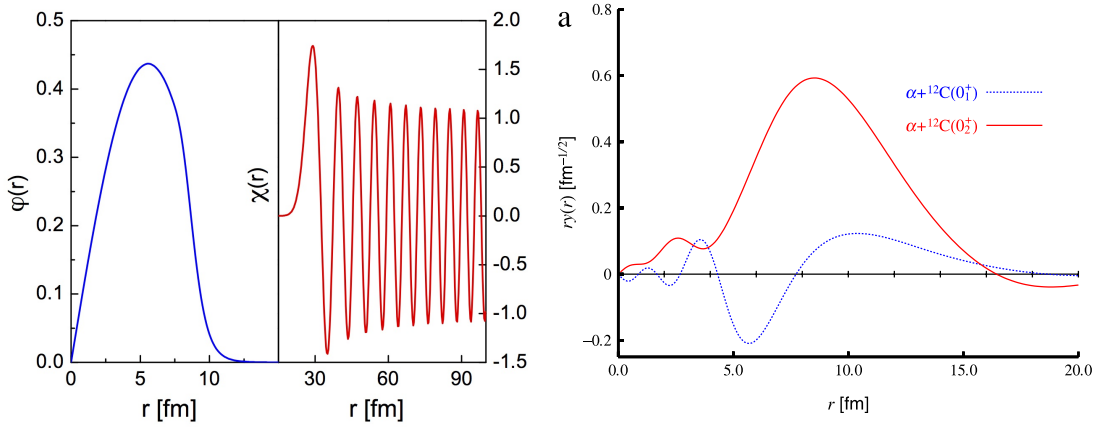

Figure 3. Left panel: c.o.m. wave function for the α particle in ^{212}Po [6]. Right: c.o.m. wave function for the α particle in ^{16}O [4].

in the standard way, see [6].

Table 1. The calculated preformation probability and decay half-life of ^{212}Po using different sets of effective c.o.m. potentials.

Pot.	c [MeV fm]	d [MeV fm]	$E_{\rm tunnel}$ [MeV]	Fermi en. μ_4[MeV]	$E_{\rm tunnel}-\mu_4$ [MeV]	Pref. fact. P_α	Decay half-life $T_{1/2}$[s]
A	13866.30	4090.51	-19.346	-19.346	0	0.367	2.91×10^{-8}
B	11032.08	3415.56	-19.346	-19.771	0.425	0.142	2.99×10^{-7}

Two sets of parameters c, d have been used as given in Tab. 1. For both sets A, B, the parameter values c, d reproduce the measured tunneling energy $E_4 = -19.52$ MeV. Furthermore, set A fulfills the condition $E_4 = \mu_4$ of the Thomas-Fermi model. The result for the decay half-life $T_{1/2}$ is too short. Instead, set B is constructed to reproduce the measured decay half-life. Then, the position of E_4 is above the Fermi energy μ_4 of the core nuclei. Note that a gap is expected to occur in a shell model when adding the four nucleons on top of the double magic core nucleus.

4. THSR and ^{20}Ne

Several approximations have been made which should be discussed and improved, if possible. We neglected gradient terms so that our approach is close to the local-density approximation. Within a rigorous approach, the c.o.m. potential $W(\mathbf{r}, \mathbf{r}')$ is non-local. A full treatment of the inhomogeneous case relevant for finite nuclei, which includes the gradient terms and the non-local potentials, is a future goal, presently not in reach.

Furthermore the simple TF model for the core nucleus has to be improved by shell model calculations. To be consistent, also few-particle correlations for the core nucleons have to be considered, not only for the quartet on top of the core.

The THSR wave function concept is a systematic approach to treat few-body correlations between all nucleons. It has been successfully applied to light nuclei. Our quartetting wave function approach is inspired by the THSR concept so that we have to compare with respect to the treatment of few-nucleon correlations going beyond an uncorrelated quasiparticle model. However, because of complexity THSR calculations are restricted to small numbers of nucleons.

