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To cite this article: M Möller *et al* 2018 *IOP Conf. Ser.: Mater. Sci. Eng.* **388** 012015

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Investigation on multiaxial strength reduction for multi-directional laminates under variable amplitude loading

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Abstract. Due to their excellent ratio of high strength and stiffness to low density, the use of composite materials in lightweight solutions is growing in widely varying industry sectors. Most of the lightweight structures are loaded cyclically and therefore there is the need for optimized computation models concerning the fatigue of composite materials. The present study mainly focuses on the fatigue life estimation of continuous fibre reinforced plastics under variable amplitude loads. The approach is based on iterative ply-wise calculation of multiaxial stress states using classical laminate theory and a subsequent analysis of the material effort with the use of the failure criteria by Puck. A number of suitable linear and nonlinear residual strength models are examined for an in-depth approach on the multiaxial strength reduction at the ply-level. The aim of this study is to calculate fatigue life of multi-directional laminates with input data from uni-directional plies only. The model is finally validated for a multi-directional laminate under variable amplitude loads with arbitrary pulsating and alternating stress ratios using experimental data from the well documented composite fatigue data base OptiDAT.

1. Introduction

Particularly in the era of lightweight manufacturing, resource conservation and sustainability, and therewith the steadily increasing demand for more efficient utilization of lightweight materials, deepening the understanding of the behavior of composite materials is becoming increasingly important. Whilst in operation, most structural parts are subjected to cyclic loads, which especially requires further research of the fatigue material behavior of composites. Therefore, the current research focuses on the calculation of fatigue life and residual strengths of multi-directional laminates via ply-wise structural analysis. The present paper mainly deals with variable amplitude loads in terms of two-block loading considering different residual strength models. Figure 1 shows a simplified routine for the lifetime estimation of composites using a ply-wise fatigue analysis based on a modified classical laminate theory (CLT). On this occasion, only data of uni-directional plies is used for fatigue life and residual strength estimation of multi-directional laminates. As shown in Figure 1 the current stiffness and strength of each ply



is calculated within every cycle of the actual block load. The status of the laminate is updated and the current stress state within each ply is then calculated subsequently via CLT. The current

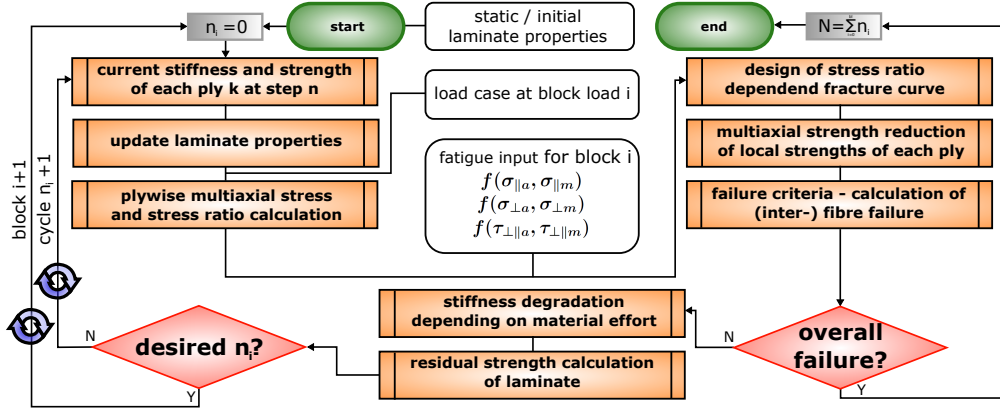


Figure 1. Routine for lifetime estimation of composites under variable amplitude loading

ply-related fracture curves are then designed with the use of S-N curves for the present stress ratios and a multi-axial strength reduction of all five in-plane strengths is carried out. After that, the failure conditions are checked on the ply-level and changes to the ply-related stiffness values are made. The procedure is repeated until failure occurred in every single ply.

