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Structure of $4n$ nuclei in a formalism of quartets

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Abstract. We show that low-energy spectra of $4n$ nuclei in the sd shell can be described with high accuracy in a formalism of four-body correlated structures (quartets). The states of all $N \geq Z$ nuclei belonging to the $A = 24$ isobaric chain are represented as a superposition of two-quartet states, with quartets being characterized by isospin T and angular momentum J . These quartets are assumed to be those describing the lowest states in ^{20}Ne ($T_z=0$), ^{20}F ($T_z=1$) and ^{20}O ($T_z=2$). We find, in particular, that the spectrum of the self-conjugate nucleus ^{24}Mg can be well reproduced in terms of $T=0$ quartets only and that, among these, the $J=0$ quartet plays by far the leading role in the structure of the ground state. These results highlight the importance of quartetting in the structure of $4n$ nuclei.

1. Introduction

Self-conjugate nuclei possess the unique property of carrying an equal number of protons and neutrons distributed over the same single particle orbits. In these nuclei, owing to the charge independence of the nuclear interaction, the isovector proton-neutron (pn) pairing is expected to come into play on equal footing as the like-particle proton-proton and neutron-neutron pairing of the more common $N > Z$ nuclei. In addition, pn pairing is also expected to occur in an isoscalar form. The competition between these two types of pn pairing in $N = Z$ nuclei has been matter of great debate in recent years [1].

We have recently carried out an analysis of the pn pairing in even-even $N = Z$ nuclei both in the isovector and in the isoscalar channels [2, 3, 4, 5]. This analysis has evidenced, on the one hand, that a description of the ground state correlations induced by this interaction in terms of a condensate of collective pairs (of various form [5]) is not satisfactory and, on the other hand, that these correlations can be accounted for to a high degree of precision by approximating the ground state as a product of identical $T = 0$ quartets. $T = 0$ quartets are four-body correlated structures formed by two protons and two neutrons that, in the case of a spherical mean field, are also characterized by a total angular momentum $J = 0$. We have also explored a more sophisticated approximation which consists in letting the quartets to be all distinct and we have verified that it leads to basically exact results in the case of pn isovector pairing in deformed systems [3]. In all cases the quartets have been constructed variationally for each nucleus.

The pn pairing is for sure a key ingredient of the nuclear force for $N = Z$ nuclei but it is nonetheless only a part of it. In the presence of a full Hamiltonian, other quartets are reasonably expected to come into play besides the $T = 0, J = 0$ ones emerging from the analysis of the pn pairing. In the following we will describe how the approach of Refs. [2, 3, 4, 5] has been



extended to include quartets with arbitrary values of isospin and angular momentum and to treat realistic interactions. As an application of the method we will provide a description of some *sd* shell nuclei.

The criterion adopted for the selection of the quartets has been that of choosing, as representative of the quartets with a given isospin T , those describing the lowest levels with that isospin in nuclei with four active particles outside the inert core of reference [6]. For applications within the *sd* shell, the inert core is represented by ^{16}O and the nuclei which have therefore been considered for the definition of the quartets are $^{20}_{10}\text{Ne}_{10}$, $^{20}_9\text{F}_{11}$ and $^{20}_8\text{O}_{12}$. The lowest states of these nuclei are characterized by $T=0, 1$ and 2 , respectively. Each of these states therefore identifies a quartet with given T, J (and a projection $T_z = (N - Z)/2$). We have carried out SM calculations for these three nuclei and, as initial sets of quartets, we have selected those formed by the lowest 6 states in ^{20}Ne ($0 \leq J \leq 6$), the lowest 5 states in ^{20}F ($1 \leq J \leq 5$) and the lowest 9 states in ^{20}O ($0 \leq J \leq 4$). The mixing amplitudes defining each collective quartet have resulted from these SM calculations. As it will be seen below, we have also explored reductions of these sets to identify the most relevant quartets in the structure of the nuclei under investigation. Throughout these calculations we have employed the USDB interaction [7].

