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The method for celestial bodies' center of mass position relative to their figures determination on the basis of harmonic analysis of the expansion in spherical functions in order to refine the physical libration parameters

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Abstract. In this paper the problem of the lunar center of mass relative to the center of its figure determination on the basis of space observations is considered, since the Moon is the most studied celestial object and there is a complete database on it. The future prospects for lunar laser ranging and radio interferometry require development of adequate theoretical support for modern technologies. The aim of these studies is the distances' measurement between the Moon and the Earth with an accuracy of 1 mm. Thus, determination of the lunar center of mass position, represented in this paper, and development of the selenocentric system will allow to solve the above mentioned problem more accurately and reliably. The new values of the lunar center of mass relative to its center of figure in orthogonal selenographic coordinate system $\Delta\xi$, $\Delta\eta$, $\Delta\zeta$ have been determined; they are: -1.75, -0.75, 0.11 km respectively.

1. Introduction

The modern software used for space researches apply methods of structural analysis of celestial bodies' internal structure demonstrating advantages and capabilities of an interdisciplinary approach: astrophysical observations and theoretical simulation are complemented by geophysical methods. The special role is played by physical librations parameters analysis, especially in terms of numerical theories construction. The basic problem of such theories is still planetary core detection and its main parameters determination, particularly a celestial body's center of mass position relative to its center of figure. In case of the Earth solution for this problem is usually obtained on the basis of significant amount of ground and space observations, but for other celestial bodies the capabilities are quite limited. In the present work the method for the position determination of the lunar center of mass (LCM) relative to the center of the lunar figure (LCF) is represented, since for our natural satellite there is the most complete database on space observations.

It should also be noted that the numerical approach to construct a theory of lunar physical libration (LPL) [1, 2] allows to consider many elements of the lunar body's internal stratigraphy. This gives an opportunity to get the high-precision modern data from Lunar laser ranging (LLR). Also this idea is worth developing for the analytical approach to LPL development as well for the unique opportunities to separate forced and free librations and to identify many subtle effects of lunar rotation with the physical nature of their origin. The similar approach to obtain some information on the basis of various models of LPL has promoted a series of papers in which these problems were solved [3, 4].

Today LLR, which has been implemented for more than 45 years, is one of the most efficient sources of information about the Moon. Precision of laser measurements has reached the sufficient level to define even relativistic effects in the «Earth-Moon» system. Analysis of laser data at definition of lunar rotation parameters has allowed to not only refine numerical characteristics of the dynamical lunar figure (nondimensional moments of inertia and coefficients of elasticity), but to clearly define amplitudes and phases similar to Chandler wobble (or variation of latitude) modes in free librations and at the same time to reveal the presence of strong dissipation of the rotation. Though the problem



of long-term free libration existence at the presence of strong rotational dissipation is not yet solved, still at the present time a number of significant steps are made. As it was proved in the series of works [1, 3], one of the reasons for dissipation is not only tidal friction, but the possible presence of lunar liquid core as well. Dissipation processes at differential core rotation and viscoelastic mantle cause a deviation of lunar rotation from Cassini laws: as a result of the dissipation the lunar axis of rotation is out of plane, in which the axis of ecliptic and the lunar equator lie, for a constant of 0.26 arc seconds [5].

Precision of LPL description is primarily determined by the accuracy of selenopotential description. The dynamical model depends on Stokes coefficients obtained from gravimetric satellite measurements and nondimensional moments of inertia defined from LLR. The prospects of laser ranging and radio interferometry of the Moon within the international cooperation of space powers both in field of observation and processing and interpretation observational data require the development of adequate theoretical support for the modern technologies of determination the distance between Earth and Moon. Thus, the determination of the lunar center of mass, represented in the present work, and as a result – the construction of the selenocentric system, will allow to solve the above mentioned problems with higher accuracy and reliability.

