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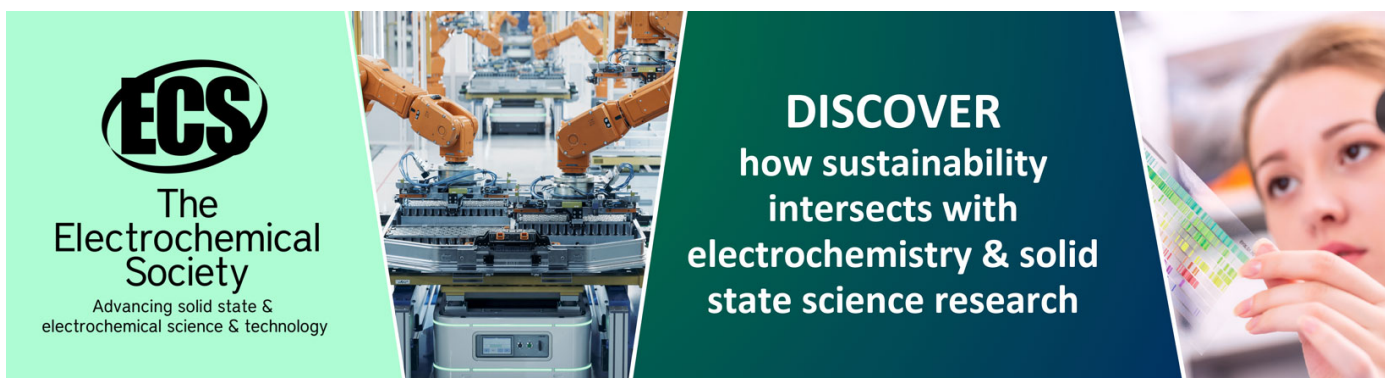
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Linear bending of functionally graded beams by differential quadrature method

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Abstract. Based on the first-order shear deflection and classical beam theory, the linear bending of functionally graded (FG) beams is analyzed by applying the differential quadrature method (DQM). First, using the principle of total potential energy, the governing equations of the FG beams subjected to a distributed lateral force are derived according to two different beam theories. To calculate the numerical solutions of static linear bending problem, the dimensionless boundary conditions and differential equation which are used to discretize by DQ method. Static deflection curves and load-central deflection curves is generated considering two different beam theories. The influences of the power law index, transverse shear deformation and the distributed lateral force on the linear bending of the FG beams under conditions of the simply supported and the clamped-clamped boundaries are analytically investigated in detail.

1. Introduction

A new type of composite materials which is called functionally graded materials (FGMs), are provided with smooth and continuous spatial changes of macroscopic characteristics, comprising density and elastic modulus. These parameters are achieved by controlling the volume fraction, size, and shape of the material components during manufacturing.

For linear analyses, Sankar [1] brought forward an explicit solution for FG beams that considers static transverse loading to assume that the material properties obey an exponential law along thickness direction. Ma and Lee [2] solved an explicit, closed form solution for the typical nonlinearity static corresponding of FGMs beams in a in-plane thermal force. Zhong and Shang [3] presented an explicit 3D analysis for a FG piezoelectric rectangular plate that was simply grounded and sustained toward four borderlines. By applying the similarity of mathematics among the derived equations, the correspondence relationship of bending results of Timoshenko FG beams and those of the congruent EB beams were obtained by Li et al. [4]. The maximum deflections of S-S and C-C TBs in an uniformly distributed load were gained from this obtained equations, and results were found by the shooting method. Thai et al. [5] reported the effect of shear deflection and power law index on the bending of FG beams using another TBT for the bending of FGMs. Applying the Ritz method model for FG material beams, which was first proposed the physical neural surface and TBT, Zhang [6]



obtained the approximate solutions about nonlinearity bending. Using TBT, Ravikiran et al. [7] carried on the static problems of the FGM beams under ambient temperature. To obtain a proper representation of the constitutive matrix for the FGM beam, a convergence research is required to choose the number of layers along the thickness. Reza et al. [8] raised an improved couple stress theory to analyze small-scale FGM beams. Applying the Airy stress function, Ding [9] et al. obtained the exact solutions for anisotropic FGM beams.

