Effect of Sample Dimension on Three and Four Points Bending Tests of Fine Crystalline Marble and its Relationship with Direct Tensile Strength

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Abstract. The indirect tensile strength test methods such as three and four points bending tests and Brazilian tests have been commonly used tensile strength determination and other mechanical properties of rocks since it is a quick, easy, and inexpensive testing methods. EN and ASTM standards suggested that sample dimensions of three and four points bending tests have been different from each other. The aim of this study, in order to analyses the effect of sample dimension on bending strength property of fine crystalline marble and is to determine the direct tensile strength (DTS) of rock by using new dumbbell shape. Indirect tensile strength of marble was determined by using three and four-point bending test methods according to related standards. In addition to experimental study, stress distribution and stress intensity on the sample was analyzed by using ANSYS. As a result, four-point bending test is more suitable for determining of tensile strength of marble.

1. Introduction

The tensile strength of rock is a key parameter for determining the load bearing capacity, their deformation, fracturing, crushing, drilling, tunnel boring, and blasting etc., and is used to analyse the stability and serviceability of rock structures [1-3]. And also the tensile strength of rocks is much lower than the compression strength [4,5]. Direct and indirect methods are used to determine the tensile strength of the rocks. Because specific tools or apparatus are needed for the application of the uniaxial tensile test and difficulties in preparing the sample, it is not preferred [5-7]. Although the Brazilian indirect tensile strength (BTS) test (splitting test) is the most commonly used indirect tensile test (ITS) in the world, in some studies it is stated that BTS does not exactly represent the true tensile strength value. Three-point bending strength (3PBS) and four-point bending strength (4PBS) tests are other indirect methods which used in determination of the tensile strength in rocks [4,5]. These tests are applied at different sample sizes according to both EN and ASTM standards. A brief summary of the literature on dumbbell-shaped DTS and bending strength test is presented below.

First, Brace [8] stated that the most suitable sample geometry to be used to determine the direct tensile strength of the materials should be the dog bone (dumbbell) shape. Hoek [9] determined the axial tensile stresses of dog bone shaped specimens in a triaxial cell and concluded that the length / diameter ratio should be 2-3. In the study, it was pointed out that when the direct tensile specimen is not in the form of a dog bone, the stress concentration occurs at the upper end of the cylindrical

specimen (at epoxy or cement connection points or at the points of connection with compression), an invalid test result is obtained by breaking near the upper end of the specimen. Colback [10] stated that the samples used in the DTS tests were difficult to prepare, that the appropriate sample could not be prepared, and that failure usually occurred outside to the center of the sample. Gorksi [11] has patented an apparatus (test setup) that is self-centering under eccentric load for use in DTS in the USA, particularly for brittle materials. The developed device is designed to be subjected to tensile stress of dog-bone shaped specimens under compression force similar to that proposed by Brace [8]. In the study conducted by Nott [12], DTS and BTS tests were performed on artificial rock samples and compared with numerical model results. The tensile strength values determined by dog bone (dumbbell-shaped) test specimens were not found valid because they were broken at the sample connection points. Klanphumeesri, in his dissertation study [13], has developed a new loading device (tool) to apply indirect tensile stress to a dog-bone shaped rock sample. In studies carried out by Unlu and Yilmaz [14], a new experimental setup conforming to ISRM standards was proposed to determine the direct tensile strength of cylindrical rocks. They determined that the values obtained from experiments using this setup were lower than the DTS values. In order to prevent the difficulties encountered in the adhesion of the sample (for attachment) in DTS test, Komurlu et al. [5] proposed a new method for dog bone shaped specimens prepared in a lathe. Tufekci et al. [3] developed a new jaw gripping mechanism to determine the DTS in their study and compared the results of the tests on dumbbell-shaped specimens with those of Brazilian indirect tensile strength.

Komurlu et al. [15] conducted numerical modelling studies using the finite element method on dog-bone shaped samples on DTS test to determine the effects of sample geometry and steel apparatus geometry that grips the sample. Then, direct tensile tests were applied for 3 different rock groups and concrete samples and compared with the model results. According to the results of the study, the dog bone specimen geometry with a minimum diameter of 3.2 cm and cutting angles of angular part of 63° was determined as the optimal geometry both numerically and experimentally. A series of experiments were conducted to examine the tensile behaviour on reinforced (steel reinforcement and resin) rock specimens by Wu et al. [16].

