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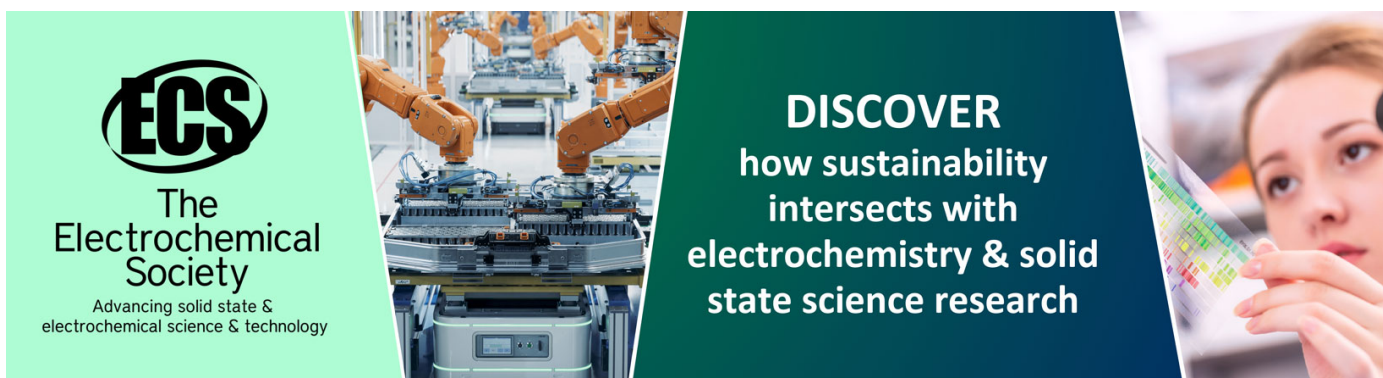
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The fluctuation theory of radio occultation signals: geometric optical approximation of the Canonical Transform method

O A Koval¹, M E Gorbunov^{1,2}, V Kan¹

¹A. M. Obukhov Institute of Atmospheric Physics RAS, Pyzhevsky per. 3, Moscow, 119017, Russia

²Hydrometeorological Research Center of Russian Federation, 11-13, Bolshoy Predtechensky per., Moscow, 123242, Russia

Corresponding author: O A Koval kov.oksana20@gmail.com

Abstract. The Canonical Transform (CT) method in different modifications is widely employed for processing radio occultation data. The leading idea of the method consists in the transformation of the measured wave field into the diffractionless representation, where the CT amplitude is only defined by the horizontal gradient of the atmospheric refractivity field. In this paper, we develop the theory of the fluctuation of the CT amplitude in the framework of the geometric optics (GO). The GO approximation is based on the Hamilton form of the ray equations, transformed into the representation of the ray impact parameter. The variations of the CT amplitude are defined by the variation of the ray tube cross-section. We derive the expression for the CT amplitude fluctuation spectrum as an operator on the 3-D fluctuation spectrum of the atmospheric refractivity described by the Kolmogorov turbulence model. We perform the numerical simulation using the split-step method, which indicates that the expression for the spectrum is in a good agreement with the simulated amplitude fluctuation. We estimate of the diffraction limit of the GO approximation.

1. Introduction

Currently, the radio occultation (RO) method is successfully applied for the numerical weather prediction and climate change monitoring. The RO method has the following advantages: stability, global coverage, absence of the necessity of calibration, low cost, all-weather capabilities, and high vertical resolution. Nevertheless, this method has its limitations: low horizontal resolution, sensitivity to residual ionospheric noise above 30 km, ambiguity of temperature retrieval in presence of humidity, and biases in the lower troposphere [1].

The impressive success of the GPS/MET experiment stimulated the further development of RO satellite and constellations, including Challenging Minisatellite Payload (CHAMP) and Constellation Observing System for Meteorology, Ionosphere, and Climate (COSMIC). RO data are employed by the leading centers for the numerical weather prediction, such as European Centre for Medium-Range Weather Forecasts (ECMWF) [1]–[5].