2. Models

2.1. Failure Criteria

In the present paper, the failure criteria by Puck [2] is used for calculations of fibre and inter-fibre failure. To do justice to the increase of strengths due to the in-situ effect of the plies within the laminate, the equations for the stress expose factors under transverse tension (Mode A) and transverse compression (Mode B or Mode C) are modified as follows

$$f_{e,IFFA}(n) = \sqrt{\left(\frac{\tau_{\perp\parallel}(n)}{X_t(n)}\right)^2 + \left(1 - p_{\perp\parallel}^+ \frac{Y'_t(n)}{S'_{\perp\parallel}(n)}\right)^2 \left(\frac{\sigma_{\perp}(n)}{Y'_t(n)}\right)^2} + p_{\perp\parallel}^+ \frac{\sigma_{\perp}(n)}{S'_{\perp\parallel}(n)} \quad (1)$$

$$f_{e,IFFB}(n) = \frac{1}{S'_{\perp\parallel}(n)} \left(\sqrt{\tau_{\perp\parallel}^2(n) + \left(p_{\perp\parallel}^- \sigma_{\perp}(n)\right)^2} + p_{\perp\parallel}^- \sigma_{\perp}(n) \right) \quad (2)$$

$$f_{e,IFFC}(n) = \left[\left(\frac{\tau_{\perp\parallel}(n)}{2(1 + p_{\perp\perp}^-) S'_{\perp\parallel}(n)} \right)^2 + \left(\frac{\sigma_{\perp}(n)}{Y_c(n)} \right)^2 \right] \frac{Y_c(n)}{(-\sigma_{\perp}(n))} \quad (3)$$

where $\sigma_{\perp}$ and $\tau_{\perp\parallel}$ are the transverse and in-plane shear stress, X_t is the parallel tension strength, Y_c is the transverse compression strength, $\{p_{\perp\parallel}^+, p_{\perp\parallel}^-, p_{\perp\perp}^+, p_{\perp\perp}^-\}$ are the inclines of the failure envelope and Y'_t and $S'_{\perp\parallel}$ are the increased transverse tensile and in-plane shear strength for embedded plies by Wang and Karihaloo [3] according to the following equation

$$Y'_t = Y_t \left[1 + \mathcal{A} f_t(\Delta\Theta) \right], \quad S'_{\perp\parallel} = S_{\perp\parallel} \left[1 + \mathcal{C} f_t(\Delta\Theta) \right] \quad (4)$$

where $\mathcal{A}$ and $\mathcal{C}$ [3] are material parameters, which take into account the number of plies of a given thickness in the embedded lamina. The functions $f_t(\Delta\Theta)$ and $f_s(\Delta\Theta)$ are given by the orientation of the neighboring plies as follows

$$\min \left[\frac{\sin^2(x)}{1 + \sin^2(x)}, \frac{\sin^2(y)}{1 + \sin^2(y)} \right] = \begin{cases} f_t(\Delta\Theta) & \text{with } x = \Delta\Theta_a \quad \text{and } y = \Delta\Theta_b \\ f_s(\Delta\Theta) & \text{with } x = 2\Delta\Theta_a \quad \text{and } y = 2\Delta\Theta_b \end{cases} \quad (5)$$

2.2. Strength Reduction

Different existing residual strength models are brought together to one procedure for the application within the computational routine. For the repetitive computations, the residual strength in every cycle is defined by

$$S_{r,n} = S_{r,n-1} - \left\{ \left[(S_{st} - \sigma_{max,i}) \left(1 - \left(\frac{n_{i-1}}{N_i} \right)^{\alpha_i} \right)^{\beta_i} \right] - \left[(S_{st} - \sigma_{max,i}) \left(1 - \left(\frac{n_i}{N_i} \right)^{\alpha_i} \right)^{\beta_i} \right] \right\} \quad (6)$$

