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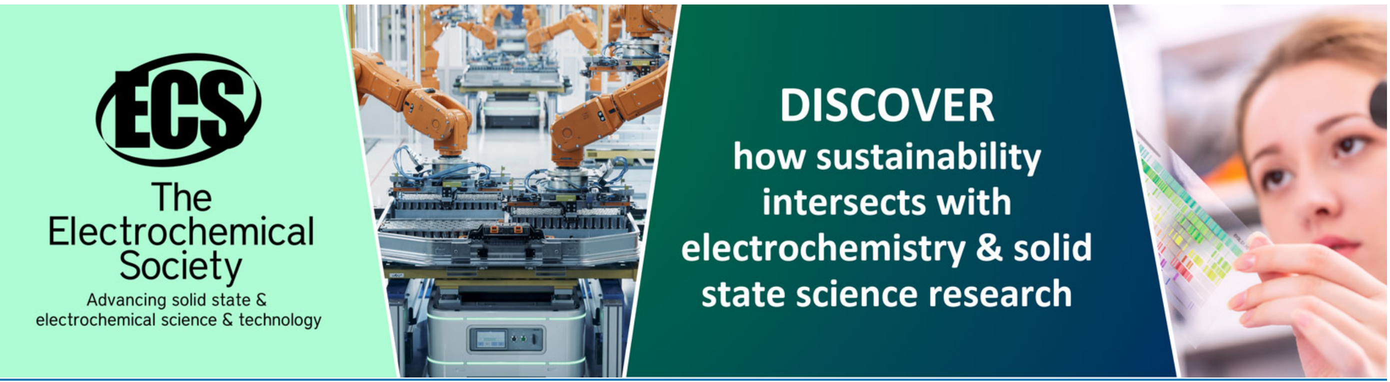

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Numerical model updating technique for structures using firefly algorithm

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Abstract: Numerical model updating is a technique used for updating the existing experimental models for any structures related to civil, mechanical, automobiles, marine, aerospace engineering, etc. The basic concept behind this technique is updating the numerical models to closely match with experimental data obtained from real or prototype test structures. The present work involves the development of numerical model using MATLAB as a computational tool and with mathematical equations that define the experimental model. Firefly algorithm is used as an optimization tool in this study. In this updating process a response parameter of the structure has to be chosen, which helps to correlate the numerical model developed with the experimental results obtained. The variables for the updating can be either material or geometrical properties of the model or both. In this study, to verify the proposed technique, a cantilever beam is analyzed for its tip deflection and a space frame has been analyzed for its natural frequencies. Both the models are updated with their respective response values obtained from experimental results. The numerical results after updating show that there is a close relationship that can be brought between the experimental and the numerical models.

1. Introduction & Literature review:

Numerical modelling of a structure is nothing but a set of mathematical equations used to represent and analyze a real test structure to reduce the experimental burden. This modelling may be performed in the form of programming using MATLAB or any other programming software or Finite element modelling etc. Updating this numerical model basically means to bring the numerical models close to experimental models. For this purpose, numerical models are validated with standard experimental responses of the structure as the main parameter such as the deflections, natural frequencies, stresses, etc. Experimental data required can be obtained by conducting the experiments or collecting the data from literature available for the experiments conducted on real or prototype structures. Then, a suitable optimization tool has to be chosen for the numerical model updation process to be carried out. The parameters that are to be considered for the optimization process might be geometrical or material properties of the experimental model. The parameters obtained after carrying out the optimization process can be used to update the numerical model and simulate the experimental model.

Mehrdad *et al.,* worked on the model updating techniques that can be applied to civil infrastructural systems. Bayesian method of updating technique is mentioned and reviewed. Advantages and disadvantages of these methods are mentioned in this work [1]. Saeed *et al.,* studied about different model updating techniques that are used in structural dynamics. There were several techniques that were used in this field depending on their application type. Some include model updating techniques involving forced vibrations, regularization techniques, etc. [2]. Kyoung-Bong-Han developed a method for structural re-analysis of dynamic systems using model updating techniques. This work mainly focuses on the noise that definitely prevails in a real experimental data even with whatever accuracy the experiment may be conducted. A modified damping, stiffness and mass matrices are developed based on the least sum of squares method [3].