Due the aforementioned advantages of the RO method, it is not only useful for the retrieval of regular profiles of meteorological variables, but also for the study of random inhomogeneities of atmospheric refractivity from the fluctuations of received signals [5]–[9]. The observations of stellar scintillations indicated that the Earth's atmosphere is characterized by the following two types of



inhomogeneities: 1) isotropic fluctuations (Kolmogorov turbulence) and 2) strongly anisotropic layered structures (internal gravity waves). Based on these observations, the empiric two-component model of the 3-D inhomogeneity spectrum is developed [10]–[12].

An increasing number of papers discuss the use of GPS RO for the study of random atmospheric inhomogeneities. In particular, in [13], a study of atmospheric turbulence is presented, based on the Canonical Transform (CT) method. The authors applied the Rytov weak fluctuation theory for the estimate of the CT amplitude fluctuations. However, the wave field transformed into the impact parameter representation, according to the CT method, is described by an equation that differ from the standard parabolic equation, for which the Rytov method is formulated.

In this paper, we develop the theory of RO signal fluctuations based on the geometric optical (GO) approximation of the CT method. The CT method [14], [14] is used for the retrieval of bending angle profiles from RO observations, based on the reconstruction of ray manifold structure of the wave field. There are different choices of the canonical coordinates in the phase space. In the CT method, the ray impact parameter is used as the unique coordinate of the ray manifold for the case of weak horizontal gradients of refractivity [14].

Meteorological parameters are retrieved from RO observations of amplitude and excess phase [14]–[17], while the ionospheric contribution is removed [18], [19]. At different stages of the RO data processing, different approximations are involved. Due to the observation geometry, there is a limitation of the maximum value of the observed bending angle [19]. This is explained by the fact that in this case it is necessary to measure very weak signals, which are sensitive to noise.

Bending angle and impact parameter can be evaluated from the Doppler frequency shift and observation geometry under the assumption of local spherical symmetry of the atmosphere. Because the real atmosphere has horizontal gradients, the concept of the effective bending angle and impact parameter is introduced [21], which are evaluated using the formulas for a spherically layered atmosphere, but are looked at as known functionals from the 3-D refractivity field.

The basic assumption in all the approaches based on Fourier Integral Operators (FIO) is that the impact parameter should be a unique coordinate of the ray manifold and, therefore, bending angle is a unique function of impact parameter. This assumption was used in the Back Propagation method [22], [23], CT method [14], [14], Full Spectrum Inversion (FSI) method [24], and Phase Matching (PM) method [24]. Random fluctuations of refractivity field that result in random fluctuations of observed amplitudes and phases can also result in biases [1]. The weak fluctuation theory was developed in [26], [27]. The idea of studying the transformed field, instead of the observed field, was first introduced in [13]. The value of this approach consists in the fact that the FIO used in the CT method removes the effects of the wave field propagation in the free space. Therefore, it mitigates the limitations due to the Fresnel zone size. Because the field in the impact parameter representation is described by a different (not the standard parabolic) equation, the Rytov weak fluctuation theory cannot be applied. In this paper, we derive the fluctuation spectrum of CT amplitude using the GO approximation of the CT method. In order to validate the derived expression, we perform a numerical simulation based on the split-step method with random phase screens for the Kolmogorov isotropic turbulence model.

2. The derivation of the CT amplitude fluctuation spectrum in the GO approximation

We are using the following model of the refractivity fluctuation spectrum:

$$\Phi_v = AC^2\eta^2 \left(\eta^2(\tilde{s}^2 + \tilde{s}'^2) + \tilde{p}^2 + K^2 \right)^{-\mu/2} \exp \left(-\frac{\eta^2(\tilde{s}^2 + \tilde{s}'^2) + \tilde{p}^2}{k^2} \right), \quad (1)$$

where $\tilde{s}, \tilde{s}', \tilde{p}$ are the 3-D components of vector κ , $\tilde{s}$ being the along-ray horizontal component, $\tilde{s}'$ being the transversal component, and $\tilde{p}$ being the vertical component, the x axis points in the direction of the incident ray; C^2 is the structure constant defining the intensity of fluctuations v , η is

the anisotropy coefficient, which equal the ratio of the horizontal and vertical scales, $K = \frac{2\pi}{L}, k = \frac{2\pi}{l}$ are the external and internal scales, respectively.

