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Calculation and analysis of critical speed of high speed motor spindle rotor system

S Huaitao [1,3*]**, Z Jizong** [1]**, Z Yu**[1] **and H Gang** [2]

[1] School of Mechanical Engineering, Shenyang Jianzhu University, National and Local Joint Engineering Laboratory of High-grade Stone Numerical Control Processing Equipment and Technology, Shenyang 110168, China, Hunnan East Road on the 9th,Hunnan New District,S henyang, China.Postcard:110168
[2] Shenyang Yadong Technology Co. Ltd., 110000, China

[*] E-mail: 651554653@qq.com

Abstract. The traditional transfer matrix method used to calculate the critical speed of high-speed spindle motor rotor system will produce large errors. Taking the 170SD30 electrospindle as the analysis object, a high-speed electric spindle dynamics theoretical model was established based on the Riccati method，using Matlab to program and calculate the kinetic parameters of the first three critical speeds and natural frequencies of the high-speed spindle. Finally, the simulation data was compared with experimental data. The results show that the Riccati transfer matrix method is 2.3% more accurate than the traditional transfer matrix method, and it verifies the accuracy and feasibility of the Riccati transfer matrix method, and is of great significance to effectively guide the analysis and design of high-performance and high-speed motor spindle.

1. Introduction

The high-speed motor spindle is the core component of CNC machining and is the carrier of advanced manufacturing technology. Its performance will directly affect the development of CNC machine tools and even the entire manufacturing industry. With the development of high-speed electric main shaft high-speed, high-precision, and high-efficiency direction, higher requirements have been put on the dynamic characteristics of the electric spindle itself and on the electric spindle[1-4].When the spindle is running at high speed, it will produce different degrees of centrifugal force, inertial force and various unbalanced responses, and when the frequency of excitation force is close to or equal to the natural frequency of the rotor system, the rotor will resonate. This speed is the critical speed of the rotor, which shows that, when the rotor system is running at critical speed, the rotor will experience severe vibration, and if the speed is greater or less than a certain speed value or speed range, the rotor will run smoothly. The rotor runs at critical speeds, which can make the rotor's vibration worse, seriously affecting the processing precision and quality of the processed parts, and causing serious accidents [5, 6].For high-speed motorized spindles, it is necessary to analyse and study the critical speed of the rotor system due to its large load, high speed, and other characteristics.

At present, there are mainly three methods for calculating the critical rotational speed of high-speed electric spindle: formula method, transfer matrix method and finite element method [7-9].The formula method is applicable to the driveshafts of equal section, and it is more accurate to calculate the critical

speed [10];the traditional transfer matrix method is widely used in the calculation of the critical speed of a chain structure, such as a rotor with the advantages of simple program, less time and memory required. However, with the trial frequency increasing, the accuracy of calculation will decrease [11]; the finite element method can quickly and accurately determine the critical rotational speed of the rotor with the development of computer technology, but the finite element method variable occupies a large amount of memory, it requires a high degree of computer hardware, and a long calculation time [12, 13].The above methods for calculating the critical rotational speed all have different degrees of inadequacy and cannot accurately and quickly calculate the critical rotational speed of the rotor system.

In this paper, the Riccati transfer matrix method is used to propose a new electrospindle dynamics model to calculate the critical rotational speed of high-speed electric spindle, and the influence of critical rotational speed on the dynamic characteristics of high-speed electric spindle is analysed. In order to carry out the above work, firstly, the principle and solving method of the Riccati transfer matrix method is introduced, then, Matlab was used to compile the Riccati transfer matrix method and the traditional matrix method program and calculate the critical speed of the electric spindle. Finally, the simulation data and experimental data were compared and analysed.

2. High-speed electric spindle dynamics modelling

The transfer matrix method first simplifies the structure to a lumped parameter mechanical model, in the process of simplification, in accordance with the principle of invariable center of mass, so as to maintain all the dynamic characteristics of the original structure as much as possible, the rotor system of the high-speed electric spindle system is simplified to a model consisting of lumped masses and several shaft sections [14].