Perras and Diederichs [17] investigated the relationship between DTS and BTS, and the predictability of tensile strength from other methods. According to literature reviews of their study, it has been found that accurate tensile strength values cannot be predicted from other laboratory tests. Efimov [7] tried to determine the tensile strength by using 4-point bending strength measurements. Biolzi et al. [6] have developed formulas to predict the tensile strength from the flexural (bending) strength in terms of shape and size effects in their study on granite specimens. Caviello et al. [18] performed various types of tests for determining the tensile strength of soft rocks, including the Brazilian, ring, Luong tests and three and 4- point bending tests. They presented an assessment of some widely used laboratory techniques on the basis of experimental data from the literature and their own investigations. In addition, they stated that 3PBS and 4PBS tests indicated acceptable standards in determining the bending strength of materials such as rocks, building materials, cement and concrete. Mixed mode crack behaviour has been investigated experimentally and theoretically by applying asymmetric 4-point bending tests on pre-cracked granite specimens by Razavi et al. [19]. Plangklang et al. [20] investigated the time-dependent tensile strength values by applying 4-point bending strength tests on the salt specimens. According to the results of the study, it was found that under the tensile stress, the salt showed elastic properties and was subject to deformations depending on the time.

In this study, bending tests were carried out on samples of different sizes in accordance with ASTM and EN standards on fine crystalline marble samples to try to reveal the differences between them and the results were compared with the results obtained from a new dumbbell-shaped direct tensile test. In addition to experimental study, stress distribution and stress intensity on the sample analyzed by using ANSYS.

2. Experimental study

The fine crystalline marble used in this study was obtained from Afyon region in Turkey. Afyon marble is entirely composed of calcite minerals and shows granoblastic texture. The calcites are interlocked and there is no clearance between them. The chemical composition of marble was given in Table 1 and the physical and mechanical rock properties were given in Table 2. The density, bulk density and porosity tests were carried out in accordance with TS EN 1936 [21], P-wave velocity, and uniaxial compressive strength (UCS) tests were carried out in accordance with ISRM (2007) suggested methods [22].

Table 1. Chemical composition (%) of fine crystalline marble

CaO	MgO	Al_2O_3	Fe_2O_3	SiO_2	K_2O	Na_2O	SO_3	LOI
55.90	0.27	0.02	0.02	0.001	0.001	0.001	0.04	43.23

Table 2. Physical and mechanical properties of rock

Specific Density (g/cm^3)	2.731 ± 0.009
Bulk density (g/cm^3)	2.720 ± 0.004
Total porosity (%)	0.381 ± 0.158
P-wave velocity (m/s)	5300 ± 250
Uniaxial compressive strength (MPa)	77.78 ± 6.74

2.1. Three point bending strength

3PBS tests were carried out in accordance with TS EN 12372 [23] (flexural strength under concentrated load) and ASTM C 99 [24]. According to TS EN 12372 standard, the thickness (h) should be between 25 mm and 100 mm, the total length (L) should be equal to 6 times the thickness, the width (b) should be 50 mm ≤ b ≤ 3h and the distance between the supporting rollers (l) should be equal to 5 times the thickness. In this study, 3PBS tests (EN standard) were carried out on specimens of dimensions 50*50*300 mm and 25*50*150 mm, provided that the specified conditions were met. Since the load must be increased uniformly at a rate of 0.25±0.05 MPa/s until the specimen breaks, the loading rate for the specimens of dimensions of 50*50*300 mm was chosen to be 66 N/s and for the specimens of dimensions 25*50*150 mm was chosen to be 41 N/s. According to ASTM standard, the specimens should be approximately 101.6*203.2*57.2 mm in size, the distance between the supporting rollers (l) should be 177.8 mm and the loading rate should be 74 N/s. 12 samples were used for all experiments performed in this section and 3PBS (σ_{3b}) is calculated from the Equation (1).

$$\sigma_{3b} = \frac{3Fl}{2bh^2} \quad (1)$$

Where the F was measured as the maximum applied force, l is the length of span, b is the width of specimen, h is the thickness of specimen.