2. Formulation of the problem and the method for selenocentric model construction on the basis of expansion in spherical functions

Currently all the data on selenophysics may be divided into 2 types. The first one is obtained by the lunar physical surface scanning with «board satellites», this well describes the physical surface, however does not allow to determine coordinates of reference objects situated on the Moon [6]. The other type is obtained on the basis of lunar objects direct binding to the stars; this allows to determine coordinate of reference objects but does not describe the lunar surface with a sufficient accuracy. The mentioned systems have different reference systems and axes of orientation. It is well known that data obtained as a result of all space missions refers to quasi-dynamical coordinate system in which the origin is the lunar center of mass with axes not coinciding with the lunar axes of inertia.

In the majority of modern selenodetic catalogues the quasi-dynamical coordinate systems are used as well. In these systems either an origin does not coincide with LCM or axes do not coincide with the lunar axes of inertia. It should also be noted that currently there is no dynamical selenocentric reference system constructed on the basis of satellite observations covering sufficient part of the lunar surface [7].

The problem of LCM determination relative to LCF is important for the lunar evolution and its structure investigation as well as for the construction of spin-orbital theory; it is also important for the increase in precision of the near-Moon navigational tasks solution. If LCF is determined by geometrical data (altitudes of objects) and LCM is defined on the basis of the external gravitational field (orbital etc.), then the position of the LCM may be defined relative to the center of a sphere approximating the Moon. Firstly, these results have been obtained in Englehardt Astronomical Observatory on the basis of processing of meridian observations made in Royal Observatory (Greenwich) [8] and analysis of the lunar marginal zone's maps data [9]. The recent space missions [10-11] have also given some large informational material to define relative position of LCM and LCF.

We suggest solving the presented problem by constructing selenocentric hypsometrical model based on the catalogue of 272 931 reference objects [7]. For the construction of the selenocentric model various observations and robust methods may be used [12]. In this work as a model describing the relief on the lunar surface the expansion of the altitude function in a series of spherical harmonics in the form of regression has been used [6]:

$$h(\varphi, \lambda) = \sum_{n=0}^N \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \cdot \bar{P}_{nm}(\cos \varphi) + \varepsilon, \quad (1)$$

where:

$h(\varphi, \lambda)$ is altitude function;

φ, λ are latitude, longitude – known parameters of the lunar objects;

$\bar{C}_{nm}, \bar{S}_{nm}$ are normalized harmonic amplitudes;
 $\bar{P}_{nm}$ are normalized associated Legendre functions;
 ε is random regression error.

With the existing software on the basis of formula (1) and harmonic analysis the Lunar Selenocentric Dynamical System (LSDS), which later was used as a work tool for the analysis of three-dimensional distribution of the center of mass position, had been constructed. At the construction of the model (1) estimates of precision and reliability of observance of the Least Square Method conditions have been carried out. In case of their violations the corresponding adaptation methods have been applied. Solution of the redefined system for various sources of hypsometrical information has been implemented within the approach of regression simulation involving in addition to ordinary stages (postulation of the model (1) and amplitudes $\bar{C}_{nm}, \bar{S}_{nm}$ estimation) the use of a number of quality statistics including external measures, diagnostics of observance of the Least Square Method basic conditions, adaptation in case of their violation. As calculation schemes of the Least Square Method the schemes of Gauss-Jordan and Householder have been used. Noise harmonics are removed by step by step regression.

3. The method of harmonic analysis for position of lunar center of mass determination

There are several methods to determine the position of the lunar center of mass based on observations:

- 1) The position of LCM gets determined relative to the center of the sphere approximating the Moon, if LCF is defined by geometrical data (objects' altitudes) and LCM – by the external gravitational field (motion of the lunar artificial satellites etc.). LCM has been fixed as a center of a system of objects' on its surface coordinates catalogue obtained from various sources, and hypsometrical data has been used to represent altitude (as a function of spherical coordinates) in the form of the expansion in spherical harmonics. After that by the amplitudes of first order the position of LCF relative to LCM is defined [13];
- 2) The second method is based on direct use of selenocentric catalogue of objects' being identified coordinates which are uniformly distributed on the surface of the Moon. In the present work this method is used. When conducting the present investigation, it is assumed that all the sources of hypsometrical data used in calculations are equally accurate.

