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Binomial tree method for pricing a regime-switching volatility stock loans

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Abstract. Binomial model with regime switching may represents the price of stock loan which follows the stochastic process. Stock loan is one of alternative that appeal investors to get the liquidity without selling the stock. The stock loan mechanism resembles that of American call option when someone can exercise any time during the contract period. From the resembles both of mechanism, determination price of stock loan can be interpreted from the model of American call option. The simulation result shows the behavior of the price of stock loan under a regime-switching with respect to various interest rate and maturity.

1. Introduction

Stock loan contracts with two parties involved, the lender and borrower, can be considered as an alternative of investment. Using the stock as the collateral, the stock owners as the borrower can get liquidity without selling their stocks. During the life of the contract, the lender holds the stock and the borrower gets the stock back as he paybacks the loan. As the stock price decreases and reaches the loan amount then the borrower fails to payback. The borrower pledges the stock to the lender and just walks away - this is called non-recourse [1].

The analytic valuation of stock loans as a perpetual contract is proposed by Xia and Zhou [1]. The perpetual scheme is used to approach a long tenor contract of stock loan. However, the tenor of stock loans contract is finite. A finite maturity of stock loan valuation has been studied by Lu and Putri [2] which presented a semi-analytic method. The study evaluated the stock loans in three dividend distributions in Dai and Xu [3]. Further, the optimal exit strategy is discussed in Dai and Xu [3] and is implemented using binomial method under the Black-Scholes model. As the Black-Scholes' assumes a constant volatility, the model is likely to inaccurate for pricing.

The change in economics is an inevitability which is represented by the volatility values in the mathematical model [4]. Therefore some attempts to present better valuations of non-constant volatility stock loans are presented: perpetual stock loans with a regime-switching volatility under the Black-Scholes model [5], a Levy process based stock loans valuation [6], and stock loans with stochastic volatility model [7].

A regime-switching volatility model has advantages for its flexibility to apply and its easier procedure than solving a non-linear PDE based model [8]. Unlike the perpetual stock loans with a regime-switching volatility which has analytic solution, the finite maturity stock loans with regime-switching has not been discussed intensively so far in literature. Therefore, this paper

discusses about the finite maturity stock loans with a regime-switching volatility. The method used for valuing such a contract is a binomial method for its simplicity and flexibility.

This paper is organized as follows. Section 2 discusses about the binomial model of stock loans with constant volatility and Section 3 presents the contracts with a regime-switching volatiliy. Numerical simulation in Section 4 and concluding remark in Section 5.

2. Binomial tree method for stock loans with constant volatility

The random dynamic of stock price as the collateral of the stock loans contract is assumed to follow a stochastic differential equation,

$$dS = (r - \delta) S \, dt + \sigma S \, dW_t \tag{1}$$

where dW_t is a Wiener process defined for $t \in [0, \infty)$, $r \geq 0$ is the risk-free interest rate, $\delta \geq 0$ is the continuous dividend yield, and σ is the volatility of the stock. Equation 1 as a continuous random walk model is approximated by a discrete random walk model in binomial method [9].

Following the common procedure to obtain the valuation of options in [10], the factor of up and down are determined as a limiting case of binomial distribution and the factors are $u = e^{\sigma \sqrt{\Delta T}}$, dan $d = e^{-\sigma \sqrt{\Delta T}}$ [11].

The risk neutral probability of the stock price's increase or decrease are defined by p and $q = 1 - p$ respectively,

$$p = \frac{e^{rT} - d}{u - d}, \, q = \frac{u - e^{rT}}{u - d}.$$

Subsequently, the discrete formula price of stock loan without a regime- switching volatility with binomial model is

$$V_0 = max\Big\{ \frac{1}{\rho} \Big(q^u V_T^u + q^d V_T^d \Big), max \left(S_n - q e^{\gamma T}, 0 \right) \Big\}$$

where q is the loan amount and γ is the loan interest.