At present, THSR calculations have been performed up to ^{20}Ne [9, 10]. Calculations for large nuclei such as ^{212}Po are not in reach.

The nucleus ^{20}Ne can also be considered as a quartet on top of a double magic nucleus ^{16}O as core nucleus. The investigation of this nucleus opens the possibility to compare our quartetting wave function approach with the THSR approach. Problems concerning different approximations inherent in both approaches can be analyzed. First exploratory calculations using the known density distribution of ^{16}O give an value $r_{\rm cr} = 3.302$ fm, an α-like bound state can exist only for $r > r_{\rm cr}$. To calculate the mean-field potential we have to fit the parameter c, d occurring in the M3Y-type effective interaction. This can be done using the difference of the binding energy of ^{20}Ne and ^{16}O (binding energy of the α particle). Assuming the Thomas-Fermi model condition $E_4 = \mu_4$ discussed above, the c.o.m. wave function can be calculated. The corresponding rms radii can be compared with the respective empirical values of both nuclei. Preliminary calculations give reasonable results.

Of interest is the comparison of the c.o.m. wave function for the quartet in the different approaches, see Fig. 3. The typical behavior of the quartetting wave function approach gives a nearly linear increase up to $r_{\rm cr}$, after this a continuous decrease. THSR calculations show also a nearly linear increase, the deviations are related to the internal structure of the core. The comparison with THSR calculations would lead to a better understanding of the microscopic calculations, in particular the c.o.m. potential, the c.o.m. wave function, and the preformation factor.

Note that the position of the chemical potential determines the preformation probability. If $\mu_4 < E_{\rm tunnel}$, the preformation probability becomes smaller. If $\mu_4 > E_{\rm tunnel}$, the preformation probability becomes larger. In the latter case, the states at the Fermi surface are also correlated. This is the case, e.g., for the Hoyle state where all core nucleons are found in correlated states.

5. Conclusions

- α-like clustering occurs in nuclear systems in the region of low density. Examples are the density tails near the surface of heavy nuclei, e.g. ^{212}Po.
- A consistent quantum statistical approach is needed to describe the formation of four-nucleon correlations and its dissolution at increasing nucleon density. We present an approach where a quartet moves under the influence of the core nucleus which is treated as a mean field.
- A simple description of the core nucleus is possible within a local density approach. It has to be improved taking into account density gradients and non-locality of Pauli blocking. An essential issue for future research is the treatment of correlations within the core nucleus.
- The investigation of ^{20}Ne where the full treatment of four-nucleon correlations within the THSR approach has been done, would be of high interest to discuss α preformation and to describe the α-like particle on top of the ^{16}O core [9].

References

[1] Delion D S, Liotta R J, and Wyss R 2015 *Phys. Rev.* C **92** 051301(R)
[2] Röpke G, Schnell A, Schuck P and Nozières P 1998 *Phys. Rev. Lett.* **80** 3177
[3] Tohsaki A, Horiuchi H, Schuck P and Röpke G 2001 *Phys. Rev. Lett.* **87** 192501
[4] FunakiY, Horiuchi H and Tohsaki A 2015 *Progr. Part. Nucl. Phys.* **82** 78
[5] Röpke G, Schuck P, Funaki Y, Horiuchi H, Zhongzhou Ren, Tohsaki A, Chang Xu, Yamada T, and Bo Zhou 2014 *Phys. Rev.* C **90** 034304
[6] Xu C *et al* 2016 *Phys. Rev.* C **93** 011306(R)
[7] Tarbert C M *et al* 2014 *Phys. Rev. Lett.* **112** 242502
[8] Misicu S and Esbensen H 2007 *Phys. Rev.* C **75** 034606; **76** 054609
[9] Bo Zhou *et al* 2012 *Phys. Rev.* C **86** 014301
[10] Bo Zhou *et al* 2014 *Phys. Rev.* C **89** 034319