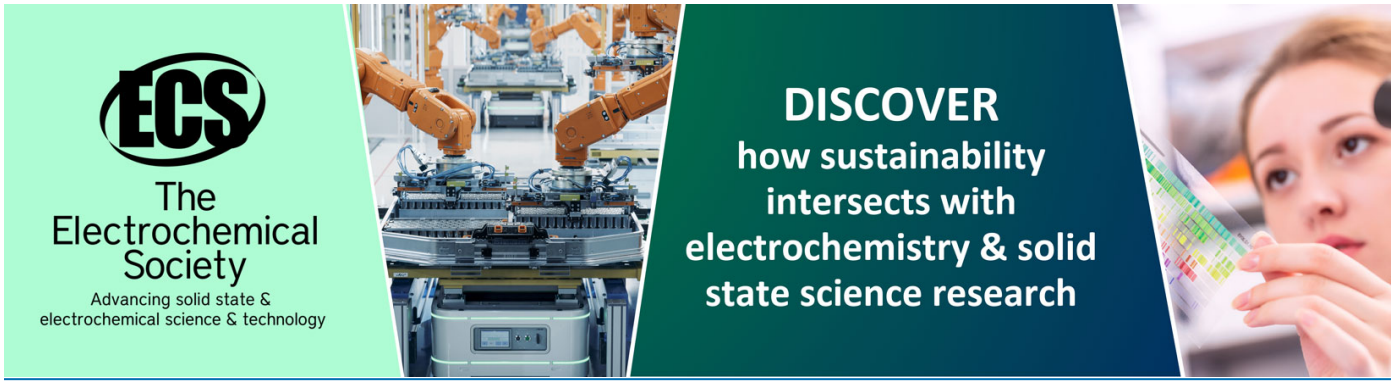
# Application of Adjusted Canonical Correlation Analysis (ACCA) to study the association between mathematics in Level 1 and Level 2 and performance of engineering disciplines in Level 2





# Application of Adjusted Canonical Correlation Analysis (ACCA) to study the association between mathematics in Level 1 and Level 2 and performance of engineering disciplines in Level 2

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**Abstract**. Mathematics plays a key role in engineering sciences as it assists to develop the intellectual maturity and analytical thinking of engineering students and exploring the student academic performance has received great attention recently. The lack of control over covariates motivates the need for their adjustment when measuring the degree of association between two sets of variables in Canonical Correlation Analysis (CCA). Thus to examine the individual effects of mathematics in Level 1 and Level 2 on engineering performance in Level 2, two adjusted analyses in CCA: Part CCA and Partial CCA were applied for the raw marks of engineering undergraduates for three different disciplines, at the Faculty of Engineering, University of Moratuwa, Sri Lanka. The joint influence of mathematics in Level 1 and Level 2 is significant on engineering performance in Level 2 irrespective of the engineering disciplines. The individual effect of mathematics in Level 2 is significantly higher compared to the individual effect of mathematics in Level 1 on engineering performance in Level 2. Furthermore, the individual effect of mathematics in Level 1 can be negligible. But, there would be a notable indirect effect of mathematics in Level 1 on engineering performance in Level 2. It can be concluded that the joint effect of mathematics in both Level 1 and Level 2 is immensely beneficial to improve the overall academic performance at the end of Level 2 of the engineering students. Furthermore, it was found that the impact mathematics varies among engineering disciplines. As partial CCA and partial CCA are not widely explored in applied work, it is recommended to use these techniques for various applications.

## 1. Introduction

The studies on the factors that influence students academic performance has received great attention among researchers. Several researchers have stated the importance of mathematical knowledge for engineering students to develop their analytical thinking [1-3]. A study by [4] revealed that mathematics in Level 1 is significantly influenced on students' overall academic performance in Level 2 irrespective of the seven engineering disciplines at the Faculty of Engineering and the impact of mathematics varies among engineering disciplines. This study is therefore to determine the individual effect of mathematics in both Level 1 and Level 2 separately on engineering performance in Level 2.

## 2. Materials and Methods

### *2.1. Data Description*

The study was conducted with engineering undergraduates from three different disciplines namely: Civil Engineeering (CE), Mechanical Engineering (ME) and Electronic and Telecommunications Engineering (EN) at the Faculty of Engineering, University of Moratuwa, Sri Lanka for the academic year, 2011/2012. Students' examination marks of mathematics modules in Level 1 (i.e. semester 1 (S1) and semester 2 (S2)) as well as Level 2 (i.e. semester 3 (S3) and semester 4 (S4)) and all compulsory engineering modules in Level 2 were used. Table 1 presents the mathematics modules followed in each semester in Level 1 and Level 2.