2. Formalism

We work in a spherically symmetric mean field and label the single-particle states by $i \equiv \{n_i, l_i, j_i\}$, where the standard notation for the orbital quantum numbers is used. The quartet creation operator is defined as

$$Q_{\alpha, JM, TT_z}^+ = \sum_{i_1 j_1 J_1 T_1} \sum_{i_2 j_2 J_2 T_2} C_{i_1 j_1 J_1 T_1, i_2 j_2 J_2 T_2}^{(\alpha)} \times \left[[a_{i_1}^+ a_{j_1}^+]^{J_1 T_1} [a_{i_2}^+ a_{j_2}^+]^{J_2 T_2} \right]_{MT_z}^{JT}, \quad (1)$$

where a_i^+ creates a fermion in the single particle state i and $J(T)$ and $M(T_z)$ denote, respectively, the total angular momentum(isospin) and the relative projections. No restrictions on the intermediate couplings $J_1 T_1$ and $J_2 T_2$ are introduced in the calculations. In order to generate the spectra of $4n$ nuclei we perform configuration interaction calculations in spaces built in terms of selected sets of the above quartets.

3. Results

We start our analysis by examining the self-conjugate nucleus ^{24}Mg . In Fig. 1, we compare the experimental spectrum of this nucleus with that resulting from a shell model (SM) calculation and with the spectrum obtained in the quartet model (QM) when all the selected $T=0, 1, 2$ quartets are taken into account. The quartet approach is seen to reproduce well both the SM ground state correlation energy and the SM excited states up to an energy of about 9 MeV. The SM is in turn able to fit well the experimental levels.

Having verified that the selected sets of $T=0, 1, 2$ quartets are sufficient to describe the low-energy spectrum of ^{24}Mg , it becomes of interest to investigate the role of the different quartets. We begin by focusing on isospin. In Fig. 1, on the right hand side, we show the theoretical spectrum obtained in the quartet formalism when only $T=0$ quartets are retained. On top of the lowest levels we also show the overlaps with the corresponding SM eigenstates. One can see that this spectrum does not exhibit relevant differences with respect to the full QM calculation and that the above overlaps are pretty large, what confirms the good quality of the QM($T=0$) wave functions. This result provides a clear evidence of the marginal role played by the $T=1$ and $T=2$ quartets in the structure of these states.

As a next step, we concentrate on the ground state by employing only $T=0$ quartets. In Fig. 2, we show how the error in the correlation energy of this state, relative to the SM result,

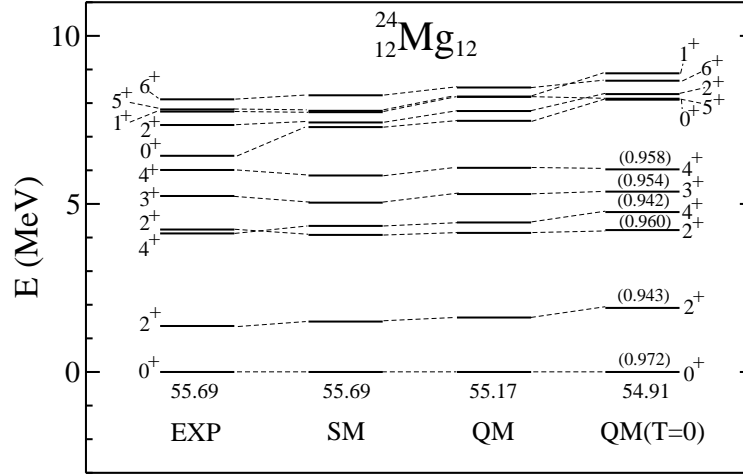


Figure 1. Spectrum of ^{24}Mg obtained in the quartet model (QM) compared to experimental data (EXP) and shell model (SM) results. QM(T=0) denotes the results obtained only with T=0 quartets; the numbers on top of these levels are the overlaps between the QM and SM eigenfunctions. The number below each spectrum gives the ground state correlation energy, namely the difference between the total ground state energy and the energy in the absence of interaction.