We evaluate the fluctuation spectrum in the straight-line approximation. The coordinates along the ray trajectory have the following form:

$$\theta(s) = \frac{s}{a}, \quad r(s) = \sqrt{p^2 + s^2} = p + \frac{s^2}{2a}, \quad x = r\Delta\theta = s, \quad (2)$$

where a is the Earth's radius.

The analysis of the fluctuations will be based on the assumption that relative fluctuations of the refractivity are statistically homogeneous in the space [31]. This allows the representation of refractivity in the form $N = \langle N \rangle (1 + v) = \langle N \rangle + \delta n$, where $\langle N \rangle$ is the average value, and v describes the random relative fluctuations, $\langle v \rangle = 0$, and variance $\langle v^2 \rangle \ll 1$. In a way similar to [31], the wave field in the transformed space is represented in the following form:

$$u'(p) = \int f(r(s), s) \exp\left(-\frac{r(s) - a}{H}\right) \exp\left(\frac{p - a}{H}\right) ds, \quad (3)$$

where H is the height scale of the homogeneous atmosphere, approximately equal to 8 km. According to [32]:

$$f(r(s), s) = \frac{\partial \delta n(r(s), s)}{\partial \theta}. \quad (4)$$

We represent function $f(r, s)$ in terms of its Fourier transform:

$$f(r(s), s) = \frac{1}{2\pi} \int \tilde{f}(\tilde{r}, \tilde{s}) \exp(ir(s)\tilde{r} + is\tilde{s}) d\tilde{r} d\tilde{s}. \quad (5)$$

Switching to the ray trajectory coordinates (2) and taking the Fourier transform according to (5), we arrive at the following expression:

$$f(r(s), s) = a \frac{\partial \delta n(r(s), s)}{\partial s} = \frac{a i \tilde{s}}{2\pi} \int \delta \tilde{n}(\tilde{r}, \tilde{s}) \exp(ir(s)\tilde{r} + is\tilde{s}) d\tilde{r} d\tilde{s}. \quad (6)$$

The Fourier transform of the wave field in the transformed space as a function of the horizontal gradient of refractivity can be inferred as follows:

$$\tilde{u}'(\tilde{p}) = i a \int \delta \tilde{n}(\tilde{r}, \tilde{s}) \tilde{s} \sqrt{\frac{aH}{1 - i\tilde{p}H}} \exp\left(-\frac{aH\tilde{s}^2}{2(1 - i\tilde{p}H)}\right) d\tilde{s} \quad (7)$$

Differentiating $\tilde{u}'(\tilde{p})$ with respect to the impact parameter $\tilde{p}$, we obtain the variation of the CT amplitude, because $\tilde{u}'(\tilde{p})$ corresponds to δp_R , and $f(r(s), s)$ is variation of gradient $\frac{\partial \delta n(r_0, \theta)}{\partial \theta}$, according to (2), with respect to s :

$$\begin{aligned} \frac{C}{2} \frac{d}{d\tilde{p}} \tilde{u}'(\tilde{p}) &= \frac{C}{2} i a \sqrt{aH} \int \tilde{s} \exp\left(-\frac{aH\tilde{s}^2}{2(1 - i\tilde{p}H)}\right) \times \\ &\times \left[\frac{d\delta \tilde{n}(\tilde{p}, \tilde{s})}{d\tilde{p}} \frac{1}{\sqrt{(1 - i\tilde{p}H)}} + \frac{\delta \tilde{n}(\tilde{p}, \tilde{s}) i H}{2(1 - i\tilde{p}H)^{3/2}} - \frac{\delta \tilde{n}(\tilde{p}, \tilde{s}) i a (H\tilde{s})^2}{2(1 - i\tilde{p}H)^{5/2}} \right] d\tilde{s} \end{aligned} \quad (8)$$

Taking the Fourier transform of the derivative and remembering Eq. (5), we infer that the factor inside the integral $\frac{\partial \delta \tilde{n}(\tilde{p}, \tilde{s})}{\partial s}$ equals $i \tilde{s} \delta \tilde{n}(\tilde{p}, \tilde{s})$, and $\frac{d\delta \tilde{n}(\tilde{p}, \tilde{s})}{d\tilde{p}}$ equals $i \tilde{p} \delta \tilde{n}(\tilde{p}, \tilde{s})$, where $\tilde{s}$ and $\tilde{p}$ are the frequency factors.