**Table 1.** Mathematics modules in Level 1 and Level 2.

| Academic Level | Semester | Module Code | Module Name |
|---|---|---|---|
| Level 1 | S1 | MA1013 | Mathematics |
| | S2 | MA1023 | Methods of Mathematics |
| Level 2 | S3 | MA2013 | Differential Equation |
| | | MA2023 | Calculus |
| | S4 | MA2033 | Linear Algebra |
| | | MA2053 | Graph Theory |
| | | MA3013 | Applied Statistics |

### *2.2. Unadjusted and Adjusted Canonical Correlation Analysis (CCA)*

In this study unadjusted CCA and adjusted CCA: partial CCA [5] and part CCA [6] were used. The CCA was used to examine the joint effects of mathematics in Level 1 and Level 2 on engineering performance in Level 2.The partial CCA was used to find the individual effect of mathematics in Level 2 on engineering performance in Level 2, when the effect of mathematics in Level 1 is removed from both groups, as the students have already completed mathematics in Level 1 at Level 2. The part CCA was used to determine the individual effect of mathematics in Level 1 on engineering performance in Level 2 when the effect of mathematics in Level 2 is eliminated from engineering performance in Level 2.

## 3. Results and Discussion

### *3.1. Correlation Analysis*

Correlation analysis confirmed data are suitable for CCA as most of the mathematics and engineering variables are significantly correlated ($p<0.05$) within their sets as well as between the two sets for all disciplines. Thus, adjusted CCA (part CCA and partial CCA) for two semesters in Level 2 (S3 and S4) were done separately for each engineering disciplines.

The marks of all compulsory engineering modules in two semesters (S3 and S4) in Level 2 are the dependent set of variables, but the number of variables in both S3 and S4 varied based the engineering disciplines. The results of unadjusted and adjusted CCA were summarized mainly focusing on the mathematics variables.

*3.2. Impact of mathematics in Level 1 and semester 3 on the engineering performance in semester 3*
The results of unadjusted and adjusted CCA for student performance in S3 by their engineering disciplines are summarized in Table 2.

*3.2.1. CCA.* Mathematics modules in S1 and S2 in Level 1 and S3 are taken as the predictor set. The p-value of Wilk's lambda test statistics confirmed that only the first canonical variate pair is statistically significant ($p < 0.05$) for all engineering disciplines. It implies that the first canonical variate pair is sufficient to explain a significant amount of variability of the predictor set and dependent variable set. According to the first canonical correlation (CC), it is clear that student mathematics performance is strongly correlated with engineering performance in S3 for all disciplines (CC >0.6). The proportion of the variance in the first canonical variate of engineering performance explained by the first canonical variate of the mathematics performance varied from 39% (in CE) to 70% (in EN). The canonical loadings of mathematics variables reflect that all mathematics variables are strongly associated with its first canonical variate except MA1013 in all disciplines. The redundancy index of engineering indicates that the explainable variability of engineering performance by the first canonical variate of mathematics varied from 12% (in CE) to 40% (in EN).

*3.2.2. Part CCA.* The two mathematics modules in Level 1 are the predictor set while mathematics modules in S3 are the control set, which eliminates its influence from the dependent set. By referring p-value of Wilk's lambda test statistics, it is clear that at least a first canonical variate pair of part CCA does not explain a statistically significant amount of variability of the predictor and dependent sets for all disciplines ($p>0.1$). It implies that the linear relationship between mathematics in Level 1 and engineering performance in S3 is not statistically significant with the effect of mathematics in S3 partialed out of the engineering performance in S3 for all disciplines. Furthermore, the first part canonical correlations are found to be less than 0.5 for all disciplines. It confirmed that mathematics in Level 1 is weakly correlated with engineering performance when the effect of mathematics in S3 is eliminated from engineering performance in S3. The results of squared canonical correlations indicate that the variation in the first canonical variate of engineering is explained by the first canonical variate of mathematics in Level 1 is less than 18% for almost all disciplines. In addition to that, the redundancy measures in all disciplines imply that amount of variability in mathematics and engineering sets explained by their opposite first canonical variate are not sufficient.