Visser worked on model updating techniques using frequency response data. The main aim of the work was to develop a practical approach for updating the errors that exist in numerical modelling. The limitations of these techniques were also presented. This technique is used with the help of the response function method. [4]. Beck *et al.*, identified some uncertainties that exist in model updating techniques. This work mainly focuses on providing accurate response predictions for a particular type of dynamic loadings and assessing its accuracy. For each uncertain structural response encountered there is a predictive possible distribution assigned which is nothing but a weighted average of the response generated and this can be used to find uncertainties [5]. Sehgal *et al.*, worked on a review on the structural dynamic model updating techniques. It covers all the model updating techniques starting from the iterative ones to the direct techniques which can be applied to real life systems are available. The aim of the work is to highlight the issues regarding the model updating techniques and the advancements that happened in this field [6]. Arora *et al.*, studied about the comparative finite element updating methods. They mainly focused on developing mathematical models for the vibration control of structures which might be sometimes catastrophic. They are studied with numerical examples and the results obtained show that iterative methods are more accurate for the model updating purpose [7].

Boukaibet *et. al.*, worked on a fuzzy based model updating technique using the meta-heuristic algorithms. It is a non-probabilistic approach in which fuzzy logic is used for updating the finite element models of structures. Uncertainties are modelled in terms of fuzzy membership functions and the associated parameters are updated by defining an objective function using two meta-heuristic algorithms: Ant colony and Particle swarm optimization techniques. Finally, these results obtained from this approach are compared with the results obtained from the Bayesian technique for model updation [8]. Abdullah *et. al.*, studied about the correlation of experimental and numerical analysis for the BIW structure. It is mentioned that the finite element analysis of such complex structures becomes difficult due to some issues in modelling such as boundary conditions, joints etc. So in this study, modal properties of the structure are determined both experimentally and numerically. A correlation is performed between both the data and a considerable error is obtained. This error is reduced based on a model updating procedure which is proved to be effective on improving the structure [9].

Park *et. al.*, developed a numerical model updating technique for estimating load carrying capacities of highway speed railway bridges. A combined stage numerical model updating technique has been used for model updating based on univariate search method. Meticulous measurements are performed using the measuring point roaming method. With this method it is possible to obtain numerical results on par with experimental results and this updated model is used for finding the load carrying capacity of the bridge [10]. Zahari *et. al.*, worked on a review on model updating technique of joint structure for dynamic analysis purpose. The aim of this study is to review all the model updation methods for joint structure and mention some guidelines for carrying out this updation purpose. The two major methods for model updation mentioned in this paper are: iterative and direct method. A mixture of geometric and material parameters is considered for updation rather than geometric alone. Iterative method of model updation is more accurate in spite of its slightly higher computational burden when compared with direct method [11].

However, these updation techniques have some limitations and uncertainties such as choosing the updation parameter which is called as parameterization and regularization. Sometimes, the finite element model updating techniques can give poor results, as there might be issues with the structural connectivity and boundary conditions. In such cases, the interpretation of structures will become difficult [12].

Keeping in view all the above mentioned problems related to model updation techniques, an indirect method of model updation has been proposed in this study. Numerical model is developed using technical computing tools like MATLAB which gives flexibility to eliminate some limitations related to numerical modelling. This numerical model developed is updated using the firefly algorithm which is one of the powerful and novel meta-heuristic algorithms. The variables (parameters) for updating can be material or geometrical properties of the experimental model or both. These variables considered for optimization are varied around a reasonable range of values depending on the properties of the experimental model. One response parameter of the structure such as deflections, natural frequencies etc. has to be chosen which helps to correlate the experimental data with the numerical data. After the optimization is run, the parameters of the experimental model that obtained are to be used for updating the numerical model. Now, this updated numerical model can be used for analyzing or simulating the experimental model. These methods generally are computationally cheaper and have a wider choice of parameters for updating. Even, the model updating assumptions also are closely related to the experimental data.

2. Methodology:

This updating process is applicable if and only if there is a significant difference between the numerical and experimental data. This difference cannot be completely eliminated, but can be reduced. The accuracy in conducting the experiment is also very important in this process as the complete updating process is based on this data. So, if there is a large difference existing between the experimental and analytical data, then there might be a problem with the experimentation and it is suggested that the experiments has to be conducted again. The proper boundary conditions to be incorporated in the numerical model to match the experiment conditions. The following are the steps involved in the model updating process, which are common to every model irrespective of their type, properties, conditions etc.:

Step-1: Choose the experimental model and collect the corresponding response data after conducting the experiment.