where the residual strengths $S_{r,n}$ and $S_{r,n-1}$ at the n th and the $n-1$ th step as well as the static strength S_{st} represent the five local strengths in terms of the parallel tensile and compression strengths (X_t, X_c), the transverse tensile and compression strengths (Y_t, Y_c) or the in-plane shear strength ($S_{\parallel}$). Equation 6 refers to different residual strength models with varying combinations of material parameters α_i and β_i . With α_i and β_i being both zero, there won't be any strength degradation of the addressed strength. For $\alpha_i = 1$ and $\beta_i = 1$ the model refers to the linear residual strength model by Broutman and Sahu [4]. As studies on the fatigue life under spectrum loads by Passipoularidis and Philippidis [5, 6] have shown, the linear model is generally the most efficient choice for conservative predictions and without the need of any residual strength data. For arbitrary values of α_i with $\beta_i = 1$ the model represents the nonlinear one-parameter model used by [7, 8]. The strength degradation is then simulated with either a steep loss of strength at the beginning or at the end of the lifetime. For all other values of α_i and β_i the model refers to the “normalized residual strength model (NRSM)” by Stojkovic et al. [9]. By using the NRSM, the typical initial loss of strength as well as the sudden decrease in strength at the end of life can be modeled very well. Equation 6 is applicable for variable amplitude loads, as shown for two exemplary two-block loads in Figure 5. Two degradation

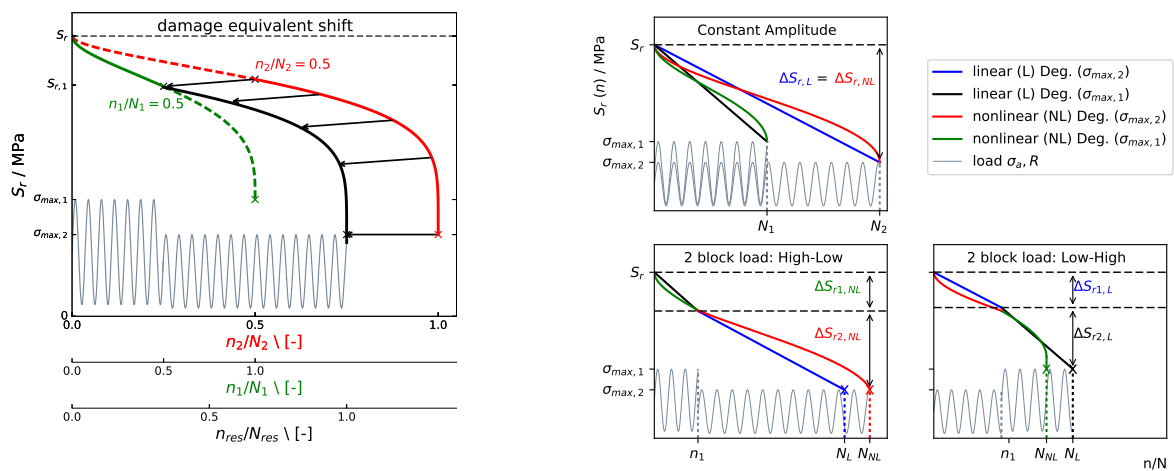


Figure 2. Life fraction equivalent shift of residual strength behaviors under block loads

behavior at $\sigma_{max,1}$ and $\sigma_{max,2}$ in High-Low and Low-High loads are illustrated exemplarily. If the maximum stress is changed, the new maximum stress is taken into consideration and the degradation starts at the equivalent life fraction of the new curve. The shape of the degradation curve stays the same while the start point of the second curve is moved to the equivalent life fraction of the first curve, as shown in Figure 2 for an exemplary High-Low block load.

2.3. Stiffness Degradation

In the current work Puck's model for the stiffness degradation of transverse and shear values due to inter-fibre fracture [10] is applied. The model is used for stiffness degradation in static failure analysis in combination with Puck's failure criteria, but has already been examined for the modeling under fatigue loads by Adden and Horst [11].