2.2. Four point bending strength

4PBS tests were also carried out in accordance with TS EN 13161 [25] (flexural strength under constant moment) and ASTM 880-98 [26]. According to TS EN 13161 standard, dimensions of specimens, the distance between the supporting rollers and loading rates are same as in TS EN 12372. According to ASTM standard, the specimens should be approximately 102*32*381 mm in size, the distance between the supporting rollers (l) should be 318 mm and the loading rate should be 70 KPa/s. 12 samples were used for all experiments performed in this section too. 4PBS (σ_{4b}) for EN standard is calculated from the Equation (2) and for ASTM standard is calculated from the Equation (3). The

notations are the same as in TS EN 12372. Figure 1 shows the dimensions of the six types of bending strength tests.

$$\sigma_b = \frac{Fl}{hh^2} \tag{2}$$

$$\sigma_b = \frac{3Fl}{4bh^2} \tag{3}$$

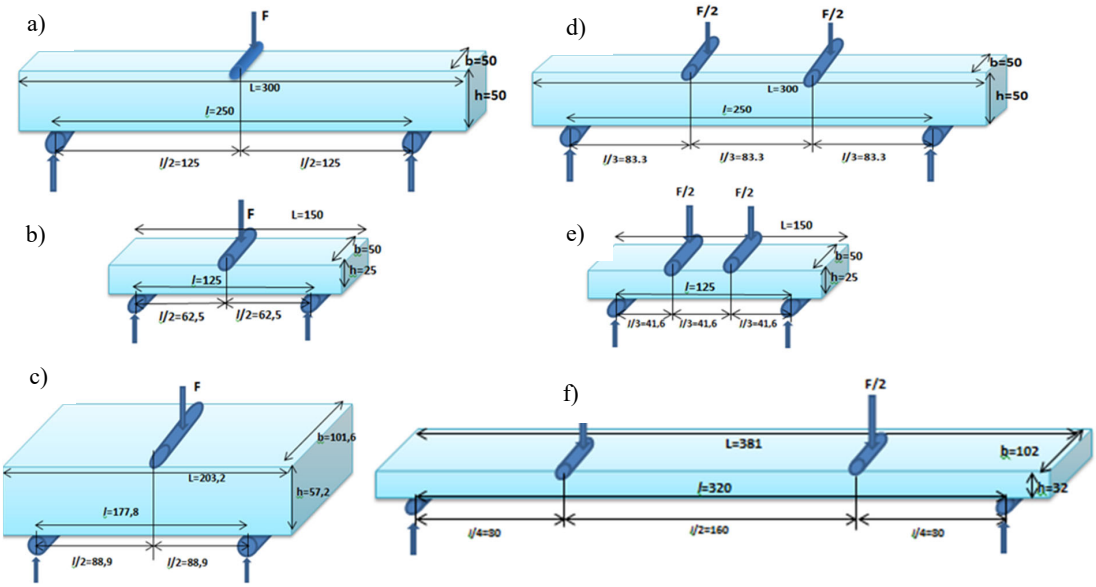

Figure 1. Test arrangement for a) EN 12372 (50*50*300mm) b) EN 12372 (25*50*150mm) c) ASTM C 99 d) EN 13161 (50*50*300mm) e) EN 13161 (25*50*150mm) f) ASTM 880-98

2.3. Direct tensile strength

The DTS tests were performed on specimens having a new dumbbell-shaped sample geometry and a jaw gripping mechanism developed by Tufekci et al. [3] given in Figure 2 (a) and in an accordance with ISRM [22] suggested methods. All samples which were free from cracks or discontinuities were tested on dry conditions. All bending tests and DTS tests were carried out on the same electromechanical universal test device which is fully automatic computer controlled and 100 N capacities (Figure 2 b). In this study, 9 marble samples were used in DTS tests. DTS is calculated by the following Equation (4).

$$\sigma_t = \frac{F}{(\pi D^2)/4} \tag{4}$$

Where F is the maximum applied force to the sample and D is the diameter of the region where the sample is failed.