$$\frac{C}{2} \frac{d}{d\tilde{p}} \tilde{u}'(\tilde{p}) = \frac{aC}{2} \int \tilde{s} \sqrt{\frac{aH}{1-i\tilde{p}H}} \delta\tilde{n}(\tilde{p}, \tilde{s}) \left[-\tilde{p} - \frac{H}{2(1-i\tilde{p}H)} + \frac{a(H\tilde{s})^2}{2(1-i\tilde{p}H)^2} \right] \exp\left(-\frac{aH\tilde{s}^2}{2(1-i\tilde{p}H)}\right) d\tilde{s} \quad (9)$$

Due to the spectral density definition:

$$\langle \tilde{u}'(\tilde{p}'), \tilde{u}'^*(\tilde{p}'') \rangle = \delta(\tilde{p}' - \tilde{p}'') \Phi_u(\tilde{p}), \quad (10)$$

The spectral density of the CT amplitude can then be expressed as follows:

$$\frac{C^2}{4} \left\langle \frac{d\tilde{u}'(\tilde{p}')}{d\tilde{p}'}, \frac{d\tilde{u}'^*(\tilde{p}'')}{d\tilde{p}''} \right\rangle = \delta(\tilde{p}' - \tilde{p}'') \Phi_u(\tilde{p}), \quad (11)$$

From this, we derive:

$$\Phi_u(\tilde{p}) = \frac{C^2 a^3 H}{16} \int \frac{\Phi_v(\tilde{p}, \tilde{s}) \tilde{s}^2}{(1+(H\tilde{p})^2)^{5/2}} \exp\left(-\frac{aH\tilde{s}^2}{1+(H\tilde{p})^2}\right) \quad (12)$$

$$\left(4\tilde{p}^2 (1+(H\tilde{p})^2)^2 + 4H\tilde{p} (1+(H\tilde{p})^2) - 4\tilde{p}a(H\tilde{s})^2 (1-(H\tilde{p})^2) + H^2 (1+(H\tilde{p})^2) - 2aH^3 \tilde{s}^2 + a^2 (H\tilde{s})^4 \right) d\tilde{s},$$

where $\Phi_v(\tilde{p})$ satisfies $\langle \delta\tilde{n}(\tilde{p}', \tilde{s}'), \delta\tilde{n}(\tilde{p}'', \tilde{s}'') \rangle = \Phi_v(\tilde{p}, \tilde{s}) \delta(\tilde{p}' - \tilde{p}'') \delta(\tilde{s}' - \tilde{s}'')$. Due to the definition of v :

$$\Phi_v(\tilde{p}, \tilde{s}) = \bar{N}(\tilde{p}') \bar{N}(\tilde{p}'') \iint \langle \tilde{v}(\tilde{p}_1, \tilde{s}_1), \tilde{v}(\tilde{p}_2, \tilde{s}_2) \rangle \exp\left(-\frac{\tilde{p}' + \tilde{p}'' - (\tilde{p}_1 + \tilde{p}_2)}{H}\right) d\tilde{s}_1 d\tilde{s}_2, \quad (13)$$

where $\tilde{p}' = \sqrt{\tilde{p}_1^2 + \tilde{s}_1^2}$, $\tilde{p}'' = \sqrt{\tilde{p}_2^2 + \tilde{s}_2^2}$. Finally, we arrive at the following expression:

$$\Phi_u(\tilde{p}) = \frac{AC^2 a^3 H}{16} \int \frac{\eta^2 \left(\eta^2 (\tilde{s}^2 + \tilde{s}'^2) + \tilde{p}^2 + K^2 \right)^{-\mu/2} \exp\left(-\frac{\eta^2 (\tilde{s}^2 + \tilde{s}'^2) + \tilde{p}^2}{k^2}\right) \tilde{s}^2}{(1+(H\tilde{p})^2)^{5/2}} \exp\left(-\frac{aH\tilde{s}^2}{1+(H\tilde{p})^2}\right) \times \quad (14)$$

$$\times \left(4\tilde{p}^2 (1+(H\tilde{p})^2)^2 + 4H\tilde{p} (1+(H\tilde{p})^2) - 4\tilde{p}a(H\tilde{s})^2 (1-(H\tilde{p})^2) + H^2 (1+(H\tilde{p})^2) - 2aH^3 \tilde{s}^2 + a^2 (H\tilde{s})^4 \right) d\tilde{s}$$

