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BDS Triple-frequency Medium and Long Baseline Ambiguity Resolution by Taking into Account of The Ionospheric Delay Residuals

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Abstract. In order to solve the shortcomings of the classic geometry-free TCAR algorithm in the process of the medium-long baseline ambiguity resolution, this paper puts forward the expression of the combined ambiguity standard deviation by considering the ionospheric delay residuals, tropospheric delay residuals, satellite orbital residuals and observation noise. It also comes up with the total noise level(TNL) of many optimal extra-wide lane(EWL), wide lane(WL) and narrow lane(NL) combinations based on the existing empirical value of each residual item under the condition of 100km, 100-200km and greater than 200km baseline length. Moreover, it proves that ionospheric delay residuals are the most dominant error source in the medium-long baseline carrier phase ambiguity resolution by analyzing the correlation between the total noise level(TNL) of each combination and its corresponding ionospheric delay residuals. Finally, it presents an ionospheric delay correction algorithm and then carries out experimental verification and feasibility analysis of the algorithm based on the beidou triple-frequency measured data. Results show that the fluctuation range of the NL float ambiguity can be reduced from 10 cycle to less than 1 cycle.

1. Introduction

With the development and improvement of the triple-frequency technology, the TCAR(Three triple-frequency Carrier Ambiguity Resolution) algorithm based on triple-frequency combination observation is widely used in the rapid ambiguity resolution for GNSS. However, the TCAR algorithm easily influenced by errors such as ionospheric delay residuals is hard to be implied in rapid ambiguity resolution under medium-long baseline conditions.

At present, relevant scholars at home and abroad have carried out a lot of research on the problems existing in the TCAR algorithm. Teunissen et al. studied the constraints of the geometry-free model on the rapid ambiguity resolution under the medium and long baseline conditions, and then deeply analyzed the effects of frequency distribution on the ambiguity resolution, and finally pointed out that the model could not realize the instantaneous solution of ambiguity under medium and long baseline conditions[4,5]. Forssell et al. first solved the extra-wide-lane(EWL) and wide-lane(WL) ambiguity that were less affected by the ionospheric delay residuals, and then solved the ionospheric delay residuals by using the combined observations with fixed ambiguity. [6]. Dai and Zhao proposed an improved algorithm for the TCAR algorithm[7,8]. Firstly, the WL ambiguity was resolved by combining the pseudo-range observation, WL combination observation and the EWL combination



observation whose ambiguity had been resolved, and then the NL combination observation was combined to resolve the narrow-lane ambiguity. Weiming Tang et al. analyzed the characteristics of different errors in triple-frequency Beidou observations, discussed three different TCAR algorithms, confirmed that the cascading rounding or step-wise AR algorithms are only applicable for the short baselines to achieve a relative high reliability, while the modified step-wise AR algorithm can be used in the medium-long baselines, retaining the same level of reliability[9]. Xiaohong Zhang et al. studied the influence of the systematic errors on wide-lane ambiguity resolution by analyzing the consistency of the fractional part of the averaged SD HMW combinations and the fixing rate of DD wide-lane ambiguity resolution[10].

Based on previous research, this paper firstly introduces the basic principle of classic TCAR algorithm, and further analyzes the influence factors that limit the long baseline ambiguity resolution. To determine the NL ambiguity under long baseline condition, it puts forward the ionospheric delay correction method and analyzes the effect of ambiguity resolution in the case of long baseline with the help of two sets of Beidou triple-frequency measured data.

2. The basic principle of the classic TCAR algorithm

The classic TCAR algorithm is a kind of geometric-free cascading rounding algorithm. The solution is that the double-differenced(DD) geometry-free combination is constructed which can get rid of all kinds of errors to the maximum extent, and then rounding the float ambiguities directly to its nearest integer values by making sure that the synthetic error of the combined observation is less than one half of the combined wavelength. Finally the original double-differenced(DD) ambiguity is determined. By omitting receiver and satellite markers and double-difference symbols, the double-difference pseudo-range observation can be expressed as follows:

$$P = \rho + T_p + \mu_p I_1 + e_p \quad (1)$$

Where ρ is the DD geometric distance between satellite and receiver; T_p is the DD tropospheric delay; μ_p is the combined first-order ionospheric scale factor(ISF); I_1 is the DD first-order ionospheric delay on the B1 carrier; e_p is the pseudo-range observation noise.