3. Stock loans with regime-switching volatility

A regime-switching Markov chain is a stochastic process which represents the state transitions. In this paper, the stock loans assume to have different states of economy those are growing economy and recession economy. Suppose that α_t is a continous Markov chain that describes difference states that occur in a realistic market and can be expressed as follows:

$$\alpha_t = \left\{ \begin{array}{l} 1, \text{ if the state of economy is growing / bull state.} \\ 2, \text{ if the state of economy is in recession / bear state.} \end{array} \right.$$

Based on a continuous Markov chain, the transition matrix probability is formed as $P_{ij}(t) = P\{\alpha(t + s) = j | \alpha(s) = i\}$ that is current state i and next state j. Let $M = \{1, 2\}$ and $\alpha_t \in M$, the transition matrix is written as $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$ and $0 \leq p_{ij} \leq 1$, $\sum_{j=1}^{2} p_{ij} = 1$ with $i, j \in M$. The economy conditions are represented by different volatilities, and the volatility value of each state is $\sigma^k = \sigma(\alpha_t = k)$ with $k = 1, 2$. Briefly, we use σ_1 and σ_2 to represent the economic in growth and in recession respectively.

3.1. Binomial model for stock loans without deviden in a regime- switching

For constructing a multistep binomial model of stock loans, we divide [0,T] into $[t_i, t_{i+1}]$, $i = 0, 1, 2, ..., N - 1$, with $t_i = i\Delta t$ and $\Delta t = \frac{T}{N}$. The dynamic of stock price is constructed in binomial tree under a regime-swicthing scheme with α^θ and $\theta = 1, 2$ is the state of market.
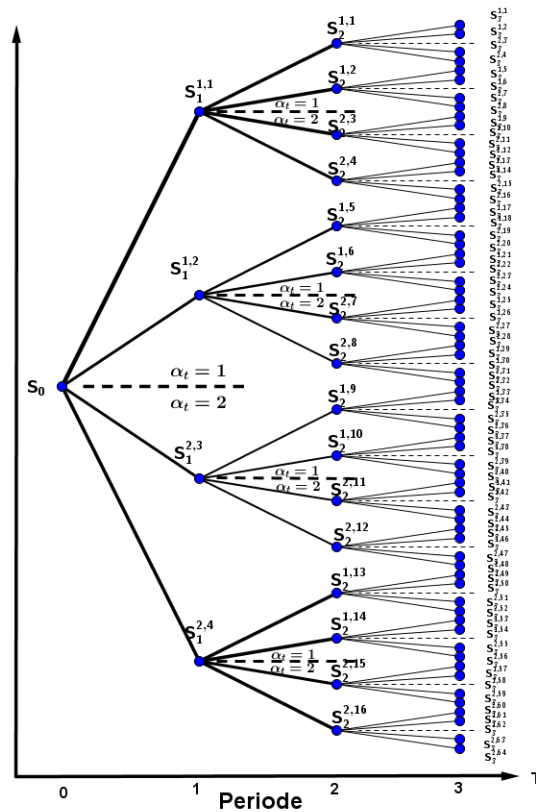
Figure 1: A multi-period binomial tree model for stock price in a two-state regime-switching

Subsequently, the stock price at time n is $S_n^{\alpha^\theta,\theta}$ with $1 \leq \theta \leq 4^n, 0 \leq n \leq N-1, \alpha^\theta = 1,2$. It means that one of four values can be taken at t_{n+1}, that is:

- If $\alpha^\theta = 1$ then obtained $S_{n+1}^{1,4\theta-3} = S_n^\theta u^1$; $S_{n+1}^{1,4\theta-2} = S_n^\theta d^1$

- If $\alpha^\theta = 2$ then obtained $S_{n+1}^{2,4\theta-1} = S_n^\theta u^2$; $S_{n+1}^{2,4\theta} = S_n^\theta d^2$

The multi-period binomial model for stock price movement in a two-state regime switching volatility scheme is described in Figure 1.

The next step is constructing the stock loans value starting with the payoff value at time n is $V_n^{\alpha^\theta,\theta}$. Moving backward from the payoff value, we can calculate each node of the tree using,

$$V_N^{\alpha^\theta,\theta} = \left(S_N^{\alpha^\theta,\theta} - qe^{\gamma T}\right)\text{with } 1 \leq \theta \leq 4^N, \alpha^\theta = 1,2,$$

and no-arbitrage assumption implies that

$$V_n^{\alpha^\theta,\theta} \geq max\left(S_N^{\alpha^\theta,\theta} - qe^{\gamma T}, 0\right) \text{ with } 1 \leq \theta \leq 4^n, 0 \leq n \leq N, \alpha^\theta = 1,2$$

The binomial tree of the stock loans value is described in Figure 2.