In connection with the fact that initial data is unevenly distributed on far and near side of the Moon, determination of mean radius requires determination of mean radius for both sides and calculation of their arithmetic mean [13]:

$$R = R_0 + \frac{1}{2}(\Delta R_{near} + \Delta R_{far}),$$

where $R_0 = 1738$ km, $\Delta R = \sum_{i=1}^n H_i p_i \cos \varphi_i / \sum_{i=1}^n p_i \cos \varphi_i$. Here H_i are altitudes of points relative to mean sphere with a center in LCM; n is number of the measured points; $p_i \cos \varphi_i$ are points' statistical weight ($p_i = 1$ or 0.1). We can write:

$$\sqrt{p_i \cos \varphi_i} \sum_{n=1}^N \sum_{m=0}^n S_{nm} i = \sqrt{p_i \cos \varphi_i} (H_i - \Delta R),$$

where $S_{nm} i = (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \cdot \bar{P}_{nm}(\cos \varphi_i)$.

$\bar{P}_{nm}(\cos \varphi_i)$ – normalized associated Legendre polynomials, which can be defined according to the formula:

$$\bar{P}_{nm}(\cos \varphi) = \sqrt{\frac{\varepsilon_m (2n+1)(n-m)!}{(n+m)!}} P_{nm}(\sin \varphi), \varepsilon_0 = 1, \varepsilon_2 = 2, m = 1, 2, \dots$$

As the system of spherical functions is orthogonal, the coefficients of each order $N = 1 - N_k$ are defined by consistent solution of N_k sets of equations of the following type:

$$\sqrt{p_i \cos \varphi_i} \sum_{m=0}^N S_{nm} i = \sqrt{p_i \cos \varphi_i} \left(H_i - \Delta R - \sum_{n=1}^{N-1} \sum_{m=0}^n S_{nm} i \right).$$

First order coefficients define the displacement of LCF relative to LCM [13]:

$$h\Delta\xi = \sqrt{3C_{11}}, \Delta\eta = \sqrt{3S_{11}}, \Delta\zeta = \sqrt{3C_{10}}, \quad (2)$$

where ξ is an axis directed towards the Earth; η – equatorial axis directed orthogonally to ξ ; ζ coincides with the axis of lunar rotation; $\bar{C}_{11}, \bar{S}_{11}, \bar{C}_{10}$ – normalized amplitudes of first order of expansion in spherical functions.

The stage of mathematical models' parameters estimation is one of the most important computational procedures at processing of observations [14]. The problem of mathematical processing correctness when combining requirements for the results accuracy and reliability exist at the process description along with the problem of choice between formal (approximating) and geometrical (cause-effect) models. Unfortunately, the traditional approach to parameters estimation involving a strictly fixed model and the least square method (LSM) application [15] does not satisfy the requirements of practice and capabilities of the methodology based on statistical computer simulation. Some attempts to go beyond the standard LSM [16] focus on solution of particular problems and do not involve any systematical approach to the task. The typical limits at ground observations and space data processing may be the presence of insignificant, uninformative and duplicate (dependent on each other) terms of the expansion, violation of the LSM conditions – the normal scheme of Gauss-Markov [14]. Noise leads to decrease of the parameters determination and the LSM estimates accuracy. The problem is getting more complicated due to the fact that, when estimating, there is no check for the model adequacy to the observations. Researchers in most cases are unaware of the actual state of affairs and do not apply any adaptive computational schemes, so the LSM conditions observance does not get checked.

At the present work, when estimating the selenocentric models, the methodology of regression modeling as an alternative to the traditional approach [14], involving in the tasks of estimation the regression analysis, check of the conditions, adaptation in case of their violation, and providing the presence of a special software, automating processes of computation and analysis, has been used. Regression modeling is a systematic approach at which correctness of the system's any element application (sampling, model, method for parameters estimation, method for structures estimation, quality measure, a set of conditions) may be questioned and checked with a corresponding adaptation application in case of the given conditions violation.

Comparison of the calculated amplitudes values and the recent results is a subject of considerable interest. The digital relief model for the full sphere of the Moon has been obtained from «Clementine» space mission [10], «ULCN 2005» [17], «KAGUYA» [11] and compared with LSDS.