2. Mathematical equation model

2.1. Material characteristics

Take over a FG beam with a rectangular section (Fig. 1), which is composed of metals and ceramics. The material characteristics change continuously along the thickness h from the pure ceramic surfaces to the pure metal surfaces. For example, Young's modulus E and the passion ratio ν , can be shown as following:

$$P(z) = P_C + (P_M - P_C)V_M \quad (1)$$

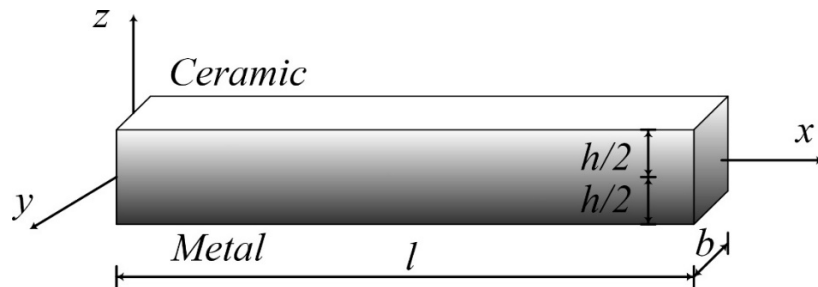


Figure 1. Geometry of FG beam and the material change of the thickness direction

Where V_M is the volume percentage of metal materials. According to below the function (Fig. 2), the equations are changed.

$$V_M = \left(\frac{1}{2} - \frac{z}{h} \right)^n \quad (2)$$

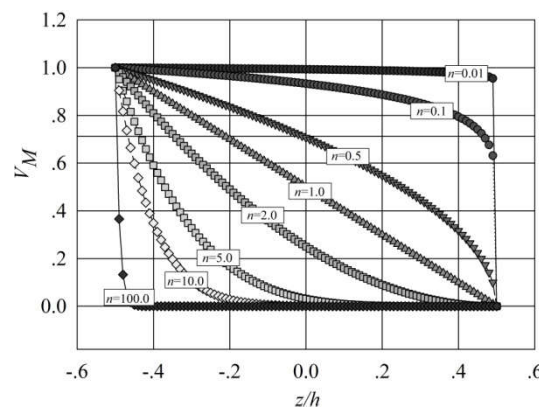


Figure 2. Curves of the volume fraction V_M and the power law index n

2.2. The governing equations

Table 1. Displacement field of the different theories

Displacement	CBT	FBT
$U_x(x, z)$	$u(x) - z$	$u(x) + z\phi(x)$
$U_z(x, z)$	$w(x)$	$w(x)$

In Table 1, x is the longitudinal axial of FG material beam; $U_x(x, z)$ and $U_z(x, z)$ are the longitudinal and transverse displacements at any points of beam, respectively; $w(x)$ indicates the deflection of FG beam; $\phi(x)$ is the rotation angle of the section.

Displacement and strain relations are provided as following:

$$\varepsilon_x = u_{,x} - \frac{4z^3}{3h^2}(\phi_{,x} + w_{,xx}) + z\phi_{,x} \quad (3)$$

$$\gamma_{xz} = w_{,x} - \frac{4z^2}{h^2}(\phi + w_{,x}) + \phi \quad (4)$$

On the basis of Hooke's law, we obtain the following displacement and stress relations:

$$\sigma_x = E(z)\varepsilon_x \quad (5)$$

$$\tau_{xz} = \frac{E}{2(1+\nu)}\gamma_{xz} \quad (6)$$

2.2.1. First-order shear deflection beam theory (FBT). The dimensionless equilibrium equations are given as follows:

$$U_{,\xi\xi} + K_1\psi_{,\xi\xi} = 0 \quad (7)$$

$$U_{,\xi\xi} + K_2\psi_{,\xi\xi} - K_3(W_{,\xi\xi\xi} + W_{,\xi}) = 0 \quad (8)$$

$$\psi_{,\xi} + W_{,\xi\xi} + K_q = 0 \quad (9)$$

The non-dimension quantities in the above equations are defined by:

$$\xi = \frac{x}{l}, W = \frac{w}{h}, \psi = \frac{\phi l}{h}, U = \frac{u}{l}, K_1 = \frac{B_{11}h}{A_{11}l^2}, K_2 = \frac{D_{11}h}{B_{11}l^2}, K_3 = \frac{A_{44}h}{B_{11}}, K_q = \frac{l^2 q}{A_{44}h}$$