2.1 Model simplification

The electric spindle is a high-speed precision spindle for machining centers in the new generation of integrated machining, electro-hydraulic and hydraulic machining, which is mainly composed of housings, spindles, bearings, stators and rotors [15]. Figure 1 shows the structure of the 170SD30 electric spindle, the front and rear bearings are all angular contact ball bearings, the front end of the spindle bearing is model 7015C, and the rear model is 7012C.
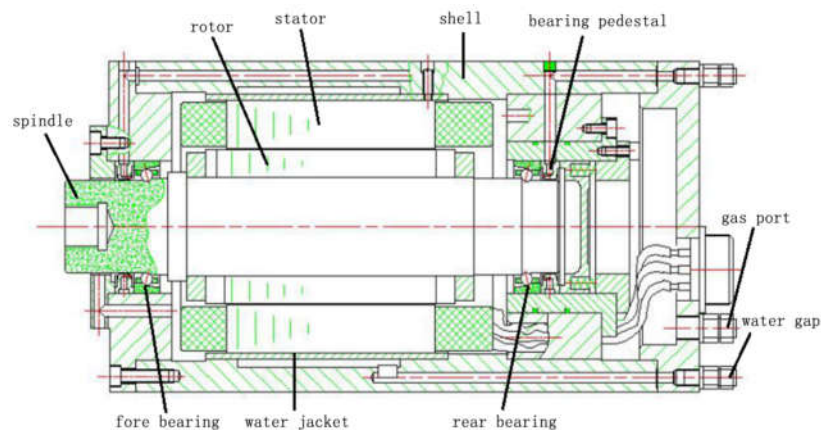
Figure 1. Spindle structure.

The support of the main shaft is generally an angular contact ball bearing. Due to the initial pre-tightening force, the working load capacity and the rigidity of the spindle can be increased. Therefore, the rigidity of the bearing needs to be taken into account in simplifying the high-speed spindle model. Bearing stiffness can be divided into radial stiffness, axial stiffness and angular stiffness, under actual conditions, with the increase of the rotational speed, both the axial stiffness and the angular stiffness will reach a relatively stable value, and the influence on the dynamic characteristics of the electric

spindle will be small [16].When simplifying high-speed motor spindle models, bearings can be reduced to four radially-distributed springs, only considering their radial stiffness, simplified as shown in figure 2. After radial preloading through the bearing, the radial stiffness is calculated as [17]:

$$K_r = \delta \times 17.7236 \sqrt[3]{Z^2 D_b} \frac{\cos^2 \alpha}{\sqrt[3]{\sin \alpha}} \sqrt[3]{F_{\alpha 0}} \qquad (1)$$

δ is the angular contact ceramic ball bearing correction factor, δ=1.8 at light preload, δ=1.9 at preload, and δ=2.0 at heavy preload; D_b is the rolling element diameter (8mm); Z is the number of bearing rolling elements (18); α is contact angle (15^0); F_{a0} is an axial preload, the electric spindle studied in this paper is δ=1.8 for light load and 400N for front and rear preloads. Substituting the above parameters into (1) K_r=2245.42N·mm^{-1}.
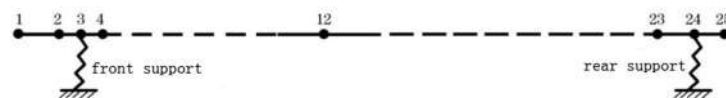

Figure 2. Simplified model of the spindle

2.2 *Node division*

When the transfer matrix method is used to simplify the model of the high-speed motor spindle, the motor spindle needs to be divided into nodes. The number of nodes will seriously affect the calculation result of the critical speed of the electric spindle. The number of nodes is too small and the result of calculation is inaccurate and too many nodes will lead to complicated calculation, long time-consuming, and increased error. According to the calculation result of the equal section beam, if the calculated critical speed error is less than 1%, then the total number of nodes N should satisfy the following relationship [18]:

$$N \geq 1 + 5.34r \qquad (2)$$

r is the highest order of the required natural frequency (or critical speed),according to the rotor model, the motor spindle is divided into 25 nodes, then r=4. The electrospindle dynamics model is shown in figure 3.

Figure 3. Simplified mechanical model of electric spindle.

3. Riccati transfer matrix method

The Riccati transfer matrix method is used to establish a new high-speed spindle model for calculating the critical speed of an electric spindle. When using matlab programming, considering the gyro torque, shear effects and cross-section coefficients and other influencing factors, the calculation results are more accurate and the calculation speed is faster [19].

The calculation principle of the Riccati transfer matrix method is: first dividing the shaft into several equal shaft sections, then according to the boundary conditions at both ends of the shaft, recursively from one end of the axis to the other to determine the state vector of the starting end and to seek the state vector of each section in reverse; in the process of increasing the speed from small to large, the point of maximum response is the critical speed [20].

