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# A Three-Dimensional Model with Adjustable Negative Compressibility

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**Abstract.** Materials with negative compressibility can expand in one or more directions when placed under compressive hydrostatic stress. In this paper, it is proved that this special property can be manifested by systems consisting of hexagonal honeycombs and particularly we show that this 3D model has adjustable negative compressibility which can be tailored for specific applications.

## 1. Introduction

Generally, common materials will contract in every direction when they are subjected to a hydrostatic pressure. However, some recent studies [1-5] show that some materials and structures may behave in a different manner, i.e. such materials and structures can expand in one or more directions when compressed uniformly. It is still an early stage for applications but their values for pressure driven actuators, force amplifiers and telecommunication line systems are clearly shown [1].

Over the past few decades, some theoretical models and structures exhibiting negative compressibility have been proposed including wine-rack-like structures [1], truss-type systems [2], bi-material strips [3], and rotating rigid triangles [4]. In view of these models, it is necessary to mention that few 3D models have been presented. Although these 2D models are simpler to analyze and sometimes they can predict the behaviors of particular projections of a complex 3D model where the linear compressibility is measured, their limitations lie in the fact that negative area compressibility (i.e. NAC), something that may obviously be more useful than negative linear compressibility (i.e. NLC), is not possible to be achieved in a 2D system. Also sometimes the simplification from 3D models to planar structures can lead to the differences between conclusions and actual results [5].

In this paper, we propose a 3D theoretical model through the combination of hexagonal honeycombs and the expressions of the mechanical properties including Young's moduli, Poisson's ratios and compressibilities are derived through an energy approach. It is shown that the structure can exhibit negative linear and area compressibility for particular conformations and all of these are detailed in the following sections.

## 2. Analytical Model

The structure taken into consideration in this paper is consisting of four hexagonal honeycombs as shown in Fig. 1. And it should be noted that the rods in the hexagonal honeycombs are connected by flexure hinges at the joints and they can only rotate around the corresponding hinges in the plane

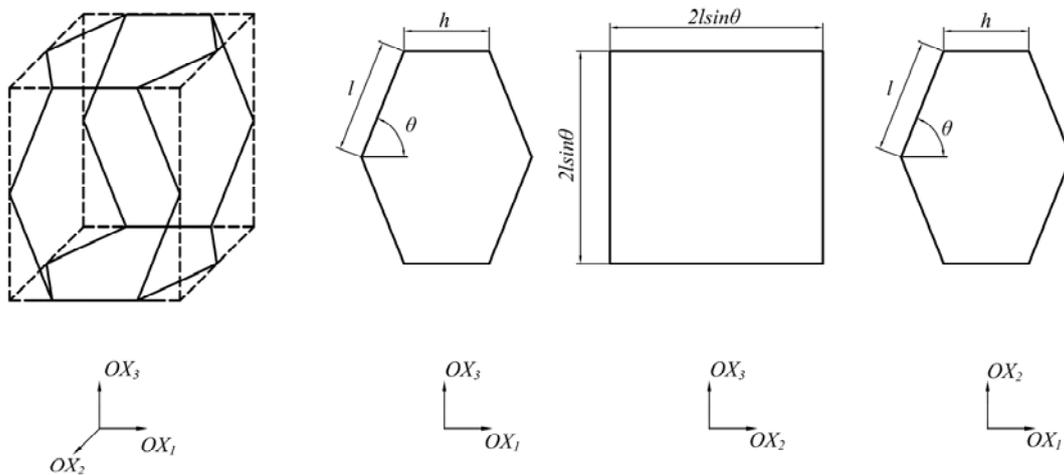


where the relevant honeycomb belongs to rather than around the  $OX_3$  axis. The structure modelled is one having dimensions  $l, h, \theta$  defined as in Fig. 1, so the projections of the model in the  $OX_i$  ( $i=1, 2, 3$ ) directions are given by:

$$X_1 = 2l \cos \theta + h \quad (1)$$

$$X_2 = 2l \sin \theta \quad (2)$$

$$X_3 = 2l \sin \theta \quad (3)$$



**Figure 1.** The 3D model and its two-dimensional projections in the three axes.

Note that apart from the normal conditions that  $l > 0$ ,  $h > 0$ , and  $0^\circ < \theta < 90^\circ$ , the structure does not allow the overlap of the rods. And then we can use the energy approach to derive the expressions of the Young's moduli. The work done by each unit cell due to changes in angles from  $\theta$  to  $\theta + d\theta^{[i]}$  for loading in  $OX_i$  directions is given by ( $i=1, 2, 3$ )

$$W = 16 \cdot \left[ \frac{1}{2} k_h \cdot (d\theta^{[i]})^2 \right] = 8k_h \cdot (d\theta^{[i]})^2 \quad (4)$$

And the work done per unit volume due to a strain  $d\varepsilon_i^{[i]}$  for loading in  $OX_i$  directions is given by ( $i=1, 2, 3$ )

$$U = \frac{1}{2} E_i (d\varepsilon_i^{[i]})^2 = \frac{1}{2} E_i \left( \frac{1}{X_i} \frac{dX_i}{d\theta} d\theta^{[i]} \right)^2 \quad (5)$$