2.2.2. Classical beam theory (CBT)

$$W_{,\xi\xi\xi\xi} = \frac{V_q}{V_1} \quad (10)$$

The dimensionless quantities in the above equations are defined by

$$\xi = \frac{x}{l}, W = \frac{w}{h}, U = \frac{u}{l}, V_1 = \frac{B_{11}}{A_{11}l} - \frac{D_{11}}{B_{11}l}, V_q = \frac{ql^2}{B_{11}}$$

2.3. The weighting coefficients

The DQM must discretize the definition domain into m points. Along the definition domain, a weighted linear summation of all of the functional values in M points approximately expresses the derivatives of any point [11].

$$\frac{d^k f(x_i)}{dx^k} = f_i^{(k)} = \sum_{j=1}^m A_{ij}^{(k)} f_j, i = 1, 2, \dots, m \quad (11)$$

Where m indicates the amount of grid points, $A_{ij}^{(k)}$ is the weighting coefficient for the k th derivative.

2.4. Selection of the sampling nodes

The sampling points in the DQ representations are selected as shifted Chebyshev-Gauss-Lobatto points, which are given as follows:

$$x_i = \frac{1}{2} \left\{ 1 - \cos \left[\frac{\pi(i-1)}{m-1} \right] \right\} (i = 1, 2, \dots, m) \quad (12)$$

2.5. Discretization of the static equilibrium equations

Linear summations substitute for the derivatives of equations, static equilibrium equations (Table 2) and boundary conditions (Table 3), which are then handled under the different beam theories as follows:

Table 2. Discretization of the static equilibrium equations

FBT	$(K_1 - K_2) \sum_{j=2}^{m-1} B_{i,j} \psi_j + K_3 \psi_i + K_3 \sum_{j=2}^{m-1} A_{i,j} W_j = 0 ; \sum_{j=2}^{m-1} A_{i,j} \psi_j + \sum_{j=2}^{m-1} B_{i,j} W_j = -K_q$
CBT	$\sum_{j=1}^m D_{i,j} W_j = \frac{V_q}{V_1}$

Table 3. The simply supported and the clamped-clamped boundary conditions

C-C	$\psi _{\xi=0} = 0, W _{\xi=0} = 0, \sum_{j=2}^{m-1} A_{1,j} W_j = 0, \psi _{\xi=1} = 0, W _{\xi=1} = 0, \sum_{j=2}^{m-1} A_{m,j} W_j = 0$
S-S	$\psi _{\xi=0} = 0, W _{\xi=0} = 0, \sum_{j=1}^m B_{1,j} W_j = 0, \psi _{\xi=1} = 0, W _{\xi=1} = 0, \sum_{j=1}^m B_{m,j} W_j = 0$

3. Numerical results and discussions

3.1. Numerical results of an Si_3N_4 -SUS304 FG beam based on FBT and CBT

For numerical solutions, we choose an Si_3N_4 -SUS304 FG beam in a uniformly distributed load q , with a length of $l=0.3\text{m}$, a height of $h=0.01\text{m}$, a width of $b=0.01\text{m}$, and the number of nodes is $m=11$. The material properties: $E_{\text{Si}_3\text{N}_4} = 348.43\text{GPa}$, $E_{\text{SUS304}} = 201.04\text{GPa}$, $\nu_{\text{Si}_3\text{N}_4} = 0.24$, $\nu_{\text{SUS304}} = 0.33$.