Step-2: Develop a mathematical tool using the standard mathematical formulation that defines the experimental model using computer program or it can be simply a FE model.

Step-3: Calculate the error between the experimental and analytical response data. If the error is very large, then reconduct the experiment. Otherwise, select a proper optimization tool.

Step-4: Vary different parameters of the model (Material or Geometric) in the optimization process within their acceptable ranges and run the optimization process.

Step-5: Input these parameters in the mathematical tool developed and calculate the error again.

Step-6: If the error reduced, then update the numerical model with the optimal parameters obtained and this updated numerical model can be used for simulating the experimental model.

Step-7: If the error is not reduced, then there might be some problem with the experimentation or computation and it is suggested to repeat those steps again.

3. Mathematical formulations:

The present study involve the calculation of deflection for a cantilever beam, natural frequencies for a space frame and an optimization technique. The mathematical formulations for each of them is as follows:

3.1 Deflection of Cantilever Beam

The deflection at the any point on a cantilever beam when a point load is applied at the tip of the beam can be calculated using the following formulas.

$$\delta = \frac{P}{6EI}(x^3 - 3x^2L) \tag{1}$$

where, ***P*** is the applied load on the beam, ***E*** is the young's modulus of the beam, ***I*** is the moment of inertia of the beam it's about neutral axis, ***x*** is the distance from the fixed end where properties are to be determined, ***L*** is the length of the beam.

3.2 *Natural frequencies of Space Frame*

All the matrix formulations considered for the development of the mathematical tool for the calculation of natural frequencies of space frame are based on finite element formulation [13]. Based on the degrees of freedom of the space frame element, a matrix based formulation is used to develop the mathematical model. An eigenvalue approach has been used to find the natural frequencies of the structure.

3.3 *Firefly algorithm as an optimization tool* [14]

Firefly algorithm is a nature inspired meta-heuristic algorithm developed by Xin-she Yang in 2007. This algorithm is developed based on the behavior of a large group of tropical fireflies and their flashing patterns. The three main idealized rules in this algorithm are:

a. Fireflies are generally uni-sexual and one gets attracted to other, regardless of their sex.
b. Their attractiveness is proportional to the brightness and attractiveness decrease with an increase in distance between them. Thus, the less bright one will get attracted towards the other brighter one. If there is no brighter firefly then it moves randomly.
c. The brightness of the firefly depends on the landscape of the objective function. The firefly which is close to the solution will have more brightness.

The attractiveness of a firefly is proportional to the intensity of light emitted by it and this intensity is inversely proportional to the distance between the fireflies. So, the attractiveness β is defined as,

$$\beta = \beta_0 e^{-\gamma r^2} \tag{2}$$

where, β_0 is the attractiveness at r = 0.
The movement of a firefly ***i*** is attracted to another more attractive (brighter) firefly ***j*** is determined by:

$$X_i^{t+1} = X_i^t + \beta e^{[-\gamma r^2]}(X_j^t - X_1^t) + \alpha_t \varepsilon_t \tag{3}$$

where, the second term is dependent on attractiveness, the third term α_t is a randomization parameter. ε_t is the vector drawn from Gaussian distribution at time t.

Parameters in firefly algorithm:

1. The term α_t governs the randomness of the algorithm. So, this parameter should be carefully modified for every iteration so that ideal randomization is introduced into the algorithm. This term every time step or iteration t is expressed as:

$$\alpha = \alpha_0 \delta^t \tag{4}$$

where, δ lies between 0 and 1 and is called as cooling factor. Generally, this is considered to be 0.95 to 0.97.
α_0 is the initial randomness of the factor and is dependent on the average scale of the system or problem of interest. Generally it is considered as 0.01 L, where L is the average scale of the system. If scaling variations can be neglected, then α_0 can be any value between 0 and 1.

2. Through many parametric studies it is proven that, the best value of β_0 i.e. the attractiveness at r = 0 can be considered to be 1.
3. The parameter γ controls the division of the fireflies. Similarly, if the scaling variations can be neglected then, γ can be considered equal to 1.
4. The population size of the fireflies n can be in between 15 to 100 and the ideal range of n is in between 25 to 40.

4. Results & Discussion:

The proposed model updating technique is validated by applying the concept on following two case studies:

4.1 Cantilever beam

Problem statement: A cantilever beam having following dimensions and properties is analyzed experimentally for its tip deflection with the point load applied at its tip. A significant error has been observed between the experimental and analytical value of tip deflection. The objective is to reduce this error.