The cell transfer matrix has four valid parameters for each node Z: radial displacement of the cross-section of the section: cross-section radial displacement X, angle A, bending moment M, and shear force Q. Marked as $Z = [X, A, M, Q]^T$. The relationship between the two nodes of the ith axis segment is Zi+1=TiZi, and Ti is the transfer matrix.

$$T_i = \begin{bmatrix} 1+\dfrac{l^3}{6EJ}(1-v)(m\omega^2 - K_{sj}) & 1+\dfrac{l^2}{2EJ}(I_P - I_d)\omega^2 & \dfrac{l^2}{2EJ} & \dfrac{l^3}{6EJ}(1-v) \\ \dfrac{l^2}{2EJ}(m\omega^2 - K_{sj}) & 1+\dfrac{l}{EJ}(I_P - I_d)\omega^2 & \dfrac{l}{EJ} & \dfrac{l^2}{2EJ} \\ l(m\omega^2 - K_{sj}) & (I_P - I_d)\omega^2 & 1 & 1 \\ m\omega^2 - K_{sj} & 0 & 0 & 1 \end{bmatrix}$$

E is the elastic modulus; J is the axial section moment ; l is the length between two units; $v = \dfrac{6EI}{k_t GAl^2}$ is the shearing coefficient between the nodes ,I is the moment of inertia of the cross section to the neutral axis, k_t is the section coefficient, ,k_t is the section modulus ,G is the shear modulus of the material, A is the cross-sectional area, and l is the length of the shaft section; m is the quality focused on the node;ω is the angular velocity of the rotor; K is the bearing stiffness at the support; I_p and I_d are the rotor's polar moment of inertia and the moment of inertia of the rotor.

The Riccati transfer matrix method divides r elements of the state vector of each node Z into two groups of f and e.

$$Z_i = \left\{ \begin{matrix} f \\ e \end{matrix} \right\}_i \tag{3}$$

In formula (3),f consists of $r/2$ elements corresponding to zero values in the starting interface state vector Z_1; e consists of the remaining $r/2$ complementary elements. Due to the fact that the left end of the spindle is free, there are boundary conditions $M_1=0$, $Q_1=0$.So $f_i = [M, Q]_i^T$, $e_i = [y, \theta]_i^T$. The relationship between the state vectors of two adjacent sections can be rewritten as:

$$\left\{ \begin{matrix} f \\ e \end{matrix} \right\}_{i+1} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}_i \left\{ \begin{matrix} f \\ e \end{matrix} \right\}_i \tag{4}$$

In formula (4), $u_{11i} = \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix}_i$; $u_{12i} = \begin{bmatrix} l(m\omega^2 - K_{sj}) & (I_P - I_d)\omega^2 \\ m\omega^2 - K_{sj} & 0 \end{bmatrix}_i$

$$u_{21i} = \begin{bmatrix} \dfrac{l^2}{2EJ} & \dfrac{l^3}{6EJ}(1-v) \\ \dfrac{l}{EJ} & \dfrac{l^2}{2EJ} \end{bmatrix}_i \quad u_{22i} = \begin{bmatrix} 1+\dfrac{l^3}{6EJ}(1-v)(m\omega^2 - K_{sj}) & L+\dfrac{l^2}{2EJ}(I_P - I_d)\omega^2 \\ \dfrac{l^2}{2EJ}(m\omega^2 - K_{sj}) & 1+\dfrac{1}{EJ}(I_P - I_d)\omega^2 \end{bmatrix}_i$$

Expand equation (4), introducing the following Riccati transformation：

$$f_i = S_i e_i \tag{5}$$

S_i in formula (5) is called Riccati matrix, which is a square matrix of $r/2 \times r/2$, substituting (4) into (3)

$$e_i = [u_{21} + u_{22}]_i^{-1} e_{i+1} \tag{6}$$

$$f_{i+1} = \left[u_{11}S + u_{12}\right]_i \left[u_{21}S + u_{22}\right]_i^{-1} e_{i+1} \tag{7}$$

Comparisons (5) and (7):

$$S_{i+1} = \left[u_{11}S + u_{12}\right]_i \left[u_{21}S + u_{22}\right]_i^{-1} \tag{8}$$

This is the recursion formula of the Riccati transfer matrix.

From the starting boundary conditions, the condition for the nonzero solution of equation 3 is:

$$\left|S_{N+1}\right| = \begin{vmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{vmatrix} = 0 \tag{9}$$

This is the frequency equation of the system. The root ω that satisfies equation (9) is the angular velocity of the critical speed required.