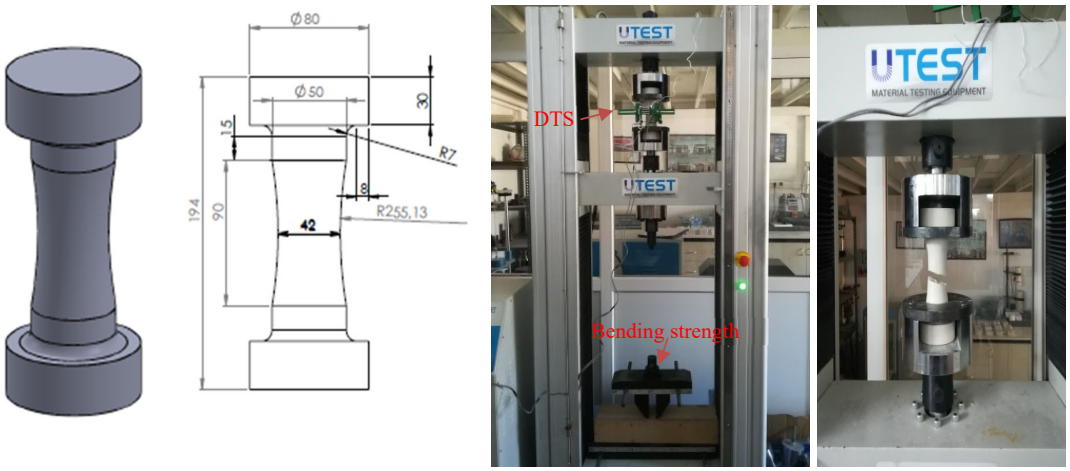

Figure 2. a) Technical drawing of dumbbell shaped specimen b) Displays of the DTS experiment

3. Numerical study

For the six different bending test methods described in the experimental study section, stress distribution and stress intensity occurring on the rock specimen were analyzed with the ANSYS Workbench R18. In finite element analysis (FEA), the samples were subjected to loads of 1 MPa which were calculated from their analytical formulas. The amount of load required to achieve a maximum stress of 1 MPa on cross-section is calculated from Equation (5).

$$\sigma = \frac{M_e}{W_e} \tag{5}$$

Where M_e is the maximum bending stress to occur in the sample, W_e is the moment of resistance. For rectangular samples; it is calculated with Equation (6).

$$W_e = \frac{bh^2}{6} \tag{6}$$

Where b is the width of the sample, h is the thickness of the sample.

In the analyzes, the modulus of elasticity was assumed to be 30 GPa and the Poisson's ratio 0.3. The graphical maximum principal stress distributions in the frontal and lower surfaces obtained at the end of the analyses for each standard test sample are given in Figures (3-8). In all graphs, positive stresses show tensile stresses and negative ones show compressive stresses. Since the shear stresses in the cross-sections are very close to zero, the maximum principal stress values equal the maximum tensile stresses in the sample. FEM analysis results conducted for different bending tests is shown Fig 3-8. As expected, the maximum tensile stress value occurs at the bottom-middle part of the sample. There isn't any stress concentration within the sample due to uniform smooth geometry.