Material and geometric properties of the beam analyzed experimentally: The material used for fabricating the experimental cantilever beam model is steel. Its elastic modulus is 2.1×10^5 N/mm^2. The length, width and thickness of the beam are 600mm, 25mm & 5mm respectively. A load of 5.886 N (0.6Kg) is applied at the tip of the beam with the help of a load cell and the tip deflection due to this applied load is measured using an LVDT.

Experimental data: The experiment that is carried out by Mahato *et al.*,[15] is considered for the updating process. The properties of cantilever beam used by them for the experimentation process are as mentioned below:

Numerical model: For calculating the tip deflections of a cantilever beam analytically, a MATLAB code is developed. This numerical model (mathematical tool) is developed based on equation (1).

The value of load applied and the corresponding experimental & analytical tip deflection values obtained for that applied load is shown in table 2. As, there is a considerable difference that is existing in between the experimental and numerical data this updating process can be applied to this model.

Objective function & parameters considered in Firefly algorithm for optimization:

i. The objective function considered for the optimization process is: $F = |\delta_{ex} - \delta_{nm}|$.

where, $\boldsymbol{\delta_{ex}}$ and $\boldsymbol{\delta_{nm}}$ are the tip deflections of the experimental model and numerical model

ii. Parameters of the cantilever beam varied in objective function of the optimization for the updating process to be carried out are E (Elastic modulus), b (Width) & t (Thickness). The variation limits of these parameters, E = 195 to 225 GPa, b = 24 to 26 mm, t = 4 to 6 mm
iii. The randomization parameters of the firefly algorithm considered are: n = 40; α = 0.5; β_0 = 1; γ = 1; Maximum generation = 400.

With the above parameters mentioned, firefly optimization is run in the form of a MATLAB code developed using equation (2), equation (3) & equation (4). These optimal parameters obtained after updating the model in comparison with initial model are shown in table 1 below:

Table 1. Initial model and final model parameters of cantilever beam

Parameter	Initial model	Updated model
Elasticity modulus [GPa]	210	195
Width of beam [mm]	25	25.8
Thickness of beam [mm]	5	5.1

Now, as the optimal parameters for the cantilever beam model considered are known, the analytical deflection values after carrying out the updation process are obtained as shown in table 2 below:

Table 2. Experimental data & Initial and Final analytical data of the cantilever beam

Case	Load Applied [N]	Experiment Deflection [mm]	Initial Numerical Model		Updated Numerical Model	
			Deflection [mm]	Error [%]	Deflection [mm]	Error [%]
1	5.886	8.12	7.749	4.57	8.087	0.41

4.2 Space frame:
Problem statement: A two storied space frame structure of the following dimensions and properties is analyzed experimentally for its natural frequencies. A significant error has been observed between its experimental and analytical values of the natural frequencies. The objective is to reduce this error.

Material and geometric properties of the space frame analyzed experimentally: The material used for fabricating the experimental space frame model is steel. Its elastic modulus is 2×10^5 N/mm^2. The square section dimension of all the beams is 12mm and that of columns is 16mm for the space frame. The plan dimension of the space frame is 0.4m × 0.3m, while each storey height is 0.35m. The density of steel used is 7860 kg/m^3 and its poisson's ratio is 0.27. To find the natural frequencies, the space frame is impacted with a hammer and allowed to vibrate freely and the response of the structure is measured with the help of accelerometers attached. From this acceleration response of the structure obtained, the natural frequencies are measured from its spectrum curves.

Experimental data: The experiment that is carried out by Mohan *et al.*,[16] is considered for the updating process. The properties of material used for fabricating this space frame is as mentioned below:

Numerical model: Similarly for calculating the natural frequencies of a space frame analytically, a MATLAB code is developed based on the matrix formulations mentioned under section 3.2.

The first eight natural frequencies of the structure obtained experimentally & the analytical natural frequencies obtained for different modes are shown in table 4. As there is a considerable difference in between the natural frequencies of the experimental and numerical models, this updating process can be applied to this model.

