IOP Conference Series: Materials Science and Engineering

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IOP Conf. Series: Materials Science and Engineering **369** (2018) 012023 doi:10.1088/1757-899X/369/1/012023

Effects of the bonded component heights on the stress intensity factors of the edge interface cracks

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Abstract. The height of the bonded component of a bonded strip changes with the scope of requirements in practical applications. Therefore, one must understand the effect of the influence of the height of the bonded component. In this research, the stress intensity factors of an edge-cracked asymmetrically bonded strip are systematically computed using the proportional crack tip opening displacement method based on FE analysis. Then, the stress intensity factors are compared for arbitrary sets of material combinations in the whole range of Dundurs' material space with varying the height of the bonded component, and the combined effects of the relative height of the bonded component and material combinations on the stress intensity factors are discussed for the typical engineering materials.

1. Introduction

Due to advances in adhesive materials with improved mechanical properties, bonded systems have found applications in a wide spectrum of industries. Interfacial cracks are usually observed within the vicinity of the bonding region since there exists a high singular stress field. The stress intensity factors (SIFs) are usually employed in predicting the stress state as well as the crack initiation and propagation. The stress intensity factors (SIFs) are usually employed in predicting the crack initiation and propagation for the interface cracks. Many researches were found to determine the stress state of an edge-cracked bonded strip accurately till recently. Lan *et al* [1] investigated the SIFs of a bi-material bonded strip for different material combinations, and the variation of SIFs with varying crack size were then discussed. Noda and Lan [2] computed the SIFs of an edge interface crack in the bonded semi-infinite planes for different material pairs in the $\alpha-\beta$ space. Lan *et al* [3] investigated the energy release rate (ERR) and mode-mixity of an edge-cracked longitudinal symmetry and asymmetry bonded components, and the effect of geometrical configurations were discussed together with the fracture toughness. As known, one may predict the onset of an interfacial crack using the SIF based fracture criterion. Therefore, in this research, the SIFs are computed for various height of the bonded component using the numerical procedure based on the proportional crack-tip opening displacement with FE analysis[4]. And the effects of the bonded component height on SIFs are examined and discussed accordingly.

2. Analysis method

The coordinate origin is set at the right crack tip and the interface crack lies along the negative x-axis. Two elastic materials are ideally bonded along the x-axis, material 1 is located above the interface and material 2 is located below. The stress singularity field in the plane stress and plane strain conditions can be expressed using equation (1) [5].

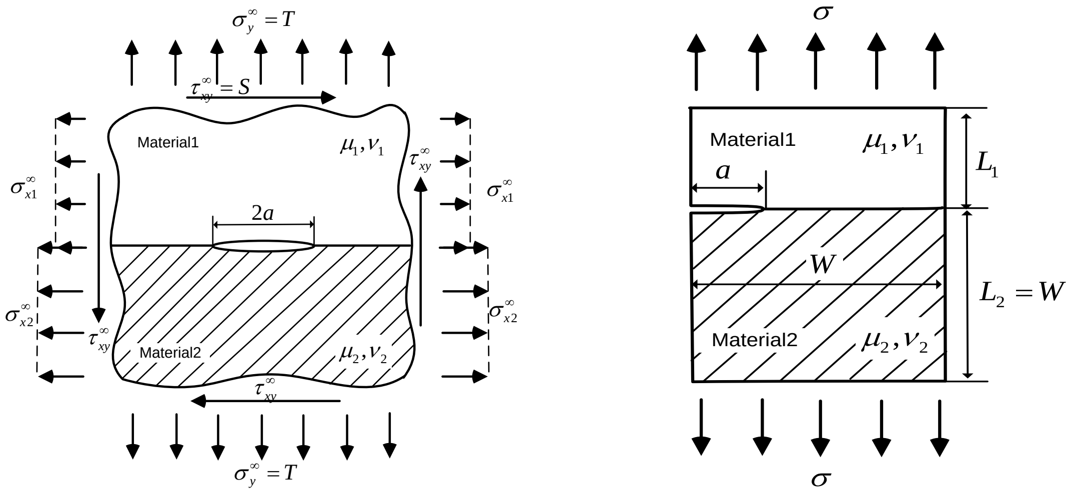


Figure 1. (a) Central interface crack in a bonded semi-infinite plane (reference problem A) and (b) edge interface crack (unknown Problem B).