varies by reducing, one quartet at a time and starting from the highest one in energy, the set of $T=0$ quartets. The correlation energy remains basically unchanged up to the point where only the lowest $J=0,2,4$ quartets are left. From this point on further reductions in the set of quartets lead to significant variations in the energy. The overlaps between the SM and QM ground states which are shown in the same figure exhibit a behavior which is consistent with that of the energies. Thus, these calculations indicate that the $T=0$ quartets with $J=0,2,4$ play a major role in the ground state of ^{24}Mg . Among these quartets, the $T=0$, $J=0$ quartet is by far the one which contributes most to the correlation energy since, as it can be seen in Fig. 2, an approximation in terms of only this quartet accounts for about 94% of the total energy. A very similar conclusion has emerged also from an analysis of the ground state of the three-quartet system ^{28}Si [6].

So far we have discussed only self-conjugate nuclei. The range of nuclei which are accessible in terms of a set of $T=0,1,2$ quartets is, however, much broader. Limiting ourselves to the case of 8 active particles (two quartets) outside the ^{16}O core, the whole isobaric chain of nuclei with $A=24$ is within reach. We have investigated to what extent the spectra of these nuclei can be described in terms of the full set of $T=0,1,2$ quartets employed in the case of ^{24}Mg . The results can be seen in Ref. [6] where we compare the low-energy spectra of $^{24}_{11}\text{Na}_{13}$, $^{24}_{10}\text{Ne}_{14}$, $^{24}_9\text{F}_{15}$ and $^{24}_8\text{O}_{16}$ obtained in the quartet model with the SM results and with the experimental data. As one can see, for all these nuclei the quartet formalism generates spectra which agree well with the SM ones. It is worth stressing that for the nuclei shown in Fig. 3 the quartets are built not only by two protons and two neutrons, as in the case of $N = Z$ nuclei, but also by one proton and three neutrons and by four neutrons. The case of quartets built by four like-particles had

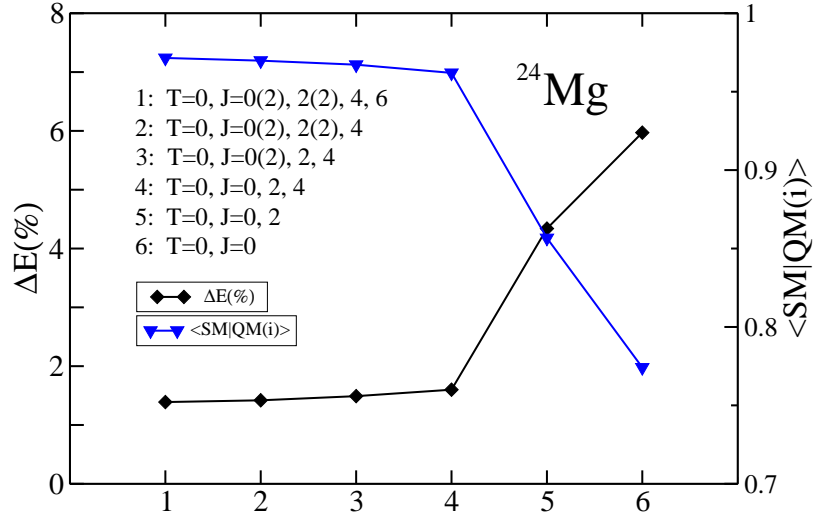


Figure 2. Relative errors (with respect to the SM value) in the ground state correlation energy of ^{24}Mg obtained within the QM with the sets of quartets indicated in the figure. For each set we also show the overlaps between SM and QM wave functions.

been already discussed in Ref. [8] in relation with a treatment of the pairing Hamiltonian.

4. Conclusions

We have provided a description of $4n$ nuclei in a formalism of quartets, i.e., four-body correlated structures characterized by total isospin T and total angular momentum J . We have analyzed in particular the whole isobaric chain of $A = 24$ nuclei. For all these nuclei the quartet formalism has provided a description of the low-energy spectra comparable in accuracy with that of shell model calculations. This fact confirms the importance of quartet degrees of freedom in any type of $4n$ nuclei and validates the present quartet formalism as the appropriate tool for treating them.

Acknowledgments

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