Objective function & parameters considered in Firefly algorithm for optimization:

i. The objective function considered for the optimization process is:

$$F = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(f_{ex} - f_{nm})^2}$$

where, $\boldsymbol{f_{ex}}$ and $\boldsymbol{f_{nm}}$ are the natural frequencies of the experimental model and numerical model

ii. Parameters of the space frame varied in objective function of the optimization for the updating process to be carried out are E (Elastic modulus), b_c (Width of column), b_b (Width of beam), d_c (Depth of column), d_b (Depth of beam), R_o (Density of material). The variation limits of these parameters E = 190 to 210 GPa, b_c = 15.5 to 16.5 mm, b_b = 11.5 to 12.5 mm, d_c = 15.5 to 16.5 mm, d_b = 11.5 to 12.5 mm & R_o = 6288 to 9432 kg/m^3.
iii. The randomization parameters of the firefly algorithm considered are: n = 40; Maximum generation = 400, $\alpha = 0.5$; $\beta_0 = 1$; $\gamma = 1$.

With the above parameters mentioned, firefly optimization algorithm is run in the form of a MATLAB code in the same way as implemented for the cantilever beam. The properties obtained after updation in comparision with initial model are as shown in table 3 below:

Table 3. Initial model and final model parameters of Space frame

Numerical model	Elastic Modulus [Gpa]	Column [mm]		Beam [mm]		Density [kg/m^3]
		b_c	d_c	b_b	d_b	
Initial model	200	16	16	12	12	7860
Updated model	190	16.3	16.1	11.9	11.8	7176

Now, as the optimal parameters for the space frame model considered are known, the analytical frequency values obtained after the updating process are as shown in table 4 below:

Table 4. Experimental data & Initial and Final analytical data of the space frame

Mode	Experiment Frequency [Hz]	Initial Numerical Model		Updated Numerical Model	
		Frequency [Hz]	Error [%]	Frequency [Hz]	Error [%]
1	34.0	32.72	3.76	33.56	1.29
2	36.5	35.11	3.80	35.80	1.92
3	47.5	46.51	2.09	47.66	-0.34
4	94.0	94.61	-0.64	94.32	-0.34
5	132.5	127.20	4.00	132.63	-0.098
6	134.5	130.38	3.07	134.59	-0.067
7	162.0	156.10	3.64	160.93	0.66
8	179.0	171.88	3.98	176.02	1.66

4.3 Discussion:

Generally, there might be some mistakes in experimentation such as fabrication errors, test setup errors or manufacturing errors, etc,. These uncertainities have to be taken into account in the model to replicate the experimental model more closely. By varying the material & geometrical properties of the numerical model, the response of the structure also varies accordingly, which can be seen clearly from the case studies considered.

In the case of cantilever beam, the parameters obtained after optimization as mentioned in table 1, can be used for updating its numerical model developed and this updated numerical model can be used for analysis and simulation of cantilever beam's experimental model tested. From table 2, it can be observed that the error between the experimental and analytical results after carrying out the updating process reduced to a great extent from 4.57% to 0.41%.

Even in the case of space frame, the parameters obtained after optimization as mentioned in table 3, are to be used for updating the numerical model of the space frame. This updated numerical model can be used for analysis and simulation of experimental space frame model. It can also be observed from table 4 that, all the analytical values came close to the experimental values and the error between the experimental and analytical data after carrying out the updating process is reduced. Hence, it can be observed that this proposed technique can be used successfully for updating numerical models of structures.

5. Conclusions:

It is observed that, the proposed updating process is efficient to reduce the difference between the experimental and analytical data in case of space frame and cantilever beam successfully. Hence, this updating technique can be generalized and be used to update any kind of structures, which are well defined.

The input parameters of the optimization process can be any number depending on the user. It is also to be noted that, the accuracy and random search capability of the optimization tool is very much important for the updating process to be efficient.

From, the percentage error observed, it can be concluded that this technique can be used to update the structure and reduce the difference between its experimental and analytical data, but cannot be used to remove the error completely. It is also important to be noted that, the percentage error difference must not be so high and in such cases this process might not work out as there might be some error in performing the experimentation or analytical calculations and it is suggested that experiment should be repeated to obtain correct values.

In case of a cantilever beam, the experimental and analytical data after updating are also very much close to each other and the error has also reduced to a great extent.

In case of space frame also, the analytical and experimental data were close enough, but in some modes they are slightly away. But, considering the structure as a whole the error reduced after the carrying out updating process.

So, these material and geometric parameters obtained after updating can be used to update the numerical model. This updated numerical model can be used for numerically simulating the experimental model and use it for further analysis.

Finally, it can be concluded that this proposed numerical model updating technique can be used on a wide range to update any model of the structure. The technique's efficiency depends on the difference that is existing between its experimental and analytical data and some of the external influencing factors.

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