Given $V_{n+1}^{\alpha^i,i}$ with $1 \leq i \leq 4^{n+1}, 0 \leq n \leq N-1, \alpha^i = 1,2$ and for each θ such that $1 \leq \theta \leq 4^n$, the recursion formula should fulfill the following requirement:
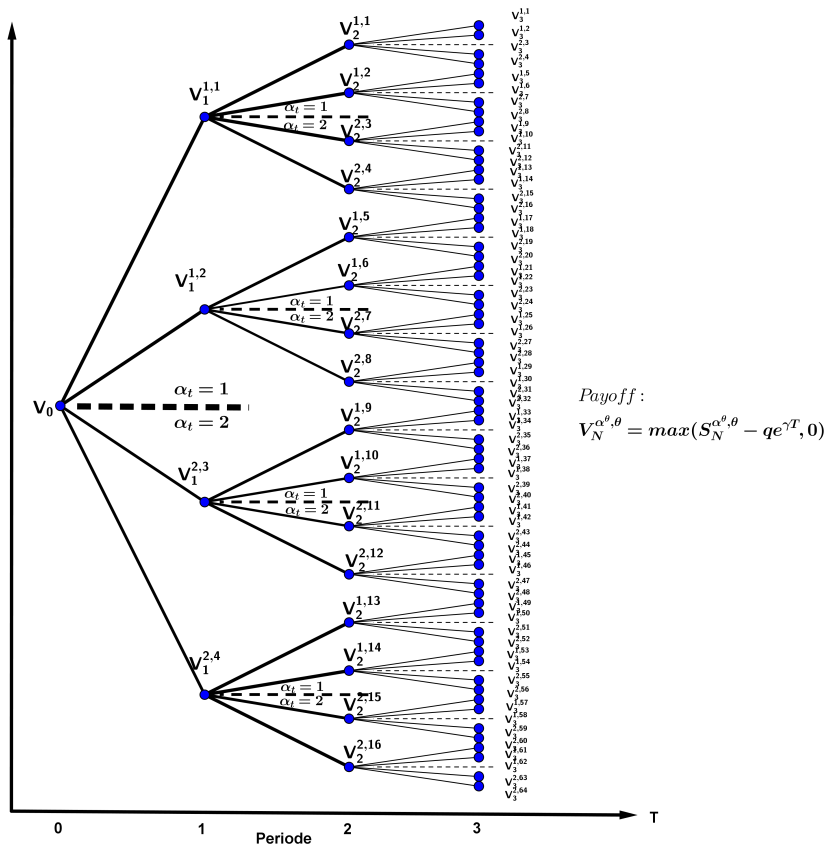
Figure 2: A multi-period binomial tree model for stock loans in a two-state regime-switching

- if $\alpha^\theta = 1$, it is obtained,

$$
V_n^{1,\theta} = max\Big\{ \frac{1}{\rho}\Big(p_{11}q^{1,u}V_{n+1}^{1,4\theta-3} + p_{11}q^{1,d}V_{n+1}^{1,4\theta-2}
$$
$$
+ p_{12}q^{2,u}V_{n+1}^{2,4\theta-1} + p_{12}q^{2,d}V_{n+1}^{2,4\theta} \Big), max\Big(S_n^{1,\theta} - qe^{\gamma n \Delta t}, 0 \Big) \Big\}
\tag{2}
$$

- if $\alpha^\theta = 2$, it is obtained,

$$
V_n^{2,\theta} = max\Big\{ \frac{1}{\rho}\Big(p_{21}q^{1,u}V_{n+1}^{1,4\theta-3} + p_{21}q^{1,d}V_{n+1}^{1,4\theta-2}
$$
$$
+ p_{22}q^{2,u}V_{n+1}^{2,4\theta-1} + p_{22}q^{2,d}V_{n+1}^{2,4\theta} \Big), max\Big(S_n^{2,\theta} - qe^{\gamma n \Delta t}, 0 \Big) \Big\}
\tag{3}
$$