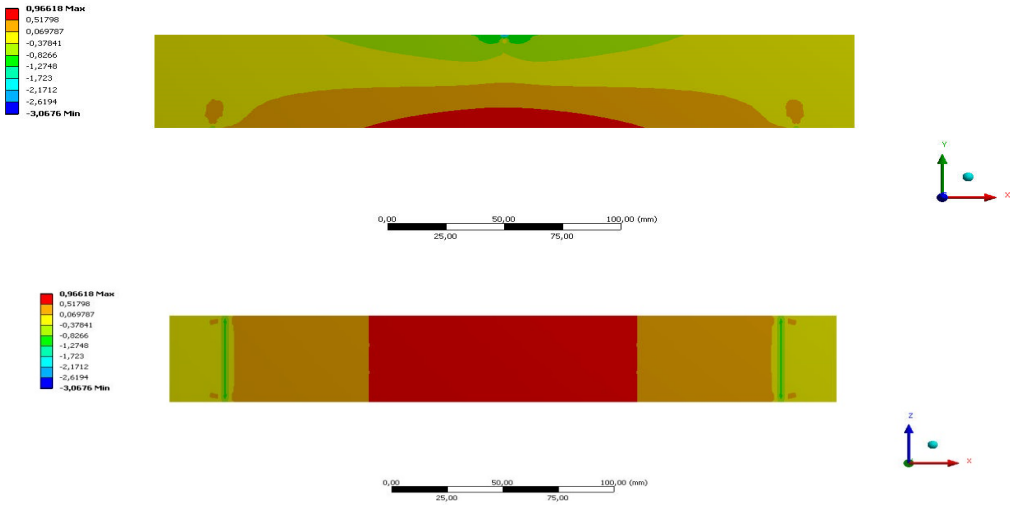

Figure 3. EN 12372_50*50*300 mm a) frontal surface b) lower surface normal stress distribution

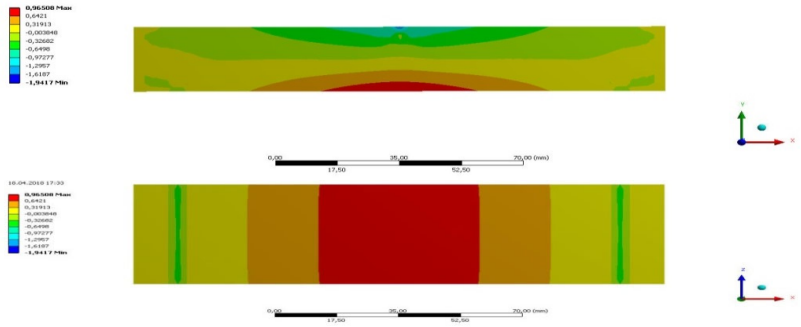

Figure 4. EN 12372_25*50*150 mm a) frontal surface b) lower surface normal stress distribution

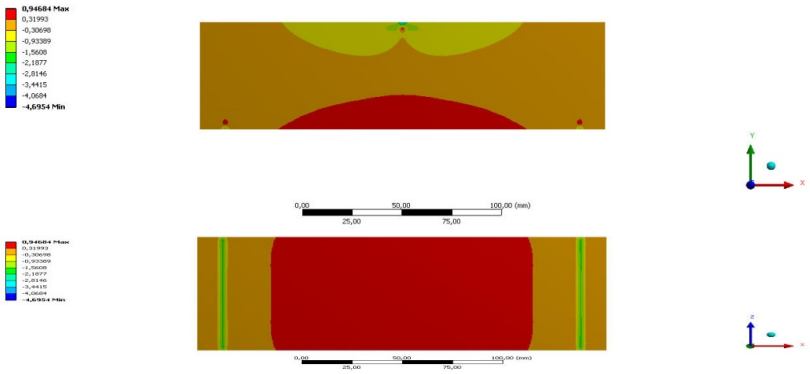

Figure 5. ASTM C99 a) frontal surface b) lower surface normal stress distribution