$$\sigma_y + \mathrm{i}\,\tau_{xy} = \frac{K_I + \mathrm{i}\,K_{II}}{(2\pi r)^{\frac{1}{2}}}\left(\frac{r}{2a}\right)^{\mathrm{i}\,\varepsilon}, \quad r \to 0 \tag{1}$$

where, σ_y, τ_{xy} are the tensile and shear stresses around the crack tip, respectively. r is the radial distance away from the crack tip, a denotes the half length of an center interface crack or the length of an edge interface crack, furthermore, ε is the bi-elastic constant which is defined in equation (2):

$$\varepsilon = \frac{1}{2\pi}\ln\left[\left(\frac{\kappa_1}{\mu_1} + \frac{1}{\mu_2}\right)\left(\frac{\kappa_2}{\mu_2} + \frac{1}{\mu_1}\right)^{-1}\right] \tag{2}$$

$$\kappa_m = \begin{cases} 3 - 4\nu_m & (\mathit{plane\ strain}) \\ 3 - \nu_m (1 + \nu_m)^{-1} & (\mathit{plane\ stress}) \end{cases} \tag{3}$$

where μ_m and ν_m are the shear moduli and Poisson's ratios of material m $(m = 1, 2)$. The associated relative COD $\delta_\mathrm{d}(d = \mathrm{x}, \mathrm{y})$ at a distance r away from the crack tip are defined by

$$\delta_y + \mathrm{i}\,\delta_x = \frac{K_I + \mathrm{i}\,K_{II}}{2(1 + 2\mathrm{i}\,\varepsilon)\cosh(\varepsilon\pi)}\left[\frac{\kappa_1 + 1}{\mu_1} + \frac{\kappa_2 + 1}{\mu_2}\right]\left(\frac{r}{2\pi}\right)^{\frac{1}{2}}\left(\frac{r}{l}\right)^{\mathrm{i}\,\varepsilon} \tag{4}$$

l is usually selected as an arbitrary length which scales with specimen size or crack length. Here, we have $l = 2a$ to coincide with the definition in equation (1). Equation (4) is rearranged and then it gives the explicit expression of K_I, K_{II} as:

$$\mathrm{K_I} = \mathrm{Z}\left\{\left(\delta_{\mathrm{y}} - 2\varepsilon\delta_{\mathrm{x}}\right)\cos\left[\varepsilon\ln\left(\frac{\mathrm{r}}{\mathrm{l}}\right)\right] + \left(\delta_{\mathrm{x}} + 2\varepsilon\delta_{\mathrm{y}}\right)\sin\left[\varepsilon\ln\left(\frac{\mathrm{r}}{\mathrm{l}}\right)\right]\right\} \tag{5}$$

$$K_{II} = Z\left\{\left(\delta_{\mathrm{x}} + 2\varepsilon\delta_{\mathrm{y}}\right)\cos\left[\varepsilon\ln\left(\frac{r}{l}\right)\right] - \left(\delta_{\mathrm{y}} - 2\varepsilon\delta_{\mathrm{x}}\right)\sin\left[\varepsilon\ln\left(\frac{r}{l}\right)\right]\right\} \tag{6}$$

where the value of S is defined as

$$Z = \frac{2\cosh\left(\varepsilon\pi\right)\left[r\left(2\pi\right)^{-1}\right]^{-1/2}}{\left(\frac{\kappa_1+1}{\mu_1} + \frac{\kappa_2+1}{\mu_2}\right)} \tag{7}$$

Equations (5) and (6) are rewritten as

$$\frac{K_I}{\delta_y} = Z\left\{\left(\cos Q + 2\varepsilon\sin Q\right) + \left(\sin Q - 2\varepsilon\cos Q\right)\frac{\delta_x}{\delta_y}\right\} \tag{8}$$

$$\frac{K_{II}}{\delta_x} = Z\left\{\left(\cos Q + 2\varepsilon\sin Q\right) - \left(\sin Q - 2\varepsilon\cos Q\right)\frac{\delta_y}{\delta_x}\right\} \tag{9}$$

and

$$Q = \varepsilon\ln\left(\mathrm{r}\left(l\right)^{-1}\right) \tag{10}$$

In equations (8) and (9), assuming $Q, \varepsilon, \delta_y\left(\delta_x\right)^{-1}$ are the same for the reference and unknown problems, then a relationship is given as equation (11).

$$K_I\left(\delta_y\right)^{-1} = const, K_{II}\left(\delta_x\right)^{-1} = const \tag{11}$$

Examining the two interface crack problems A and B shown in figure 1(a) and (b) respectively, when equations (12) and (13) are satisfied, then K_I, K_{II} are proportional to δ_y, δ_x as given in equation (14). Here, the relative COD δ_y, δ_x can be calculated using FE analysis. The SIFs of the reference problem A are analytically solved in advance, therefore, the SIFs of the given unknown problem B can be obtained from equation (14).