Expression (14) can be approximated for large $\tilde{p}$ as follows:

$$\Phi_u(\tilde{p}) = \frac{C^2}{16} a^3 H A \exp\left(-\tilde{p}^2 \frac{3l^2}{4\pi^2}\right) \times \quad (15)$$

$$\times \left[\left(\frac{1}{3H} - \frac{2a}{5H^2} \right) \tilde{p}^{-11/3} + \frac{4}{3H^2} \tilde{p}^{-8/3} + \frac{a^2}{7H} \tilde{p}^{-5/3} + \frac{4}{3H} \tilde{p}^{1/3} + \frac{4aH}{5} \tilde{p}^{4/3} \right]$$

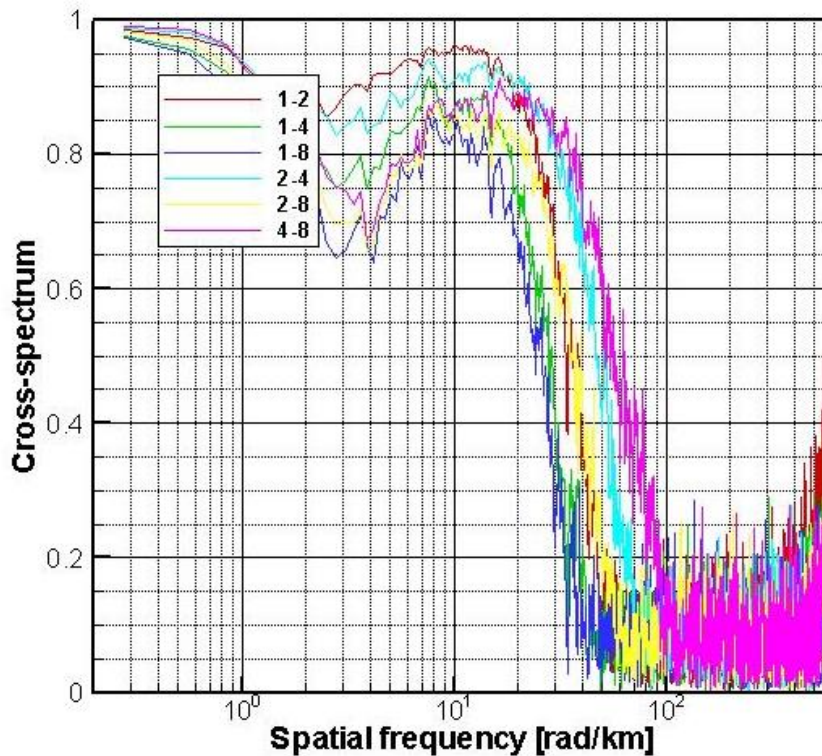


Figure 1. Cross-spectra of CT amplitude evaluated for the wave field simulated by the split-step method with screen-to-screen step of 5 km, for frequency channels of 1, 2, 4, and 8 GHz.

3. Numerical Simulations

In order to validate the above expression for the fluctuation spectrum of the CT amplitude (15), we performed a numerical simulation using the split-step method with random phase screens [1] and evaluated the normalized spectral density for the frequency channels of 1, 2, 4 и 8 GHz. For the simulation, we used the Kolmogorov spectrum with $\eta = 1$, $\mu = 11/3$, $A = 0.033$.

Figures 1 and 2 present the cross-spectra of the CT amplitude for different frequency channels for the scree-to-screen distance of 5 and 10 km. The differences between these two plots are insignificant. Currently, there is no any reliable theoretical estimate of the diffraction limit for the wave field in the transformed space. It can be inferred from these plots, because in the GO approximation, the CT amplitude should be independent on the frequency channel, and, therefore, its fluctuations should be identical for different channels. For the frequency channels in question, the diffraction effects become visible for spatial scales below 50–100 m.