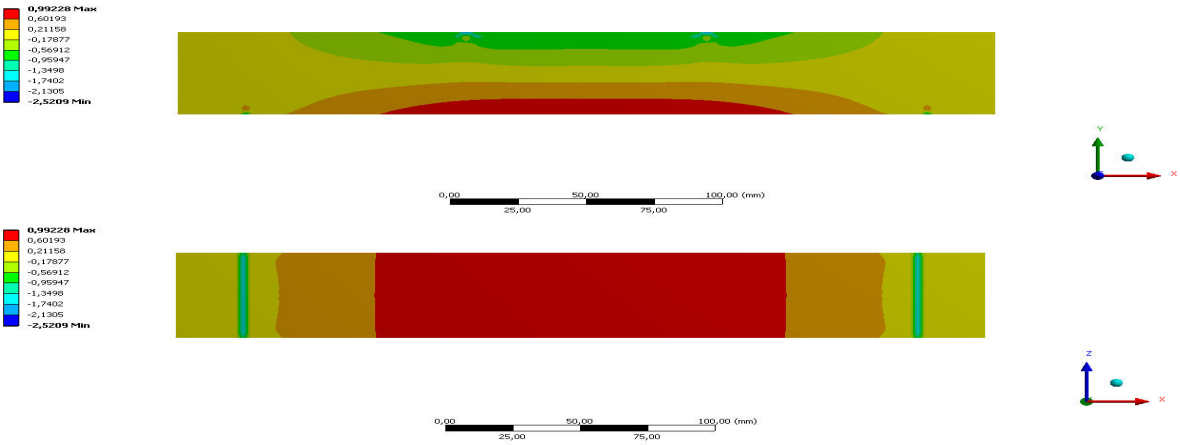

Figure 6. EN 13161_50*50*300 mm a) frontal surface b) lower surface normal stress distribution

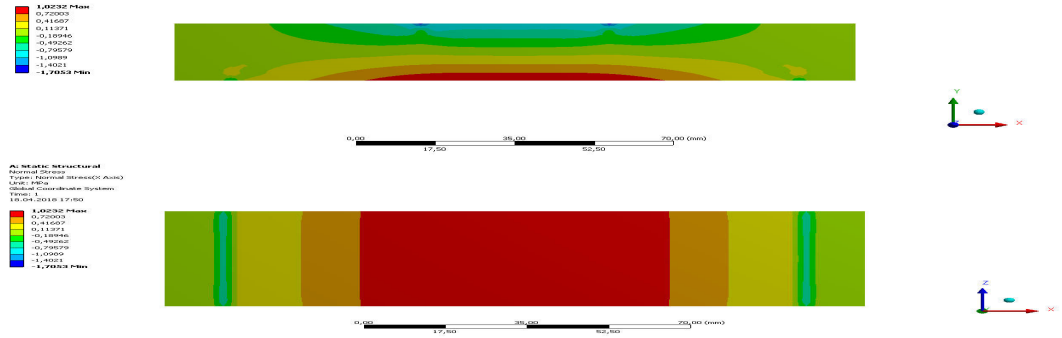

Figure 7. EN 13161_25*50*150 mm a) frontal surface b) lower surface normal stress distribution

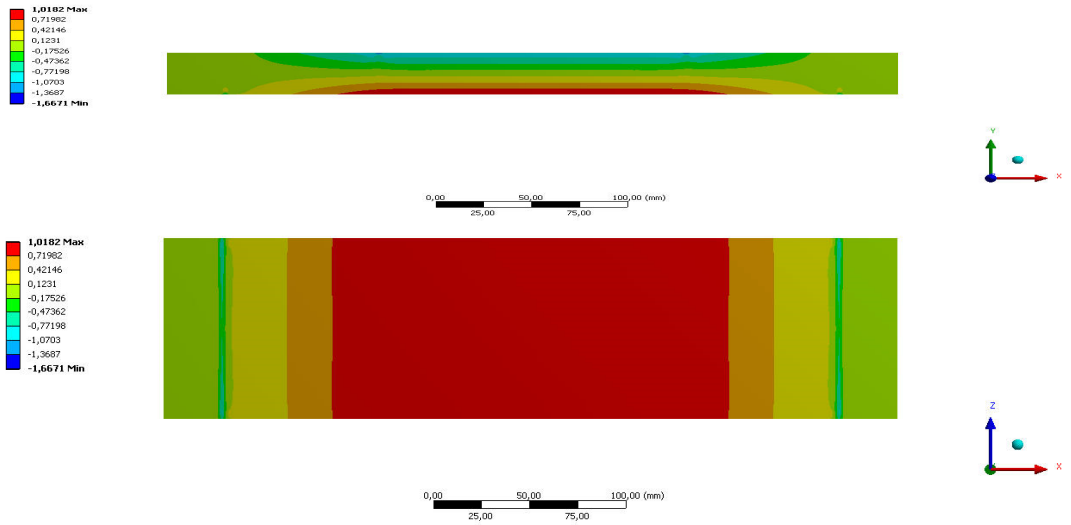

Figure 8. ASTM C880 a) frontal surface b) lower surface normal stress distribution