$$\begin{pmatrix} Q_A = Q_B \\ \varepsilon_A = \varepsilon_B \end{pmatrix} \rightarrow \begin{pmatrix} \left[\varepsilon\ln\left(r\left(l\right)^{-1}\right)\right]_A = \left[\varepsilon\ln\left(r\left(l\right)^{-1}\right)\right]_B \\ \varepsilon_A = \varepsilon_B \end{pmatrix} \tag{12}$$

$$\left[\delta_y\left(\delta_x\right)^{-1}\right]_A = \left[\delta_y\left(\delta_x\right)^{-1}\right]_B \tag{13}$$

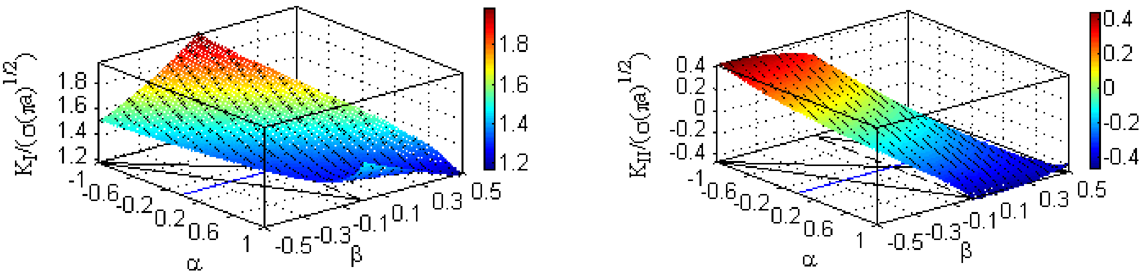
$$\left[K_I\left(\delta_y\right)^{-1}\right]_A = \left[K_I\left(\delta_y\right)^{-1}\right]_B, \left[K_{II}\left(\delta_x\right)^{-1}\right]_A = \left[K_{II}\left(\delta_x\right)^{-1}\right]_B \tag{14}$$

In this research, the bi-material bonded half-planes with a central interface crack subjected to uniform tension and shear as shown in figure 1(a) is employed as the reference problem A of the current numerical procedure. Therefore, the SIFs of the given unknown problem B can be computed using equation (14). The detailed information of the application of the current method as well as its robustness discussions could be found in [4].

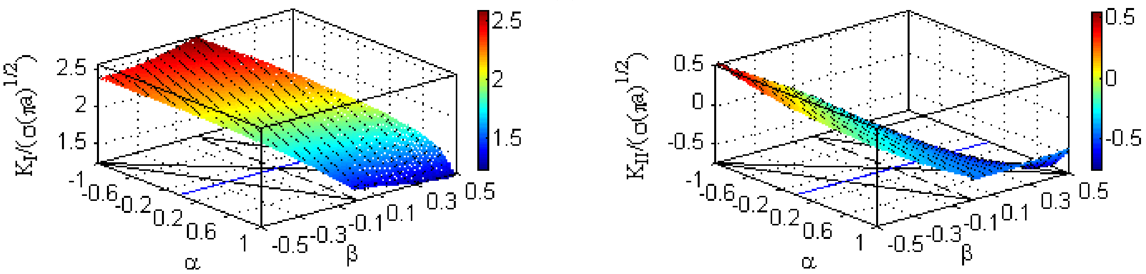
3. Results and discussions

3.1. Solutions of the bonded strip for different component heights

The longitudinal asymmetrically bonded strip shown in figure 1(b) is computed for various heights of component 1. The relative crack size is fixed $aW^{-1}=0.1$, the height of component 1 changes from $L_1=0.333W{\sim}2W$, and the height of the lower component 2 is fixed to $L_2=1W$. Here, given space limitations, only the SIFs for $L_1=0.2W, 0.1W$ are plotted in figures 2(a) and 2(b), respectively. Similar varying tendencies as in figure 2(a) can also be obtained for $L_1 \geq 0.2W$, and then they gradually transfer to the pattern shown in figure 2(b) with the continuous reduction of L_1. As can be seen from figure 2(a), the Maximum value of K_I is always located in the upper left corner $(\alpha=-1, \beta=0)$,when material 1 is extremely stiff compared to material 2, $\alpha \to 1$, K_I reaches its minimum value, and K_{II} comes closer to its minimum one. However, when material 1 is extremely compliant comparing with material 2, $\alpha \to -1$, K_I and K_{II} reach their maximum values.