Figures 3 and 4 present the CT amplitude fluctuation spectra for screen-to-screen steps of 5 and 10 km. Similar to figures 1 and 2, the results for the two screen-to-screen steps are practically identical, which confirms that the step is chosen small enough for obtaining stable results. The plots clearly indicate the diffractive decay of the spectrum. The spatial frequency, where the decay starts, is proportional to the square root of the channel frequency. The result for the analytical formula (15) are in a good agreement with the low-frequency part of the fluctuation spectra. The decay of the analytical spectrum (15) is explained by the influence of the internal scale, which was chosen to be 100 m.

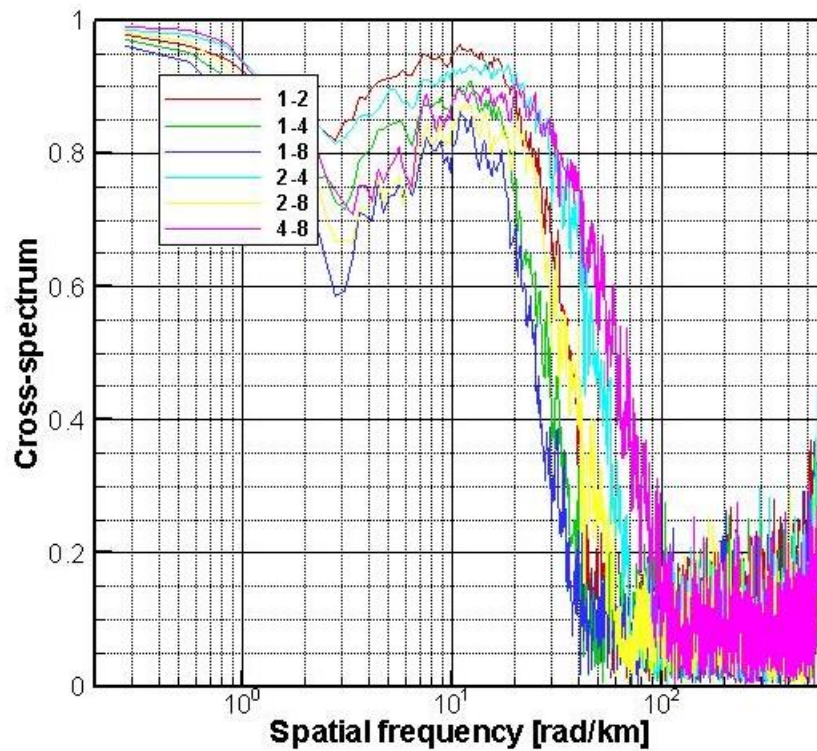


Figure 2. Cross-spectra of CT amplitude evaluated for the wave field simulated by the split-step method with screen-to-screen step of 10 km, for frequency channels of 1, 2, 4, and 8 GHz.