$$\begin{bmatrix} E_{\perp}(n) \\ G_{\perp\parallel}(n) \\ \nu_{\perp\parallel} \\ \nu_{\parallel\perp}(n) \end{bmatrix} = \begin{bmatrix} E_{\perp}^0 \left(\frac{1 - \eta_{r,\perp}}{1 + c_{\perp}(f_{e,IFF}(n) - 1)\xi_{\perp}} + \eta_{r,\perp} \right) \\ G_{\perp\parallel}^0 \left(\frac{1 - \eta_{r,\perp\parallel}}{1 + c_{\perp\parallel}(f_{e,IFF}(n) - 1)\xi_{\perp\parallel}} + \eta_{r,\perp\parallel} \right) \\ const. \\ \nu_{\perp\parallel} \cdot (E_{\perp}(n)/E_{\parallel}(n)) \end{bmatrix} \quad (7)$$

The following parameters are used for stiffness degradation under inter-fibre failure mode A, B and C for all further investigations:

- $\eta_{\perp A} = 0.3$ and $\eta_{\perp B,C} = 0.5$
- $\eta_{\perp\parallel A,B,C} = 0.25$, $\xi_{\perp A,B,C} = 1.31$, $c_{\perp A,B,C} = 5.34$ and $\xi_{\perp\parallel A,B,C} = 1.5$, $c_{\perp\parallel A,B,C} = 0.7$

3. Experimental data

Fatigue data from experimental tests is taken from the OptiDAT¹ database [12]. All of the examined materials are glass fibre reinforced plastics (GFRP). The uni-directional material consists of non-woven unidirectional glass rovings with a minor amount of off-axis reinforcement, made of polyester (PES) yarn, and the bi-axial material consists of non-woven glass rovings in 2 layers ($\pm 45^\circ$). The dry fibers are infused with the epoxy system "Prime 20" by vacuum assisted resin transfer molding and the system is post-cured at $80^\circ C$ for 4 hours [13].

Mat.#1-#3 The basic material configurations are the uni-directional laminate $[0]_5$ ("UD2" [12]), transverse tested uni-directional laminate with stacking sequence $[0]_7$ ("UD3" [12]) and biaxial laminate with stacking sequence $[\pm 45]_5$ ("MD3" [12]).

Mat.#4 The multi-directional laminate with stacking sequence $[(\pm 45/0)_4/\pm 45]$ ("MD2" [12]) is used for the validation of the simulation method.

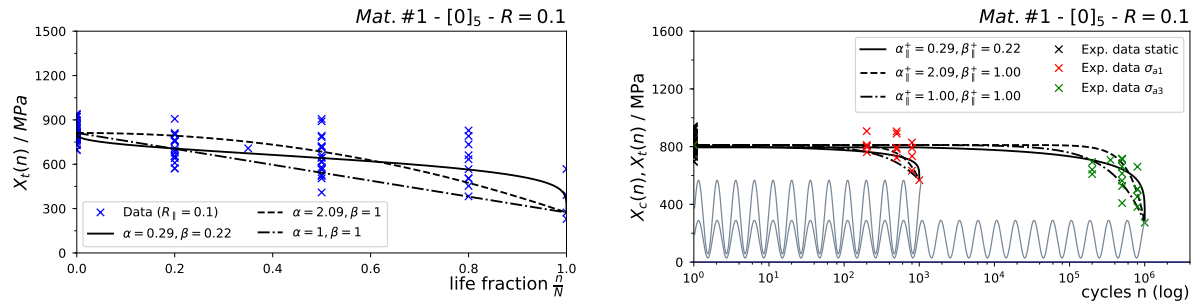
Only input data for static and fatigue tests of uni-directional (Mat.#1 and Mat.#2) and bi-axial (Mat.#3) specimen is used as input data for the simulations. Table 1 shows the static input values from static tests of flat specimen, which consist of fibre volume contents between 51,94 % and 53,73 %. Figure 3 exemplarily shows the fit of equation 6 to experimental residual

¹ The OPTIMAT BLADES project is a European research project, supported by the European Union. Project Coordinators: Wind Turbine Materials and Constructions (WMC, Netherlands) and Energy Research Centre of Netherlands (ECN - Netherlands). Database Contributors: Center for Renewable Energy Sources (CRES - Greece), German Aerospace Center (DLR - Germany), Rutherford Appleton Laboratory (RAL - United Kingdoms), National Research Laboratory (RISOE - Denmark), University of Patras (UP - Greece), Technical Research Center (VTT - Finland), Free University Brussels (VUB - Belgium) and Wind Turbine Materials and Constructions (WMC - Netherlands).