(a)

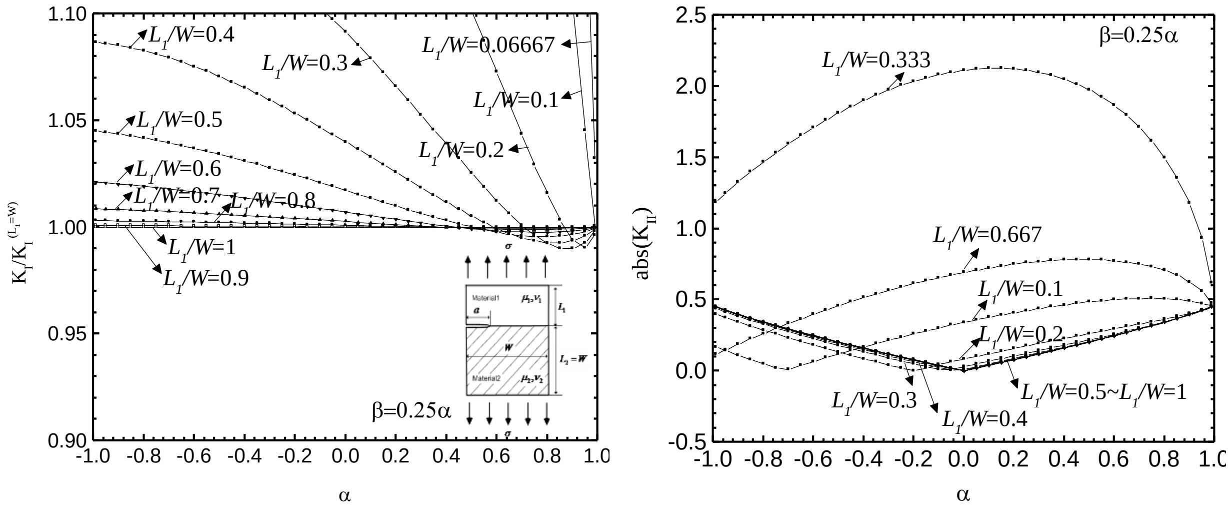


(b)

Figure 2. 3-D normalized SIFs of (a) $L_1=0.2W$ and (b) $L_1=0.1W$.

3.2. Effect of the bonded component heights

The SIFs are examined and compared for the whole physical admissible typical engineering material combinations distributed in the vicinity of $\beta=0.25\alpha$ by varying $L_1=0.333\sim 2W$. Figure 3(a) shows the values of $K_{I,nor}=K_I\left(K_I^{L_1=W}\right)^{-1}$, all the values are normalized using those of $L_1=W$. It is found that $K_{I,nor}>1$ when $L_1W^{-1}\leq 0.1$, however, for the case of $L_1W^{-1}\geq 0.2$, $K_{I,nor}<1$ can also be observed when $\alpha \to 1$, and the region of $K_{I,nor}\leq 1$ reduces with decreasing L_1. It means that the SIFs are bigger than those of $L_1=W$ for most the cases of decreasing L_1, and they can also be smaller when material 1 is extremely stiff comparing with material 2. The absolute values of K_{II} are plotted in figure 3(b). The curve of $L_1=W$ are axial symmetry around $\alpha=0$ with the minimum point of $K_{II}=0$ at $\alpha=0$. When the height of the bonded component 1 decreases, the minimum $abs(K_{II})=0$ moves to the left lower part of $\alpha-\beta$ space ($\alpha\to-1$), say, the minimum $abs(K_{II})=0$ is located around $\alpha\to-0.68$ for $L_1W^{-1}=0.1$. However, $abs(K_{II})$ grows monotonically with decreasing L_1 when component 1 is extremely thinner than component 2 ($L_1W^{-1}\le 0.06667$).



(a) (b)

Figure 3. (a) $K_I\left(K_I^{L_1=W}\right)^{-1}$ and (b) $abs(K_{II})$ for $\beta=0.25\alpha$.

4. Conclusions

In this research, the SIFs of an edge interface crack in the bi-material bonded strip subjected to remote tensile loads were calculated with varying various heights of the bonded components. It was found that K_I generally increase with the decrease of L_1/W, and K_{II} monotonically decrease with decreasing L_1/W. However, some exceptions of the SIFs which are smaller than those of $L_1=W$ for decreasing L_1 also exists when material 1 is extremely stiff compared to material 2 ($\alpha\to1$).

Acknowledgments

This work was supported by grants from National Natural Science Foundation of China (NSFC) (No. 11702160), Natural Science Foundation of Shandong Province, China (No. ZR201702170418) and the Fundamental Research Funds of Shandong University (No. 2016TB009)).

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