Figure 1 and 2 show a more complicated tree than standar binomial tree which implies a longer computational time. In order to save the computational time, we propose a modification to the tree in the mentioned figures. The same nodes in Figure 1 and 2are packed or recombined into a single node. The recombined tree can be rearranged as shown in Figure 3. The recombined tree then is used in the simulations to show the behavior of the stock loans for various parameters' values.
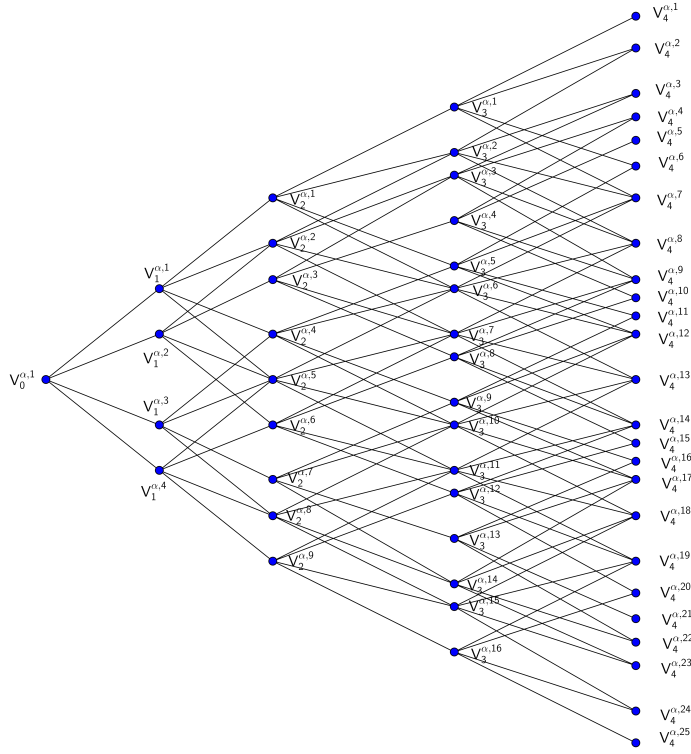
Figure 3: A multi-period binomial tree model for stock loans in a two-state regime-switching

For the recombined tree, the value of stock loans then are calculated based on the following formulas. Knowing that the stock price at time t with $1 \le \theta \le (t+1)^2$ is,

$$S_{t+1}^{\theta+\lfloor \frac{\theta-1}{t+1} \rfloor} = S_t^\theta u^1$$

$$S_{t+1}^{\theta+1+\lfloor \frac{\theta-1}{t+1} \rfloor} = S_t^\theta u^2$$

$$S_{t+1}^{\theta+t+2+\lfloor \frac{\theta-1}{t+1} \rfloor} = S_t^\theta d^2$$

$$S_{t+1}^{\theta+t+3\lfloor \frac{\theta-1}{t+1} \rfloor} = S_t^\theta d^1$$

Subsequently, the stock loan values can be calculated by Equation 4

$$V_t^{1,\theta} = max \Big\{ \frac{1}{\rho} \Big(p_{11} q^{1,u} V_{t+1}^{1,\theta+\lfloor \frac{\theta-1}{t+1} \rfloor} + p_{12} q^{2,u} V_{t+1}^{2,\theta+1+\lfloor \frac{\theta-1}{t+1} \rfloor}$$
$$+ p_{12} q^{2,d} V_{t+1}^{2,\theta+t+2+\lfloor \frac{\theta-1}{t+1} \rfloor} + p_{11} q^{1,d} V_{t+1}^{1,\theta+t+3+\lfloor \frac{\theta-1}{t+1} \rfloor} \Big), max \left(S_t^{1,\theta} - q e^{\gamma t}, 0 \right) \Big\}$$

(4)