At the same time, the subscripts "E", "W", and "N" are used to represent the extra-wide-lane (EWL), wide-lane (WL) and narrow-lane (NL) carrier phase combination respectively. Consequently, the meter-based EWL, WL and NL carrier phase combination observations Φ_E , Φ_W and Φ_N can be written as:

$$\Phi_E = \lambda_E \varphi_E = \rho + T_E - \mu_E I_1 + \lambda_E N_E + \lambda_E \varepsilon_E \quad (2)$$

$$\Phi_W = \lambda_W \varphi_W = \rho + T_W - \mu_W I_1 + \lambda_W N_W + \lambda_W \varepsilon_W \quad (3)$$

$$\Phi_N = \lambda_N \varphi_N = \rho + T_N - \mu_N I_1 + \lambda_N N_N + \lambda_N \varepsilon_N \quad (4)$$

Where φ_E , φ_W and φ_N respectively represents the cycle-based observation values of the EWL, WL and NL carrier phase combination; μ_E , μ_W , and μ_N respectively stands for the ionospheric scale factor(ISF)of the EWL, WL, and NL combination; ε_E , ε_W and ε_N respectively means the combined observation noise of EWL, WL and NL.

The specific steps of the classical TCAR algorithm are as follows.

In the first step, the pseudo-range observation with the minimum noise and the EWL phase combination are selected for difference to obtain the expression of the EWL integer ambiguity:

$$N_E = \varphi_E - \frac{P}{\lambda_E} + \frac{T_p - T_E}{\lambda_E} + \frac{(\mu_p + \mu_E)I_1}{\lambda_E} + \frac{e_p}{\lambda_E} - \varepsilon_E \quad (5)$$

For the above equation, the tropospheric delay, ionospheric delay and observation noise are omitted, and the estimation of the EWL integer ambiguity can be expressed as:

$$N_E = \varphi_E - \frac{P}{\lambda_E} \quad (6)$$

The above equation is directly rounded and fixed, and the EWL integer ambiguity is obtained:

$$\overline{N_E} = \text{round}(N_E) \quad (7)$$

Where $\text{round}(\bullet)$ is a rounding symbol.

In the second step, the EWL integer ambiguity is substituted into formula(2), and then the EWL phase combination and the WL phase combination are differentiated to obtain the expression of WL ambiguity:

$$N_W = \varphi_W - \frac{\lambda_E(\varphi_E - N_E)}{\lambda_W} + \frac{T_E - T_W}{\lambda_W} - \frac{(\mu_E - \mu_W)I_1}{\lambda_W} + \frac{\lambda_E \varepsilon_E}{\lambda_W} - \varepsilon_W \quad (8)$$

Similarly, the estimation of WL ambiguity and its integer solution can be expressed as follows:

$$N_W = \varphi_W - \frac{\lambda_E(\varphi_E - N_E)}{\lambda_W} \quad (9)$$

$$\overline{N_W} = \text{round}(N_W) \quad (10)$$

In the third step, the WL integer ambiguity is substituted into formula(3), and then the WL phase combination and the NL phase combination are differentiated to obtain the expression of NL ambiguity:

$$N_N = \varphi_N - \frac{\lambda_W(\varphi_W - N_W)}{\lambda_N} + \frac{T_W - T_N}{\lambda_N} - \frac{(\mu_W - \mu_N)I_1}{\lambda_N} + \frac{\lambda_W \varepsilon_W}{\lambda_N} - \varepsilon_N \quad (11)$$

Similar to the first step, the estimation of NL ambiguity and its integer solution can be expressed as follows:

$$N_N = \varphi_N - \frac{\lambda_W(\varphi_W - N_W)}{\lambda_N} \quad (12)$$

$$\overline{N_N} = \text{round}(N_N) \quad (13)$$

Up to now, we have obtained three different integer ambiguity of different combined observations. Since the EWL, WL and NL combination are linearly combined by three original carrier phase observations, the integer ambiguity can be obtained from the three original phase observations by solving the equations by the three combined integer ambiguity. According to the above analysis, in order to ensure that the systems of equation has a unique solution, the combination coefficients of the selected three combined observations need to be linearly independent.