In the finite element analysis applied to the samples of the different standards, the normal stresses occurring at the interface between the front surface and the lower surface of the sample are shown in Figure 9. As a result of the analyses, the numerical results obtained from the 4PBS experiments are closer to the analytical results than the numerical results obtained from the 3PBS experiments. The 1 MPa stress calculated analytically in the 3PBS experiments is a single point in the two-dimensional plane. In the finite element model, the element with a certain dimension is placed to this point and the average stress is calculated approximately by determining the forces acting on both surfaces of this element. Since the stress distribution along the length is not constant and changes linearly, the forces in the finite element model are lower than the calculated forces. In the 4PBS experiments, the 1 MPa stress, calculated analytically in a two-dimensional plane, is formed along a line in the range of two forces. Because the forces acting on both two surfaces of the finite element placed in the mid-section are uniform, it is closer to the analytical results.

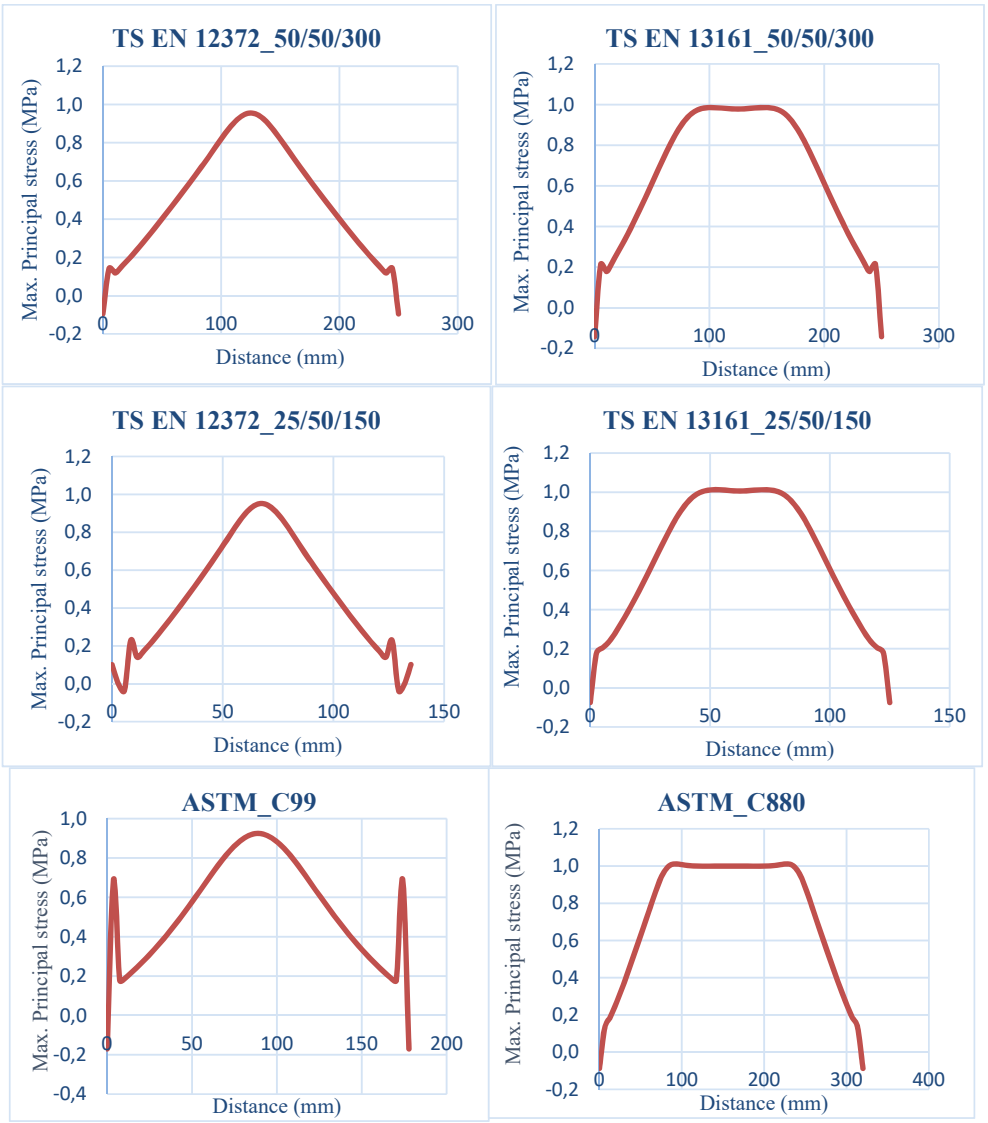

Figure 9. Maximum principal stress

4. Results and discussions

3PBS, 4PBS and DTS test results are presented in Table 3 and graphically shown in Figure 10. According to the results, the average of 4PBS values was found to be 32.4% lower for samples with dimensions of 50*50*300 mm, 35.6% lower for samples with dimensions 25*50*150 mm and 6.2% lower for samples with dimensions conforming to the ASTM standard, according to the average of 3PBS values. The highest 3PBS results were observed in samples of sizes 50*50*300 mm in accordance with EN 12372 standard, the highest 4PBS results were observed in samples of sizes 102*32*381 mm in accordance with ASTM 880-98 standard.

Table 3. Bending strength and direct tensile strength test results

3 Point Bending Strength (MPa)						Direct Tensile Strength (MPa)	
(EN)				**(ASTM)**			
50*50*300 mm		25*50*150 mm		101.6*203.2*57.2 mm			
Mean	SD	Mean	SD	Mean	SD	Mean	SD
15.80	4.04	12.94	1.17	14.26	1.69	3.19	0.61
4 Point Bending Strength (MPa)							