*3.2.3 Partial CCA.* The two mathematics variables in S3 as the predictor set and two mathematics variables in both S1 and S2 (in Level 1) as the control set, which eliminates its influence from both predictor and dependent sets are comprised in partial CCA. With reference to p-value of Wilk's lambda test statistics, it is clear that the first canonical variate pair is sufficient to explain a significant amount of variability of the predictor set and dependent variable set for all disciplines. Based on the results of first partial canonical correlations, it can be seen that the mathematics in S3 has moderately strong linear relationship with the engineering performance in S3 ($CC > 0.5$) for all disciplines except CE discipline, when the effect of mathematics in Level 1 is removed. The squared canonical correlations illustrate that the first canonical variate of mathematics accounted for 20% (in CE) to 55% (in EN) of the variance in the first canonical variate of engineering and it reflects that mathematics in S3 is significantly influenced on engineering performance in S3, even after the effect of mathematics in Level 1 is removed. Moreover, the canonical loadings reveal that mathematics variables are strongly correlated (>0.75) with their first canonical variates for all disciplines. The redundancy index of engineering reflects that the proportion of variance in engineering performance in S3 explained by the first canonical variate of mathematics also varied from 5% (in CE) to 23% (in EN).

*3.3. Impact of mathematics in Level 1 and Level 2 on the engineering performance in semester 4*
The summary of results of CCA, Partial CCA and Part CCA for academic performance in S4 is presented in Table 2 for the same three engineering disciplines.

*3.3.1. CCA.* As in Section 3.2.1, mathematics in S1 and S2 in Level 1 as well as S3 and S4 in Level 2 are taken as the predictor set. By referring the p-value of Wilk's lambda test statistics, it can be said that a significant amount of variability of predictor and dependent sets can be explained by the first canonical variate pair. The first canonical correlations reveal that mathematics in both Level 1 and Level 2 has a significantly strong linear relationship (CC > 0.7) with the engineering performance in S4. According to the canonical loadings, mathematics in S1 (MA1013) is weakly correlated with its first canonical variate whereas the remaining mathematics variables are significantly correlated with their first canonical variate for all disciplines. The amount of variance in engineering performance in S4 explained by the first canonical variate of mathematics in both Level 1 and Level 2 varied from 25% (in EN) to 34% (in CE) and it can be concluded that a considerable amount of variability in engineering performance in S4 can be explained by the mathematics performance in both Level 1 and Level 2.

*3.3.2. Part CCA.* The two mathematics variables in Level 1 are considered as the predictor set and the control set which removes its effect from dependent set, contains mathematics variables in both S3 and S4 in Level 2. With respect to the p-value of Wilk's lambda test statistics, the first pair of canonical variate in Part CCA is not statistically significant ($p > 0.05$) for all disciplines. This implies that at least a first canonical variate pair of Part CCA does not explain a statistically significant amount of variability of the predictor and dependent sets. Based on the results of part canonical correlation, it is clear that mathematics in Level 1 has a weak association with engineering performance in S4, after eliminating the effect of mathematics in S3 and S4. It is confirmed by the redundancy indices of engineering performance, which found less than 5% of the total variance of engineering performance that can be explained by the first canonical variate of mathematics in Level 1.

*3.3.3. Partial CCA.* The mathematics modules in S3 and S4 in Level 2 are the predictor set while mathematics modules in Level 1 are considered as the control set. The first canonical variate pair of Partial CCA is statistically significant ($p < 0.05$) as revealed by the p-value of Wilk's lambda test statistics. That is, the first canonical variate pair is sufficient to explain a significant amount of variability of the predictor set and dependent variable set when the effect of mathematics in Level 1 is eliminated from both mathematics and engineering performance in Level 2. As the effect of mathematics in Level 1 is statistically controlled by partial correlation, the results confirmed that the mathematics in S3 and S4 has a significant relationship with the engineering performance in S4 (>0.55). The squared canonical correlations show that the first canonical variate of mathematics accounted for 31% (in EN) to 46% (in CE) of the variance in the first canonical variate of engineering. Furthermore, the proportion of variance in engineering performance in S4 explained by the first canonical variate of mathematics in both S3 and S4 varied from 13% (in EN) to 24% (in CE) after adjusting for mathematics in Level 1.