3. Factors affecting the medium-long baseline ambiguity resolution

From the first section analysis, the classical TCAR algorithm ignores the effects of tropospheric delay error, ionospheric delay error and observed noise in the ambiguity resolution process. When the baseline is long, these error terms are less correlated. The residual values in the double-difference observation model are large. If these error terms are not considered, the success rate and reliability of the fixed ambiguity will be seriously affected. Therefore, the application range of the algorithm is limited by the baseline length.

Considering the definition of combined observation and the influence of satellite orbital error, the expression of the standard deviation(STD) of ambiguity is given as follows:

$$\sigma = \frac{\sqrt{\mu^2 \delta_l^2 + \delta_{trop}^2 + \delta_{orb}^2 + [(\alpha \lambda_1)^2 + (\beta \lambda_2)^2 + (\gamma \lambda_3)^2] \sigma_\varepsilon^2}}{\lambda} \quad (14)$$

Where μ represents the ionospheric scale factor(ISF) in the combined observation, δ_I stands for the ionospheric delay residuals, δ_{trop} means the tropospheric delay residuals, δ_{orb} is the satellite orbital residuals, and σ_{ε} is STD of the observed noise.

According to the setting of empirical values of residuals within 100km, 100-200km, and greater than 200km baseline length(Huang 2015), the Total errors of different carrier phase combinations in table 1, also known as TNL, Total Noise Level, are analyzed.

Table 1. TNL of many optimal combinations

Combination Type	(i, j, k)	Wave		$\sigma(\text{cycle})$			
		Length (m)	μ				
				$\left\{ \begin{array}{l} \delta_I = 0.1m \\ \delta_{trop} = 0.05m \\ \delta_{orb} = 0.01m \\ \delta_{\varphi} = 0.01\text{cycle} \end{array} \right.$	$\left\{ \begin{array}{l} \delta_I = 0.2m \\ \delta_{trop} = 0.1m \\ \delta_{orb} = 0.02m \\ \delta_{\varphi} = 0.01\text{cycle} \end{array} \right.$	$\left\{ \begin{array}{l} \delta_I = 1m \\ \delta_{trop} = 0.15m \\ \delta_{orb} = 0.08m \\ \delta_{\varphi} = 0.01\text{cycle} \end{array} \right.$	
EWL	(0,-1,1)	4.884	-1.591	0.037	0.07	0.328	
	(1,4,-5)	6.371	0.652	0.066	0.07	0.124	
	(-1,-5,6)	20.932	-8.963	0.09	0.116	0.436	
WL	(1,0,-1)	1.025	-1.231	0.131	0.26	1.213	
	(1,-1,0)	0.847	-1.293	0.165	0.329	1.54	
	(3,11,-14)	1.48	-0.028	0.184	0.193	0.215	
NL	(4,-3,0)	0.114	0.072	0.453	0.902	1.615	
	(1,0,0)	0.192	1.000	0.585	1.169	5.282	
	(0,1,0)	0.248	1.672	0.704	1.408	6.769	
	(0,0,1)	0.236	1.514	0.676	1.352	6.449	

Table 1 analyzes the TNL of some optimal carrier phase combination observations in the case of three baselines of different lengths. The following conclusions can be drawn from the data in the table:

- Regardless of the length of the baseline, the error of the EWL combination ambiguity is relatively small, even when the tropospheric delay residuals, ionospheric delay residuals and orbital errors reach the maximum, the EWL combination(-1,-5,6) with the biggest TNL is also within 0.5cycle, which shows that the EWL ambiguity can still be resolved by directly rounding the ambiguity floating point under long baseline conditions. Given that the combination (0, -1, 1) has a relatively small TNL in the case of a short baseline, and in the case of a long baseline, the combination (1, 4, -5) has a relatively small total noise level, in the first step of solving the EWL ambiguity, different EWL combinations can be selected according to different baseline lengths.