The table 1 provides the normalized coefficients of the first order of the lunar relief's function expansion for 4 sources of hypsometrical information. The line "0, 0" contains amendment to the accepted mean radius of the Moon. The fifth and sixth table lines contain: The first horizontal cell there are n and m values responsible for order and degree of the expansion (1) respectively; the second horizontal cell of the table contains estimates of harmonic amplitudes $\bar{C}_{nm}$ and $\bar{S}_{nm}$ obtained on the basis «Clementine» space mission data ([6] provides expansion up to the 40th order); the third horizontal cell contains estimates of harmonic amplitudes $\bar{C}_{nm}$ and $\bar{S}_{nm}$ obtained on the basis of the project «ULCN» data ([17] provides expansion up to 359th order); the fourth horizontal cell contains estimates of harmonic amplitudes $\bar{C}_{nm}$ and $\bar{S}_{nm}$ obtained on the basis of «KAGUYA» space mission ([18] provides expansion up to the 180th order); the fifth horizontal cell contains estimates of

harmonic amplitudes $\bar{C}_{nm}$ and $\bar{S}_{nm}$ obtained on the basis of «LSDS» model (as a result, expansion of 5th order is model). For «LSDS» model the fifth order of the expansion is sufficient, since its increase does not lead to any significant change of the mean square error.

Table 1. Normalized coefficients of the first order of the lunar relief's function expansion for four sources of hypsometrical information, km.

n, m	CLEMENTINE [6]		ULCN [17]		KAGUYA[18]		LSDS	
	$\bar{C}_{nm}$	$\bar{S}_{nm}$	$\bar{C}_{nm}$	$\bar{S}_{nm}$	$\bar{C}_{nm}$	$\bar{S}_{nm}$	$\bar{C}_{nm}$	$\bar{S}_{nm}$
1	2		3		4		5	
0, 0	-0.83		-1.07		-0.94		-0.87	
1, 0	-0.37	0	0.15	0	0.14	0	0.11	0
1, 1	-1.04	-0.43	-0.99	-0.42	-1.02	-0.45	-0.86	-0.40

Based on equation (1) and data from Table 1 the coordinates of the center of mass relative to the lunar center of figure have been determined (Table 2). In order to obtain mean square errors of displacements, the amplitudes in Table 1 should be multiplied by $\sqrt{3}$.

Table 2. Coordinates of the Moon's center of mass relative to its center of figure for four sources of hypsometrical information.

	CLEMENTINE (km)	ULCN (km)	KAGUYA (km)	LSDS (km)
$\Delta\xi$	-1.80	-1.71	-1.77	-1.75
$\Delta\eta$	-0.74	-0.73	-0.78	-0.75
$\Delta\zeta$	-0.64	0.26	0.24	0.11

In Table 2 values of $\Delta\xi, \Delta\eta, \Delta\zeta$ are the differences in positions of the lunar center of mass relative to its center of figure in orthogonal selenographic coordinate system. Analysis of this data shows that selenocentric model LSDS, whose system is brought to the center of mass and to main axes of inertia of the Moon, is in good agreement with the results of the recent studies. In this work the method of direct harmonic analysis of the selenocentric catalogue of the lunar objects' coordinates in order to determine the position of the lunar center of mass relative to its center of figure has been used. This approach may be applied for an analysis of other celestial objects.

4. Summary and conclusions

During this work the data on mutual positions of the lunar center of mass and its center of figure has been obtained as a basis for construction a numerical theory of the Moon. At the same time the new method of direct harmonic analysis of the investigated systems on the basis of the analysis of the known reference points' radius vectors has been developed. It should also be noted, time series of observations of celestial systems have a complex dynamics, that is why at the analysis the methods of non-equilibrium statistical physics [19–21] as well as comparative stochastic analysis [22] may be used.

In prospect, the obtained results and methods will be used at the analysis of other bodies' of the Solar system spin-orbital characteristics. In particular, at the present time a problem of Mars' and its 2 moons' Phobos and Deimos spin-orbital characteristics analysis is a subject of great interest, which is confirmed by launch of global multibillion projects in Russia as well as in ESO, NASA and other space agencies of Japan and China.

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