**Table 2.** Results of unadjusted and adjusted CCA for S3 and S4 for the three selected disciplines.

| Semester | Discipline | | CC | Sq. CC | P-value | Mathematics performance | | | | | | | | Engineering performance | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Canonical Loadings | | | | | | Variance extracted | Red. | Variance extracted | Red. |
| | | | | | | MA1013 | MA1023 | MA2013 | MA2023 | MA2033 | Extra module | | | | |
| S3 | CE | CCA | 0.623 | 0.388 | <.0001 | 0.428 | 0.765 | 0.758 | 0.862 | - | - | 52.12 | 20.26 | 30.39 | 11.81 |
| | | Part CCA | 0.292 | 0.085 | 0.217 | 0.045 | 0.966 | - | - | - | - | 46.74 | 3.99 | 27.48 | 2.35 |
| | | Partial CCA | 0.448 | 0.200 | 0.002 | - | - | 0.762 | 0.929 | - | - | 72.19 | 14.46 | 26.23 | 5.26 |
| | EN | CCA | 0.834 | 0.696 | <.0001 | 0.373 | 0.698 | 0.838 | 0.941 | - | - | 55.38 | 38.53 | 56.90 | 39.59 |
| | | Part CCA | 0.339 | 0.115 | 0.312 | 0.055 | 0.958 | - | - | - | - | 45.99 | 5.29 | 18.80 | 2.16 |
| | | Partial CCA | 0.739 | 0.547 | <.0001 | - | - | 0.783 | 0.909 | - | - | 71.94 | 39.34 | 42.96 | 23.49 |
| | ME | CCA | 0.769 | 0.591 | <.0001 | 0.338 | 0.641 | 0.860 | 0.915 | - | - | 52.54 | 31.04 | 37.10 | 21.92 |
| | | Part CCA | 0.415 | 0.173 | 0.167 | -0.189 | 0.891 | - | - | - | - | 41.43 | 7.15 | 29.61 | 5.11 |
| | | Partial CCA | 0.684 | 0.467 | <.0001 | - | - | 0.835 | 0.897 | - | - | 75.11 | 35.11 | 24.55 | 11.47 |
| S4 | CE | CCA | 0.766 | 0.587 | <.0001 | 0.374 | 0.602 | 0.612 | 0.693 | 0.736 | 0.865 | 44.10 | 25.90 | 57.29 | 33.66 |
| | | Part CCA | 0.146 | 0.021 | 0.962 | -0.260 | 0.842 | - | - | - | - | 38.82 | 0.83 | 26.18 | 0.56 |
| | | Partial CCA | 0.679 | 0.461 | <.0001 | - | - | 0.516 | 0.579 | 0.654 | 0.825 | 42.75 | 19.72 | 51.13 | 23.59 |
| | EN | CCA | 0.700 | 0.490 | <.0001 | 0.203 | 0.773 | 0.666 | 0.865 | 0.846 | - | 50.90 | 24.95 | 43.3 | 24.74 |
| | | Part CCA | 0.315 | 0.099 | 0.146 | 0.941 | 0.403 | - | - | - | - | 44.29 | 4.40 | 27.73 | 3.86 |
| | | Partial CCA | 0.559 | 0.312 | 0.000 | - | - | 0.518 | 0.866 | 0.773 | - | 53.85 | 16.81 | 33.55 | 12.67 |
| | ME | CCA | 0.758 | 0.575 | <.0001 | 0.329 | 0.773 | 0.562 | 0.791 | 0.546 | 0.624 | 38.92 | 22.36 | 52.80 | 30.34 |
| | | Part CCA | 0.284 | 0.081 | 0.416 | -0.134 | 0.914 | - | - | - | - | 42.70 | 3.44 | 28.82 | 2.32 |
| | | Partial CCA | 0.592 | 0.350 | <.0001 | - | - | 0.369 | 0.728 | 0.330 | 0.633 | 29.38 | 10.30 | 43.62 | 15.29 |

### *3.4. Comparison*

According to the results of unadjusted and adjusted CCA for both academic performance in S3 and S4, it can be seen that the level of adjusted canonical correlations; partial canonical correlations and part canonical correlations are reduced due to the relevant adjustments. This implies that the joint effect of mathematics in Level 1 and Level 2 on engineering performance in Level 2 is significantly higher compared to the individual effects of mathematics in Level 1 and Level 2. By comparing the results of partial CCA and part CCA, it is clear that the individual effect of mathematics in Level 2 is significantly higher than the individual effect of mathematics in Level 1 on the students' engineering performance in Level 2. Moreover, redundancy measures of partial CCA indicate that the individual effect of mathematics in Level 2 on engineering performance is significant, even after adjusting for mathematics in Level 1. Conversely, the individual effect of mathematics in Level 1 on engineering performance is not sufficient after eliminating the effect of mathematics in Level 2. Though the individual effect of mathematics in Level 1 is not significant, it can be a sufficient indirect effect of mathematics in Level 1 on engineering performance.

## 4. Conclusion

The joint effect of mathematics in Level 1 as well as Level 2 is significant on engineering performance in Level 2 irrespective of the engineering disciplines. As expected, the joint effect of mathematics in Level 1 and Level 2 on engineering performance in Level 2 is significantly higher compared with both individual effects of mathematics in Level 1 and Level 2. Moreover, the individual effect of mathematics in Level 1 is extensively lower compared with the individual effect of mathematics in Level 2 on the students' engineering performance. This reveals that it is not worth considering only the individual effect of mathematics in Level 1 on engineering performance. However, there exists a significant indirect effect of mathematics in Level 1 on engineering performance in Level 2.

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