	$E_{\parallel}$ GPa	$E_{\perp}$ GPa	$G_{\perp\parallel}$ GPa	$\nu_{\perp\parallel}$	$R_{\parallel}^{+}$ MPa	$R_{\parallel}^{-}$ MPa	$R_{\perp}^{+}$ MPa	$R_{\perp}^{-}$ MPa	$R_{\perp\parallel}$ MPa
mean value	38,43	14,07	4,23	0,2893	810,56	469,96	55,88	164,95	56,1

Table 1. Calculated static values for experimental data from [12]

strength data of Mat.#1 and the estimation of $\alpha_{\parallel}^{+}, \beta_{\parallel}^{+}$ for a description of the fibre parallel residual strength behavior under tension-tension load. Equation 6 is fitted to residual strength

**Figure 3.** Estimation of $\alpha_{\parallel}^{+}, \beta_{\parallel}^{+}$ for $R_{\parallel} = 0.1$

data of all five local strengths X_t , X_c , Y_t , Y_c or $S_{\perp\parallel}$ at the ply-level for a deeper investigation on multiaxial fatigue. Table 3 shows the values for α_i and β_i concerning each strength strength for tension-tension and tension-compression loads.

	ratio	$\alpha_{\parallel}^{+}, \beta_{\parallel}^{+}$	$\alpha_{\perp}^{+}, \beta_{\perp}^{+}$	$\alpha_{\perp\parallel}, \beta_{\perp\parallel}$	$\alpha_{\parallel}^{-}, \beta_{\parallel}^{-}$	$\alpha_{\perp}^{-}, \beta_{\perp}^{-}$
conf. #1	<i>all</i>	1.00 , 1.00	1.00 , 1.00	1.00 , 1.00	0.00 , 0.00	0.00 , 0.00
conf. #2	0.1	0.29 , 0.22	1.02 , 0.66	0.70 , 0.37	0.00 , 0.00	0.00 , 0.00
	-1	0.44 , 0.29	0.32 , 0.24	0.70 , 0.37	0.00 , 0.00	0.00 , 0.00

Table 2. Coefficients for residual strength simulation with experimental data from [12]

4. Results

The results for predicted fatigue life and residual strength of the multi-directional laminate Mat.#4 and related experimental data for constant amplitude loads are shown in Figure 4. The Figure illustrates the predictions for a low and a high amplitude load regarding the further considered block load simulations. As can be seen, the fatigue life of the multi-directional laminate is estimated very accurately for constant amplitude loads. There a growing deviations between predicted and experimental fatigue life at higher number of endured cycles for $R=0.1$, but the estimations are on the safe side. In the initial phase, the use of the nonlinear model leads to slightly lower residual strength values than the linear model, because of the initial loss in strength. While there are nearly no differences between the estimations until approximately 30-40% lifetime, there are rapidly growing deviations between the two models in the late stage, particularly in the region of 80% lifetime. There is no big difference in the predicted fatigue life, but earlier strength degradation leads to slightly earlier failure in the $\pm 45^\circ$ -plies and, consequently, to earlier stress redistribution into the 0° -plies. Taking into account the