4. Numerical results and discussions

We present some numerical simulations for determining the price of stock loans without dividend in a regime-switching volatility. Different from the American call options which has no optimality if there is no dividend paid, the stock loans still has optimal exercise boundary without the dividend paid in the stocks. The parameter values used for the following simulations are stock price $S_0 = 100$, loan amount $q = 100$, stock loans maturity $T = 1$, transition probability matrix
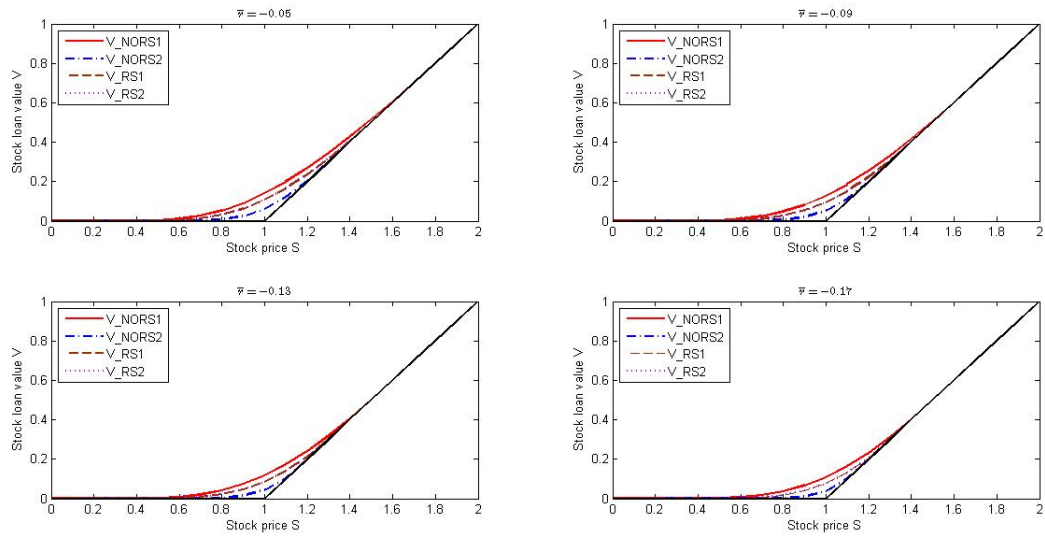
Figure 4: Stock loans value for various accumulated interest rate $\bar{r}$
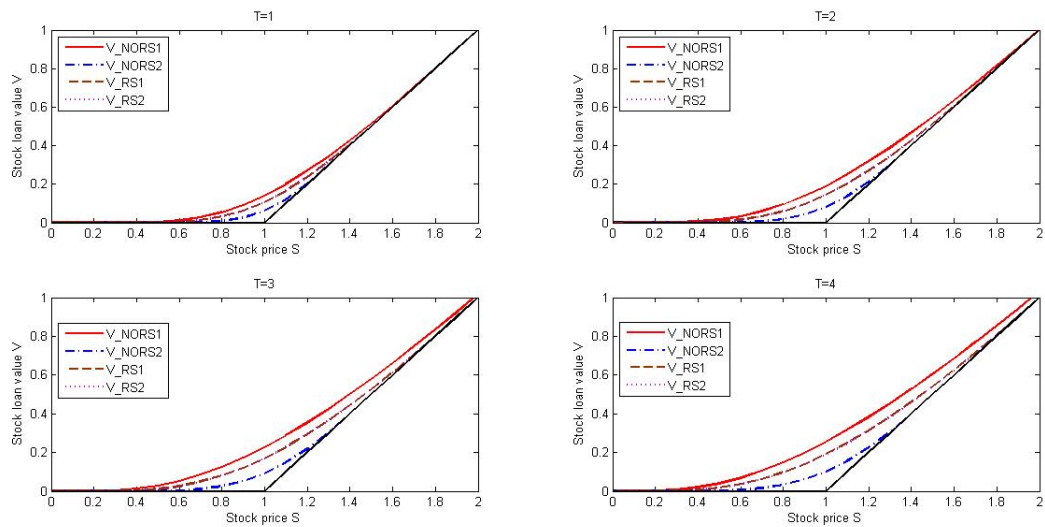
Figure 5: Stock loans value for various maturity T

$$P = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}.$$

For various accumulated interest rate ($\bar{r} = r - \gamma$) values, the stock loans value are shown by Figure 4. The results show that the values increase with respect to the riskless interest rate. The non-switching stock loan values bound the switching ones. In financial, the stock loans contract obviously has lower values as the accumulated interest rate has increased (driven by rising loan interests).

Stock loans contract values with different maturity is presented in Figure 5. A longer stock loan contract seems to have higher values than the shorter ones. The stock loan values with a regime-switching lies between the non-switching ones.

5. Conclusion

The behavior of the stock loans values under a regime-switching framework shows that the values decrease with respect to the accumulated interest rate, driven by loan interest rate. For a longer contract, the value of a stock loan is higher.

The stock loans value with switching are in between the non-switching ones. The reason is that the regime-switching framework implies a reduction to the over-valued or under-valued valuation in the non-switching scheme caused by different economic conditions.

6. Acknowledgments

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