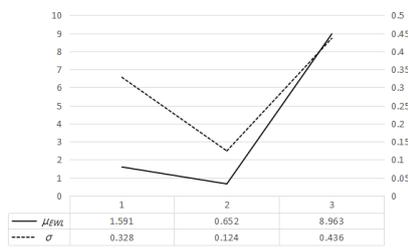
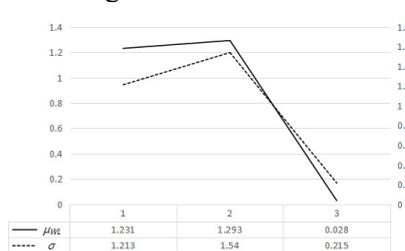
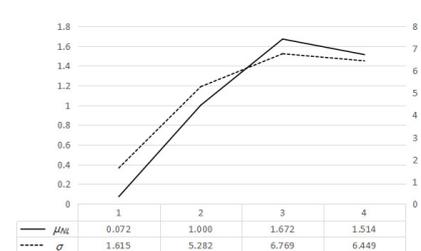
- When the baseline length is less than 100km, the TNL of the WL combination is at a low level, and its size is no more than 0.2cycle. Therefore, the correct ambiguity fixed solution can be obtained by directly rounding the WL ambiguity floating-point solution. However, when the baseline length exceeds 200km, the TNL of the combination (3,11,-14) is kept at a low level with a size of 0.215cycle, and the TNL of the other WL combinations is enlarged to more than 1.2cycle. The main reason that the TNL of the combination (3,11,-14) can maintain a low level at the long baseline is that, compared with other WL combinations, the combined ISF is very small, as shown in the table, the value is -0.028, only 1/43 of the other WL combinations, which shows that the ionospheric delay residuals is the most dominant error source in the long baseline carrier phase AR.

- The TNL of each NL combination is similar in the case of short baseline, and the size is between 0.4 cycle and 0.8cycle. However, as the baseline length increases, the TNLs of the two

original combinations (0,1,0) and (0,0,1) with larger ISF increase sharply. Under condition of baseline length exceeding 200km, the TNL has exceeded 6cycle, which is more than 4 times as many as that of most other WL combinations. That further illustrates that the ionospheric delay residuals is a major factor in the long-baseline AR.

• By analyzing the combination coefficients of each combination in Table 1, it can be known that any WL combination in the table can be linearly expressed by two EWL combinations. Therefore, in the long baseline AR, the two EWL ambiguities can be resolved first and then the linear combination can be used to solve the WL ambiguity in order to alleviate the difficulty of the WL AR.

Combined with the data in Table 1, FIG. 1-3 respectively shows the relationship schematic diagram between the ISF and the TNL of the EWL, WL and NL under the condition of more than 200km baseline. It can be seen from the figure that the TNL of each combination is positively correlated with the combined ISF under the condition of long baseline.

Figure 1. $\mu_{EWL} - \sigma$ Figure 2. $\mu_{WL} - \sigma$ Figure 3. $\mu_{NL} - \sigma$

4. Ionospheric delay correction algorithm

From the analysis in the previous section, it is known that the ionospheric delay residuals is a major influence factor for the difficulty of the medium-long baseline AR. However, the EWL and WL ambiguity can still be resolved accurately even in the case of long baseline. Therefore, both of them can be resolved first, and then the double-difference ionospheric delay residual can be inversely calculated, and the obtained result is substituted into the original observation equation to correct the ionospheric delay residuals in the original observation, and finally the success rate of the original carrier phase AR can be improved.