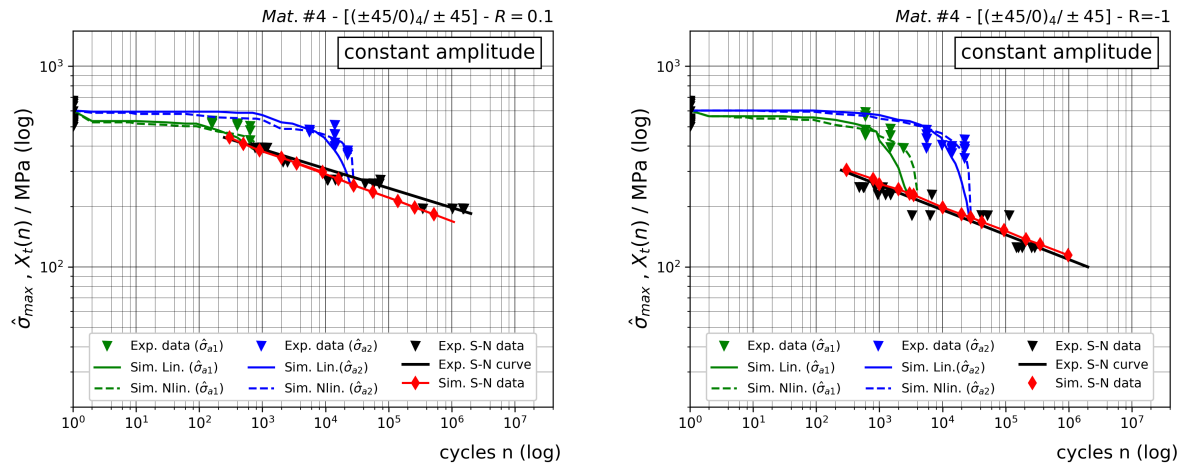


Figure 4. Predicted S-N curves and residual strength for constant amplitude loads

logarithmic scale, the differences in residual strength estimation between the use of linear and nonlinear model are significant and the nonlinear model leads to considerably better results. On the other hand, an enormous amount of data is needed for a holistic approach on multiaxial strength degradation with the nonlinear model. Nonetheless, it might also be beneficial to only use the nonlinear model for fibre parallel strength reduction and reduce the amount of data needed. Figure 5 shows the predicted life and residual strength for a High-Low and a Low-High load compared to experimental data. In the right pictures, the x-axis is divided into two parts, for a better view on the differences at the late stages. As shown in the box-plots, the interquartile range is very small at Low-High loads compared to the box-plot in High-Low loads and would therefore be difficult to examine within the plot. The four lower pictures of Figure 5

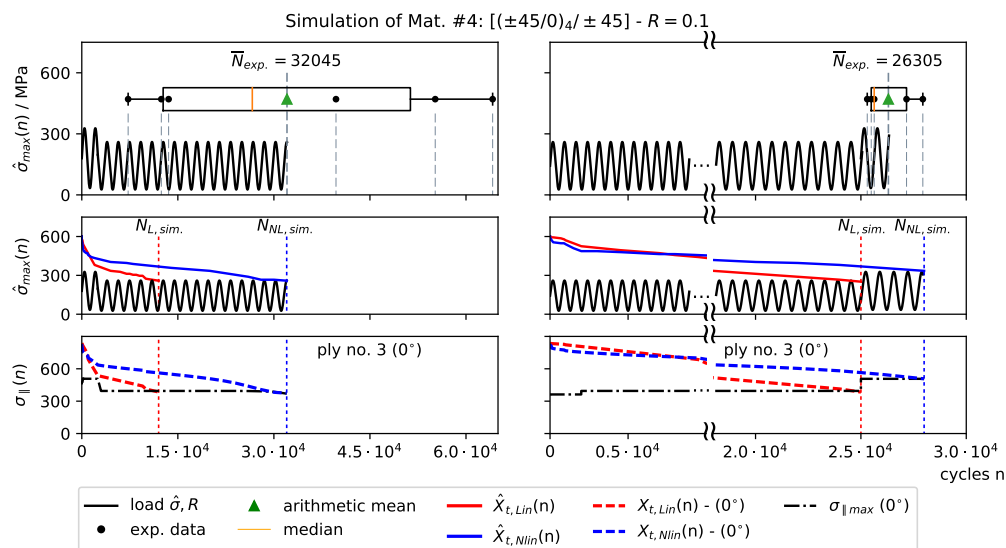


Figure 5. Predicted fatigue life and strength for High-Low and Low-High block loads at R=0.1

show, that the laminate failure is of course accompanied by the failure of the 0° plies. The use of the nonlinear model leads to more precise and less conservative predictions of fatigue life for the High-Low block load, when compared to the simulations with the linear model. Concerning

the Low-High tension-tension block loads, both models lead to a very early failure after the start of the second block load, which matches the experimental observations. Figure 6 shows the predictions for a High-Low and a Low-High block load with a alternating stress ratio $R=-1$. Similar to the findings in Figure 5, the predictions of fatigue life are improved with the use of