4. Experiment and result analysis

4.1 Experimental analysis

In order to verify the correctness of natural frequency and critical speed of high-speed motorized spindle obtained by the Riccati transfer matrix method, a dynamic performance testing system for high-speed spindle was built. In this experiment, the 170SD30 high-speed electric spindle was struck with hammering and the input and output signals were excited and collected. Experimental equipment includes hammers, charge amplifiers, acceleration sensors, force sensors, and signal collectors. The experimental platform is shown in figure 4 below.
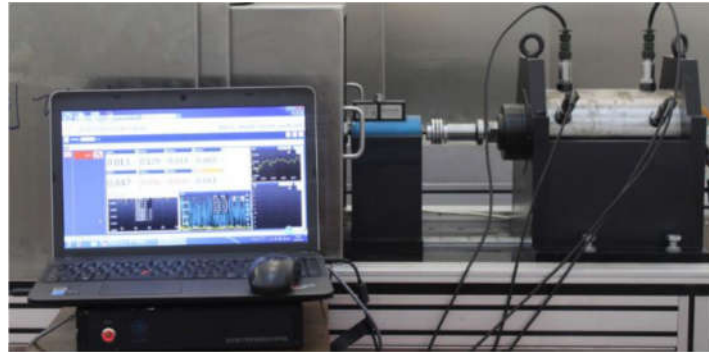
Figure 4. Spindle dynamic performance test system.

In order to reduce the experimental error and obtain more accurate experimental data, this experiment mainly selects three hammering points, namely the front section of the main shaft, the middle part of the main shaft and the rear end of the main shaft. The experimental method is single-point response and multi-point vibration. The specific steps are as follows: first, start the spindle and start running at a certain low speed; then, tapping each point with the hammer 6 times, the impact of the hammer on the front end of the spindle is a force of 126 N. Secondly, the speed is increased step by step in certain stages. At the same time, the vibration response value of the motor spindle is measured and recorded for each rotation speed. Finally, the experimental frequency and mode shape of the motor spindle are obtained, as shown in table 1 below.

Table 1. Natural frequency and critical speed of electric spindle.

order	frequency/HZ	critical speed/(r/min)
1	1767.6	106056
2	2214.0	132840
3	3186.6	191196

4.2 Simulation calculation and result analysis
Using Matlab to compile the traditional transfer matrix method and the Riccati method to calculate the critical rotational speed of the high-speed spindle, the rotor's natural frequencies and critical speeds can be obtained. The initial input spindle configuration parameters are as follows:outer diameter of stator D_1=130mm, stator inner diameter D_2=80mm,Shaft length l=11mm,Support stiffness K=2245.42N·mm,unit quality m=6.0×10^{-9}t/mm^3 .Shaft material parameters down: elastic modulus E=20600N·mm^{-2}, poisson's ratio μ=0.3,density ρ=7.85g·cm^{-3}. The first three primary natural frequencies and critical speeds are obtained by solving the equations, as shown in tables 2 and 3 below.

Table 2. Comparing the natural frequency obtained by the riccati method with the experimental results.

order	Riccati method calculation result		Experimental results		error/%
	frequency/HZ	critical speed/(r/min)	frequency/HZ	critical speed/(r/min)	
1	1667.9	100074	1767.6	106056	5.6
2	2058.1	123486	2214.0	132840	7.0
3	2956.5	177390	3186.6	191196	7.2

Table 3. Comparing the natural frequency obtained by the prohl method with the experimental results.

order	Prohl calculation results		Experimental results		error/%
	frequency/HZ	critical speed/(r/min)	frequency/HZ	critical speed/(r/min)	
1	1661.5	99690	1767.6	106056	6.0
2	2032.5	121950	2214.0	132840	8.2
3	2890.2	173412	3186.6	191196	9.5

Comparing the data from table 2 and table 3, it can be clearly seen that the critical speed that can be calculated by the Riccati transfer matrix method is 2.3% higher than the critical speed accuracy calculated by the traditional transfer matrix method, which reduces the error between the actual operating condition and the critical speed of the high-speed motor spindle using the transfer matrix method, which verifies the accuracy and feasibility of the Riccati transfer matrix method. It is of great significance to guide the development of high-performance high-speed spindles.

5. Conclusion

(1) The Riccati transfer matrix method for calculating the critical speed is by 2.3% more accurate than the traditional transfer matrix method for calculating the critical speed accuracy, which overcomes the problem of the reduced accuracy of traditional transfer matrix methods.

(2) The Riccati transfer matrix method preserves all the advantages of the traditional transfer matrix method and improves the computational speed and computational stability of the transfer matrix method.

(3) The Riccati transfer matrix method can accurately and rapidly solve the critical rotation speed and natural frequency of the electric spindle and analyze the dynamic characteristics of the electric spindle.

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