4.1 Theory of ionospheric delay correction

With the help of analysis results of the above section on the factors influencing the AR, the combination of EWL(0,-1,1), WL(1,-1,0) and NL(0,0,1) are selected for the combined AR. The carrier phase combination observation of EWL and WL with fixed ambiguity can be expressed as:

$$\Phi_E = \lambda_E \varphi_E = \rho + T_E - \mu_E I_1 + \lambda_E \overline{N_E} + \lambda_E \varepsilon_E \quad (15)$$

$$\Phi_W = \lambda_W \varphi_W = \rho + T_W - \mu_W I_1 + \lambda_W \overline{N_W} + \lambda_W \varepsilon_W \quad (16)$$

The meanings of the symbols in the formula are the same as those described in the first section. Thus, the double difference ionospheric delay residuals I_1 (in cycle) on the B1 carrier can be calculated by the simultaneous equations (15) and (16), and the calculation formula is as follows:

$$I = \frac{\lambda_E (\varphi_E - \overline{N_E}) - \lambda_W (\varphi_W - \overline{N_W})}{\mu_W - \mu_E} \quad (17)$$

The standard deviation is:

$$\sigma_I = \sqrt{\lambda_E^2 \sigma_{\varphi_E}^2 + \lambda_W^2 \sigma_{\varphi_W}^2} / (\mu_W - \mu_E) \quad (18)$$

The ionospheric delay correction obtained by equation(17) is substituted into equation(4) to solve the NL ambiguity, and the NL ambiguity floating-point solution which eliminates the influence of ionospheric delay residuals can be expressed as:

$$N_3 = \varphi_3 - \frac{\lambda_W(\varphi_W - \overline{N_W}) + I(\mu_W - \mu_3)}{\lambda_3} \quad (19)$$

the standard deviation of the NL ambiguity floating-point solution is:

$$\sigma_{N_3} = \frac{\sqrt{\lambda_3^2 \sigma_{\varphi_3}^2 + \lambda_W^2 \sigma_{\varphi_W}^2 + (\mu_W - \mu_3)^2 \sigma_I^2}}{\lambda_3} \quad (20)$$

It can be seen that the ionospheric delay correction algorithm eliminates the influence of the ionospheric delay residuals in the process of the original frequency AR, which makes the ambiguity resolution more accurate. However, equation(17) shows that the ionospheric delay correction algorithm further amplifies the observation noise because the observation noise is random. For this reason, noise can be removed by data smoothing.

4.2 Experiment and analysis

In order to verify the actual effect of the theoretical algorithm proposed above in the Beidou triple-frequency long baseline AR, a total of two sets of long-baseline Beidou triple-frequency measured data are collected. The specific data of each group is shown in Table 2.

Table 2. Data information

No.	Dist.	Date	Receiver	Interval(s)	Epoch	Location
A	131.6km	2016.09.06	Trimble	30	800	Australia
B	392.6km	2016.09.06	Trimble	30	800	Australia

C01 satellite is selected as the reference star and then is paired with C02 and C03 respectively. After that, data sets A and B are calculated with ambiguity to get ionospheric delay correction value, and then B3-frequency float ambiguity is obtained.

In order to weaken the influence of observation noise on the NL AR, it is necessary to smooth the observation data for denoising. The specific method is to take 200 epochs forward and backward for each epoch data between the 201st and 800th epochs. After that, the filtering process is performed, and then the filtered data is used to solve the NL AR, and finally the ambiguity estimation of the B3-frequency is calculated by using the linear relationship between the three combinations. Calculated results are shown in Figures 4 to 7.