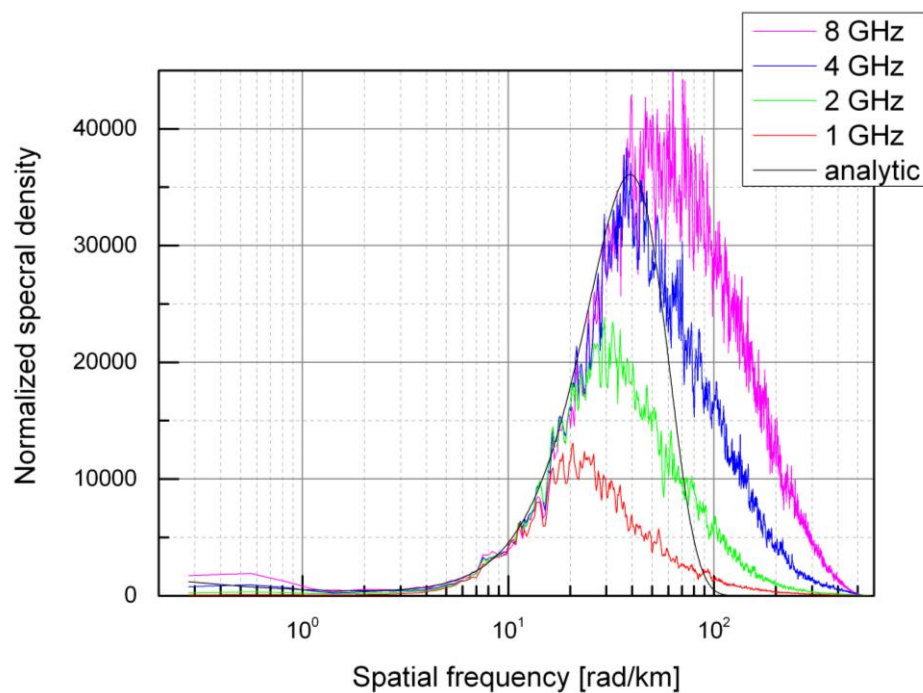


Figure 3. Spectra of CT amplitude evaluated for the wave field simulated by the split-step method with screen-to-screen step of 5 km, for frequency channels of 1, 2, 4, and 8 GHz, in comparison to the analytical expression (15) with the internal scale $l = 100$ m.

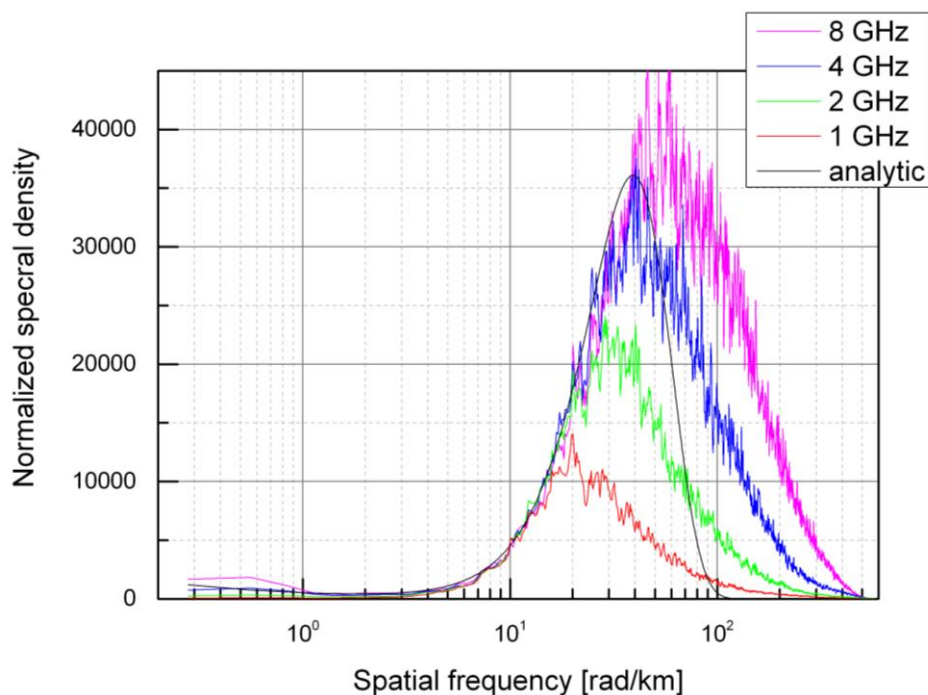


Figure 4. Spectra of CT amplitude evaluated for the wave field simulated by the split-step method with screen-to-screen step of 10 km, for frequency channels of 1, 2, 4, and 8 GHz, in comparison to the analytical expression (15) with the internal scale of 100 m.

4. Conclusions

In this study, we, for the first time, derive the expression for the spectral density of amplitude, of wave field observed in RO experiments and transformed into the impact parameter representation. We used a general model of the refractivity fluctuation spectral density obeying the power law with the constant anisotropy. We applied the geometric optical approximation of the Canonical Transform method [5], [26], [27], [31]. We performed a numerical simulation of RO sounding of the atmosphere with random fluctuations of the refractivity and evaluated the fluctuation spectra of the CT amplitude. The comparison of the simulation results with the analytical formula indicated their good agreement below the diffractive limit of the spatial frequencies. The results of this paper can be useful for the study of the atmospheric refractivity fluctuations. The CT amplitude is more convenient than the observed amplitude, because the former is insensitive to regular spherically layered structures.