(EN)				**(ASTM)**			
50*50*300 mm		25*50*150 mm		32*102*381 mm			
Mean	SD	Mean	SD	Mean	SD		
10.68	0.93	8.33	0.93	13.38	0.98		

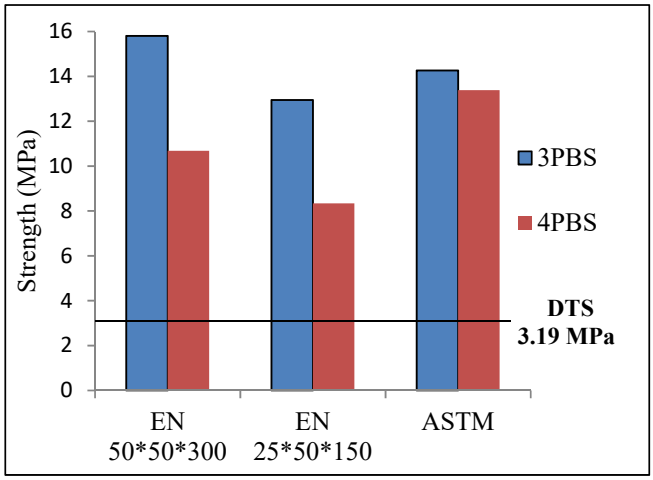

Figure 10. Comparison of bending and tensile strength results of marble

According to the results of the dumbbell shaped DTS tests, the average tensile strength of the fine crystalline marble investigated in this study were found to be 3.19 MPa. In some studies [4-7], researchers mention that the bending strength tests (both 4 and 3 point) in rocks are one of the indirect methods used to predict the tensile strength. However, in this study, the three point bending strength results were determined 4.95, 4.06 and 4.47 times higher than the DTS result, respectively. Similarly, the four point bending strength results were found to be 3.35, 2.61 and 4.19 times higher than the DTS result, respectively.

5. Conclusions

Three and four bending testing methods are most commonly performed according to ISRM, ASTM, and EN standards. The effect of sample dimension for a consistent three and four points bending value has not been well defined in previous works. The aim of this study, in order to analyses the effect of sample dimension on bending strength property of fine crystalline marble and is to determine the direct tensile strength (DTS) of rock by using new dumbbell shape. It was also investigated that whether the relation exists between direct and indirect tensile test results. In this study, in both 4PBS and 3PBS experiments, it has been found that the dimensions of the test sample play an important role in determining the value of the bending strength. As a result of the FEM, the numerical results obtained from the 4PBS experiments are found to be closer to the analytical results than the numerical results obtained from the 3PBS experiments. Moreover, four-point bending test is more suitable for determining of tensile strength of marble. According to the results obtained from this study, it is suggested that new scientific studies (including correction coefficients) should be carried out on more rock types in order to be able to estimate tensile strengths from bending experiments.

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