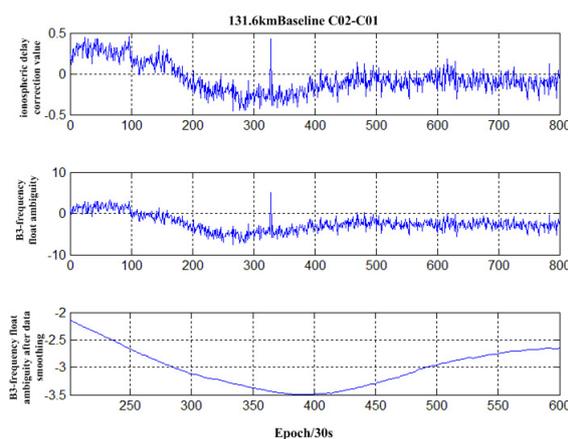


Figure 4. 136.1km baseline C02-C01

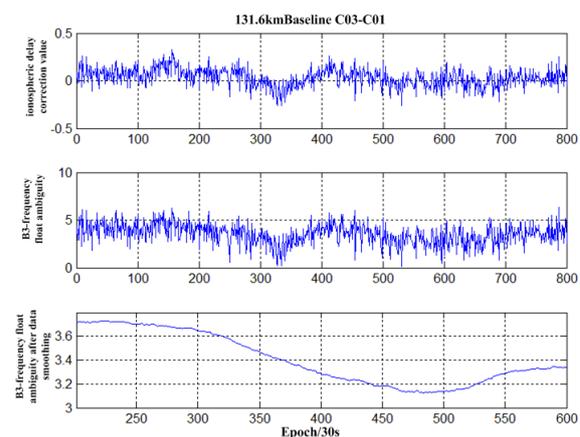


Figure 5. 136.1km baseline C03-C01

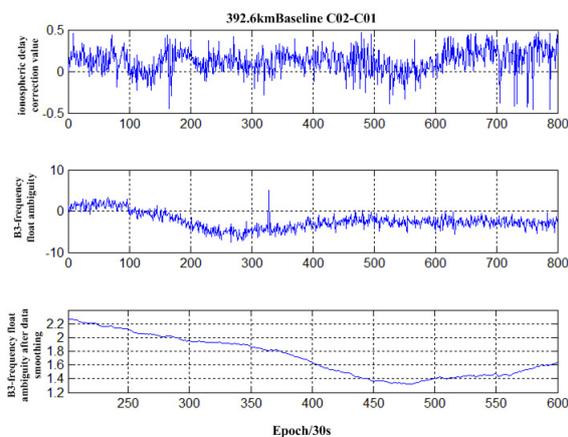


Figure 6. 392.6km baseline C02-C01

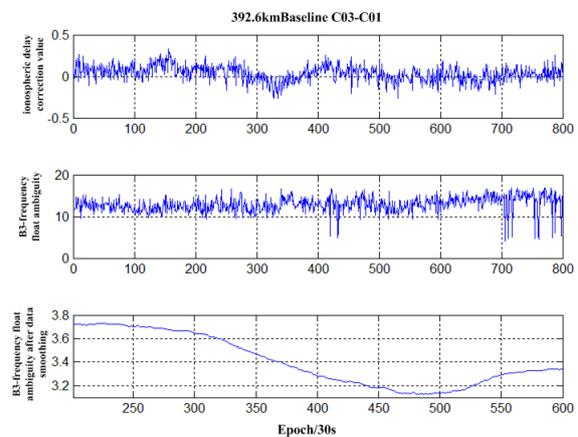


Figure 7. 392.6km baseline C03-C01

It can be seen from Fig.4-7 that the fluctuation range of the B3-frequency float ambiguity after the ionospheric delay correction is still large, and the reason is that although the ionospheric delay correction eliminates the influence of ionospheric delay residuals in the B3-frequency AR, it also amplifies the observation noise, making the B3-frequency float ambiguity seriously polluted and unable to be fixed. After the data smoothing process, the fluctuation degree of the B3-frequency float ambiguity is greatly reduced, and the fluctuation range is reduced from 10 cycles before smoothing to about 1 cycle, indicating that the data smoothing processing has a significant effect.

5. Conclusions

Based on the measured data of Beidou triple-frequency, this paper presents a medium-long baseline ambiguity resolution method that takes into account ionospheric delay correction. The main work and conclusions are as follows:

- The classical triple-frequency geometry-free TCAR algorithm was introduced, and the calculation steps of the TCAR algorithm under beidou signal constitution were presented.
- The influence factors of the Beidou triple-frequency ambiguity resolution under the medium-long baseline conditions were analyzed, and the optimal scheme of the combined measurement coefficient of each lane under the specific length baseline conditions was also analyzed.
- The ionospheric delay correction method was introduced. Firstly, the EWL and WL ambiguity with a relatively high success rate of AR were selected for calculation. Then, the double-difference ionospheric delay correction values were calculated. Then, the B3-frequency ambiguity was fixed by the B3-frequency observation measured by the ionospheric delay correction. It can be seen from the analysis that although this method eliminates the influence of double-difference ionospheric delay residuals, it amplifies the observation noise, and the fluctuation range of the original float ambiguity is greatly reduced by smoothing the observed data.

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