Acknowledgments

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References

- [1] Gorbunov M E , Vorob'ev V V and Lauritsen K B 2015 *Radio Sci* **50** 656
- [2] Gorbunov M E 2014 Statistical analysis of systematic errors in RO measurements *ROM SAF Visiting Scientist Report No. 20* (Copenhagen: Danish Meteorological Institute)
- [3] Rocken C, Kuo Y-H, Schreiner W S, Hunt D, Sokolovskiy S and McCormick C 2000 *COSMIC System Description, Terrestrial, Atmospheric and Oceanic Science* **11** 21
- [4] Poli P, Healy S B and Dee D P 2010 *Q. J. R. Meteorol. Soc.* **136** 1972

- [5] Rennie M P 2010 *Quarterly Journal of the Royal Meteorological Society* **136** 116
- [6] Belloul M B and Hauchecorne A 1997 *Radio Sci.* **32** 469
- [7] Gurvich A S, Kan V and Fedorova O V 2000 *Izv. Atmos. Ocean. Phys.* **36** 300
- [8] Tsuda T, Nishida M, Rocken C and Ware R H 2000 *J. Geophys. Res.* **105** 7257
- [9] Wang L and Alexander M J 2010 *J. Geophys. Res.* **115** D21122
- [10] Gurvich A S and Brekhovskikh V L 2001 *Waves in Random Media* **11** 163
- [11] Gurvich A S and Kan V 2003 *Izv. Atmos. Ocean. Phys.* **39** 300
- [12] Gurvich A S and Kan V 2003 *Izv. Atmos. Ocean. Phys.* **39** 311
- [13] Shume E and Ao C 2016 *Atmos. Meas. Tech.* **9** 3175
- [14] Gorbunov M E 2002 *Radio Sci.* **37** 5 1076
- [15] Gorbunov M E and Lauritsen KB 2004 *Radio Sci.* **39** RS4010
- [16] Gorbunov M E, Lauritsen K B, Rodin A, Tomassini M and Kornblueh L 2005 *Izv. Atmos. Ocean. Phys.* **41** 726
- [17] Gorbunov M E, Gurvich A S and Bengtsson L 1996 Advanced algorithms of inversion of GPS/MET satellite data and their application to reconstruction of temperature and humidity *Tech. Rep. Rep. No. 211* (Hamburg: Max Planck Institute for Meteorology)
- [18] Vorob'ev V V and Krasil'nikova T G 1994 *Izv. Atm. Ocean. Phys.* **29** 602
- [19] Gorbunov M E 2002 *Radio Sci.* **37** 9-1
- [20] Sokolovskiy S, Rocken C, Schreiner W and Hunt D J. *Geophys. Res.* **115** D22111
- [21] Gorbunov M E and Kornblueh L 2003 Principles of Variational Assimilation of GNSS Radio Occultation Data *Report No. 350* (Hamburg: Max Planck Institute for Meteorology)
- [22] Gorbunov M E and Gurvich A S *J. Geophys. Res.* **103** D12 13,819
- [23] Gorbunov M E and Gurvich A S 1998 *Int. J. Remote Sens.* **19** 12 2283
- [24] Jensen A S, Lohmann M S, Benzon H H and Nielsen A S 2003 *Radio Sci.* **38** 3 1040
- [25] Jensen A S, Lohmann M S, Nielsen AS and Benzon H H 2004 *Radio Sci.* **39** RS3009
- [26] Tatarskii V I 1971 The Effects of the Turbulent Atmosphere on Wave Propagation (Jerusalem: Israel Program for Scientific Translations)
- [27] Rytov S M, Kravtsov Y A and Tatarskii V I 1989 Principles of Statistical Radiophysics: Wave Propagation Through Random Media (Berlin: Springer) vol. 4
- [28] Kravtsov Y A and Orlov Y I 1990 Geometrical Optics of Inhomogeneous Media (New York: Springer-Verlag)
- [29] Egorov Y V 1985 Lectures on Partial Differential Equations: Additional Chapters (in Russian), (Moscow: Moscow State Univ. Press)
- [30] Arnold V I 1978 Mathematical Methods of Classical Mechanics (New York: Springer-Verlag)
- [31] Gurvich A S 1984 *Radiophysics and Quantum Electronics* **27** 665
- [32] Gorbunov M E and Kornblueh L 2001 *J. Geophys. Res.* **106** D15 17,161