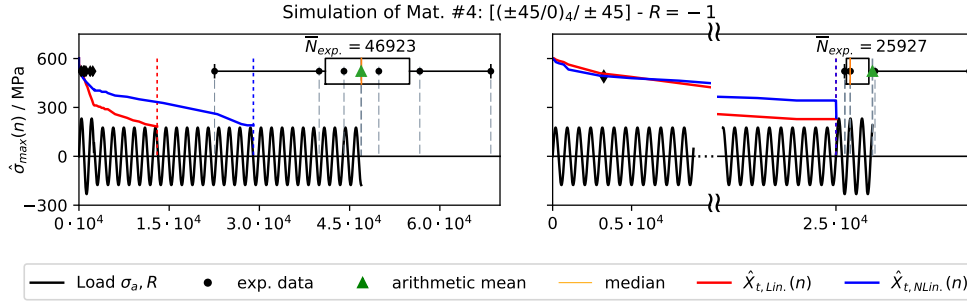


Figure 6. Predicted fatigue life and strength for High-Low and Low-High block loads at $R=-1$

the nonlinear model. While the use of both models still yield very conservative predictions, both predictions for fatigue life yield large differences from the arithmetic mean and are outside of the interquartile range for the experimental data. Anyway, there is quite a few experimental data with prematurely failed specimen in the first block load in Figure 6, which have not been taken into consideration for statistical evaluations and box-plot design. Predictions for the Low-High block load are also very similar to the results in tension-tension loads. Both models lead to conservative predictions with early failure in the first cycle after the start of the second block load. As shown in Figure 7, the simulation results for block loads with mixed stress ratios $R=0.1$ and $R=-1$ are largely a projection of those seen in Figure 5 and 6. The high tension-tension load

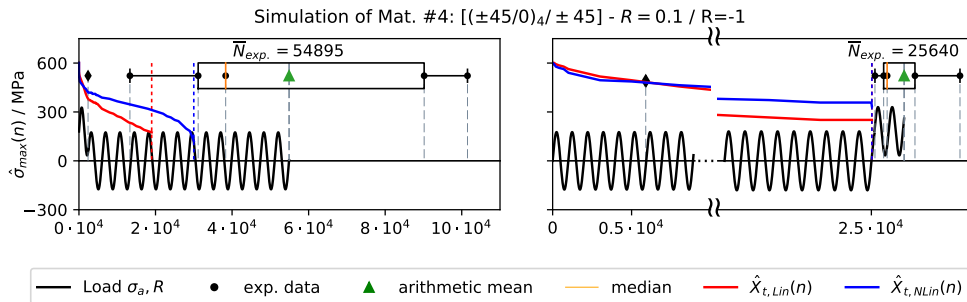


Figure 7. Fatigue life and strength for High-Low and Low-High block loads at $R=0.1$ and $R=-1$

followed by the low tension-compression load leads to the same conservative predictions as seen before. Still the nonlinear model is improving the predictions and yields less large deviations from the arithmetic mean on the safe-side. Similar to the results before, the start of the high tension-tension load after the initial low tension-compression load leads to a sudden failure in the first cycle of the second load block. Compared to the experimental data, the simulations are on the safe side, although the standard deviations of the experimental data are very small in that case. For a better comparison of the results, the model error function used by Post et al. [14] is used to compare the predicted fatigue life to the mean life from experimental data as follows

$$\text{Model Error} = M_e = \log \left(\frac{N_{\text{model}}}{N_{\text{experiment}}} \right) \quad (8)$$

The model errors for the investigated block loads are illustrated in Table 3.