Then from the principle of conservation of energy, the expressions of the Young's moduli  $E_i$  ( $i=1, 2, 3$ ) can be given by

$$E_i = \frac{16k_h}{V} \frac{X_i^2}{(dX_i / d\theta)^2} \quad (6)$$

And for such systems, the Poisson's ratio for any value of  $\theta$  may be defined by ( $i, j=1, 2, 3$ ):

$$\nu_{ij} = -\frac{d\varepsilon_j^{[i]}}{d\varepsilon_i^{[i]}} \quad (7)$$

Where  $d\varepsilon_j^{[i]}$  is an infinitesimally small change in  $OX_j$  direction due to the loading in  $OX_i$  direction. And hence the Poisson's ratio can be given by:

$$\nu_{ij} = -\frac{d\varepsilon_j^{[i]}}{d\varepsilon_i^{[i]}} = -\frac{dX_j / d\theta}{dX_i / d\theta} \cdot \frac{X_i}{X_j} \quad (8)$$

Having determined Young's moduli and Poisson's ratios, one may obtain the expressions for  $\beta_L$  ( $OX_i$ ):

$$\beta_L(OX_1) = \frac{1}{E_1} - \left( \frac{\nu_{21}}{E_2} + \frac{\nu_{31}}{E_3} \right) = \frac{l^3 \sin^2 \theta}{k_h} \cdot \left( \frac{\sin^2 \theta}{2 \cos \theta + h/l} - \cos \theta \right) \quad (9)$$

$$\beta_L(OX_2) = \frac{1}{E_2} - \left( \frac{\nu_{12}}{E_1} + \frac{\nu_{32}}{E_3} \right) = \frac{l^3 \cos \theta}{2k_h} \cdot \left( 2 \cos^2 \theta - \sin^2 \theta + \frac{h}{l} \cos \theta \right) \quad (10)$$

$$\beta_L(OX_3) = \frac{1}{E_3} - \left( \frac{\nu_{13}}{E_1} + \frac{\nu_{23}}{E_2} \right) = \frac{l^3 \cos \theta}{2k_h} \cdot \left( 2 \cos^2 \theta - \sin^2 \theta + \frac{h}{l} \cos \theta \right) \quad (11)$$

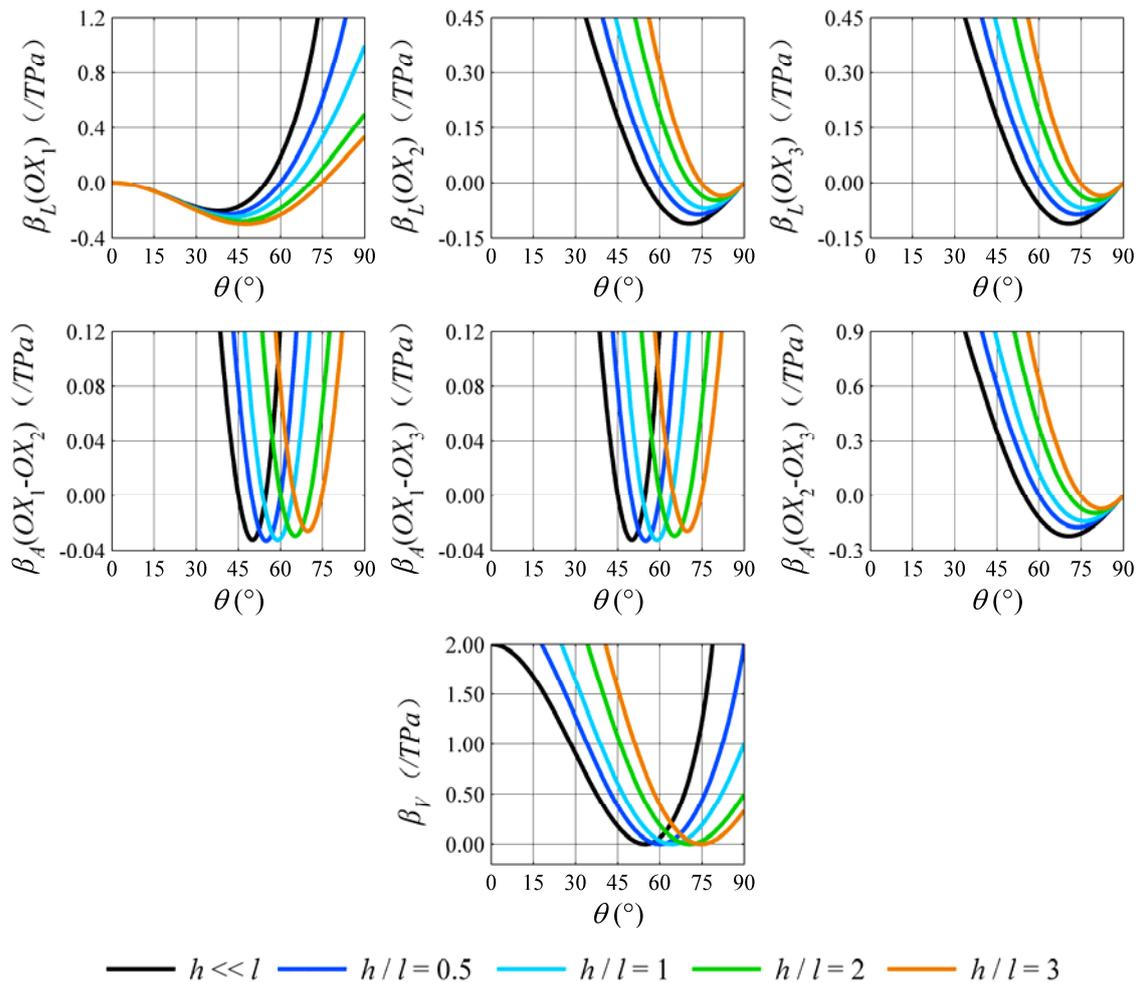
And then the area compressibility and the volume compressibility can be found from the linear compressibility:

$$\beta_A(OX_i - OX_j) = \beta_L(OX_i) + \beta_L(OX_j) \quad (12)$$

$$\beta_V = \beta_L(OX_1) + \beta_L(OX_2) + \beta_L(OX_3) \quad (13)$$

### 3. Discussion

These above equations suggest that in general, if we assume that the parameters  $l$  and  $h$  are constant, the values of the compressibility for this system can be positive, zero and negative with the actual sign and magnitude relying on the geometry of the system (i.e. the magnitude of  $\theta$ ).



**Figure 2.** The linear, area and volume compressibility of this model across various angles of  $\theta$  with  $k_h = 2 \text{ KJ/rad}^2$ ,  $l = 1 \text{ mm}$ .

Equations (9-11) and plots in Fig. 2 indicate that this system can exhibit NLC when the following conditions are satisfied:

- (1) For negative  $\beta_L(OX_1)$ :  $\sin^2\theta < 2\cos^2\theta + (h/l) \cdot \cos\theta$
- (2) For negative  $\beta_L(OX_2)$  and  $\beta_L(OX_3)$ :  $2\cos^2\theta + (h/l) \cdot \cos\theta < \sin^2\theta$

Which clearly indicates that NLC in the  $OX_2$  and  $OX_3$  directions will occur simultaneously due to the symmetry of the structure, while NLC in  $OX_1$  direction will arise at the exclusion of NLC in the  $OX_2$  and  $OX_3$  directions, and vice-versa. Moreover, the effect of NLC in the  $OX_2$  and  $OX_3$  directions can be maximized (i.e. widening the region of NLC and increasing the magnitude of the most negative value) to a greater degree by decreasing the magnitude of  $h/l$ . However, the effect of NLC in  $OX_1$  direction can be maximized by increasing the magnitude of  $h/l$ . In particular, small  $h/l$  ratios favour a larger range of negative  $\beta_L(OX_2)$  and  $\beta_L(OX_3)$ . However, there is a limit to the range of  $\theta$ -values over which such behaviour can be observed, and at most it can only cover a range of  $54.74^\circ < \theta < 90^\circ$  when  $h$  becomes zero. While large  $h/l$  ratios can lead to a larger range of NLC in  $OX_1$  direction and it seems that with the increasing of  $h/l$ , the range of negative  $\beta_L(OX_1)$  may cover a range of  $0 < \theta < 90^\circ$  when  $h$  becomes infinite. And from Fig.2 we can know that the variational laws of the effect of NAC in the  $OX_2$ - $OX_3$  plane is same to that of NLC in  $OX_2$  and  $OX_3$  directions so decreasing the magnitude of  $h/l$  can strengthen this effect. And the effect of NAC in the  $OX_1$ - $OX_2$  and  $OX_1$ - $OX_3$  planes can also be

changed by choosing the appropriate magnitude of  $h/l$  and  $\theta$ , but fallaciously it is not conspicuous. And from the plot of  $\beta_V$ , we can also know the volume compressibility can never be negative.

It is important to highlight that the work presented here indicates that the extent of NLC and NAC of this model can be fined to specific values by choosing the magnitude of  $\theta$  (which has a close bearing on the sign of the compressibility) and  $h/l$  (which influences the magnitude of the compressibility) carefully, so it would be useful when one needs to use this model to attain a material for particular application. It should be noted that for this model to exhibit negative compressibility, it is required that the pressure is exerted on the system only from the outside and the points of application of the stress are on the vertexes of the structure. Also the rods of the model are required to be rigid enough and the flexure hinges should be flexible enough to make a clear response to the sufficiently small stress.

#### 4. Conclusion

In summary, it has been shown through analytical modelling that this 3D structure consisted of four hexagonal honeycombs can exhibit the special property of negative linear compressibility in certain directions and negative area compressibility in certain planes. Given how rarely the property of negative compressibility is exhibited, we hope that this model proposed in this paper can serve as a blueprint for the design of novel man-made materials.

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