3.2. Numerical results of an Si_3N_4 -SUS304 FG beam based on FBT

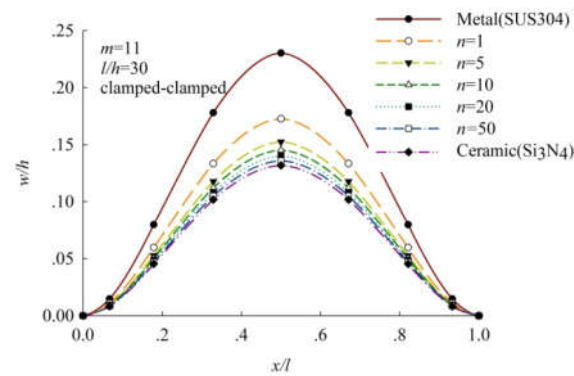


Figure 3. Deflection curves of a C-C FG beam with different power law indices and $q=18\text{KN/m}$.

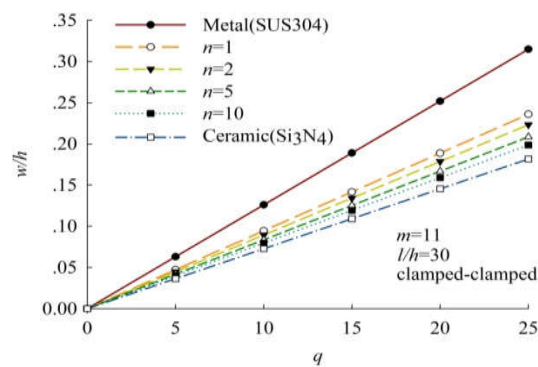


Figure 4. Central deflection-loading curves of a C-C FG beam with different power law indices.

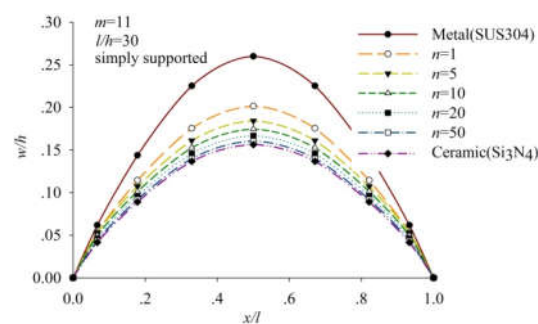


Figure 5. Deflection curves of an S-S FG beam with different power law indices and $q=4\text{KN/m}$.

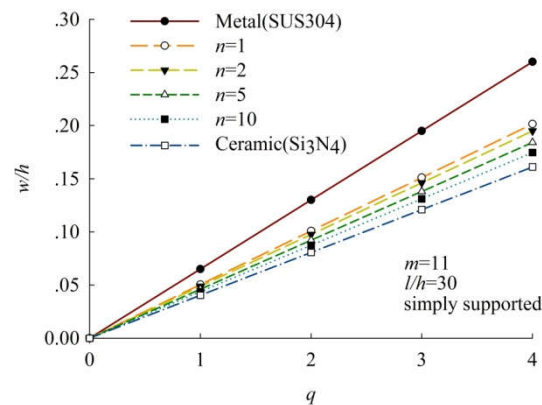


Figure 6. Central deflection-loading curves of an S-S FG beam with different power law indices.

4. Conclusion

In this article, based on first-order shear deflection beam theory, and classical beam theory, respectively, the static bending of FG beams exerted under the conditions of two different supporting boundaries is analytically investigated. The model of an FG beam subjected to a distributed lateral force with material properties is given, and it takes into account variations along the thickness dimension. These results reveal that the deflections of the beams by the first-order shear theory are always smaller than those created by the classical theory. In additional, an increase in the power law index leads to a drop in the deflection. Furthermore, viewed in terms of the deflection curves, the convergence of the deflection curves obtained by exerting a simply-supporting boundary is faster. The conclusions for the two theories differ slightly, with the numerical result of the classical theory being the smaller, and of the first-order shear theory being the bigger. This is due to the shear deformation in cross section being taken into consideration under the first-order shear theory, which makes the result more precise.

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