	$\bar{N}_{exp}$	R=0.1		$\bar{N}_{exp}$	R=-1		R=0.1 / R= -1		
		Lin .	Nonlin.		Lin .	Nonlin.	$\bar{N}_{exp}$	Lin.	Nonlin.
H-L	32045	-0.44	0.00	46923	-0.55	-0.21	54895	-0.46	-0.26
L-H	26305	-0.02	0.02	25927	-0.01	-0.01	25640	-0.01	-0.01

Table 3. Coefficients for residual strength simulation with experimental data from [12]

5. Conclusions

In summary, the use of the nonlinear model improves the predictions of residual strength in constant amplitude loads and the predictions of fatigue life in High-Low block loads. There is nearly no difference in the simulations results for the Low-High block loads, since the failure almost always occurs right at the beginning of the second cycle.

6. Acknowledgments

The authors wish to thank the contributors to the OptiDAT database [12] for the possibility to validate their investigations with the extensive amount of provided fatigue data.

References

- [1] T.P. Philippidis and V.A. Passipoularidis: *Residual strength after fatigue in composites: Theory vs. experiment*. International Journal of Fatigue, Vol. 29, pp. 2104-2116, 2007.
- [2] A. Puck and H. Schürmann: *Failure analysis of FRP laminates by means of physically based phenomenological models*. Composites Science and Technology, Vol. 62, pp. 1633-1662, 2002.
- [3] J. Wang and B.L. Karihaloo: *Optimum in situ strength design of composite laminates. Part I: in situ strength parameters* Journal of Composite Materials, pp.1314-1337, 1996.
- [4] L. Broutman and S. Sahu: *A new theory to predict cumulative fatigue damage in fiberglass reinforced plastics*. Composite Materials: Testing and Design (2nd Conference), ASTM STP497, Corten HT (1972) 170188.
- [5] V.A. Passipoularidis and T.P. Philippidis: *A study of factors affecting life prediction of composites under spectrum loading*. International Journal of Fatigue, vol. 31, no. 3, pp. 408 - 417, 2009.
- [6] V.A. Passipoularidis and T.P. Philippidis and P. Brondsted: *Fatigue life prediction in composites using progressive damage modelling under block and spectrum loading*. International Journal of Fatigue, vol. 33, no. 2, pp. 132 - 144, 2011.
- [7] J.R. Schaff and B.D. Davidson: *Life prediction methodology for composite structures. part i - constant amplitude and two stress level fatigue*. Journal of Composite Materials, pp. 170188, 1997.
- [8] K. Reifsnider and W.W. Stinchcomb: *A critical-element model of the residual strength and life of fatigue-loaded composite coupons*. Composite Materials: Fatigue and Fracture K.L., ASTM STP-907, American Society for Testing and Materials, Philadelphia, PA (1986), pp. 298-313, 1986.
- [9] N. Stojkovic and F. Radomir and H. Pasternak: *Mathematical model for the prediction of strength degradation of composites subjected to constant amplitude fatigue*. International Journal of Fatigue, pp. 478487, 2017.
- [10] A. Puck and M. Mannigel: *Physically based non-linear stress-strain relations for the inter-fibre fracture analysis of FRP laminates*. Composites Science and Technology, vol. 67, pp. 1955 - 1964, 2007.
- [11] S. Adden and P. Horst: *Stiffness degradation under fatigue in multiaxially loaded non-crimped-fabrics*. International Journal of Fatigue, pp. 108-122, vol. 32, 2010.
- [12] R. Nijssen: *Optidat - fatigue of wind turbine materials database*. Knowledge Centre WMC, Available at https://wmc.eu/optimatblades_optidat.php.
- [13] T. K. Jacobsen: *Material Specification, Reference Material (OPTIMAT) - Glass-epoxy material*. Optimat Blades, Material Specification, Doc. No: 10004, revision 1, May 7th, 2002.
- [14] N.L. Post, S.W. Case, J.J. Lesko: *Modeling the variable amplitude fatigue of composite materials: A review and evaluation of the state of the art for spectrum loading*. International Journal of Fatigue, vol. 30, no. 12, pp. 2064 - 2086, 2008.