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Optimization of cross section of mobile crane boom using lagrange multiplier's method

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Abstract. This paper discusses the optimization of cross section of telescopic boom of mobile cranes. The extruded section is taken into consideration for the optimization problem. The problem has been solved using Lagrange Multipliers method. The area of cross section of the boom has been taken as the objective function, so as to minimize the mass, whereas the constraint function has been taken as a general function of hardness and stability. The above parameters allow us to form a mathematical model for numerical analysis and thus obtain the optimum dimensions for the cross section.

1. Introduction

A crane is a machine which is used to lift and drop heavy materials or to move them in a horizontal path. The basic principle of a crane is to create a mechanical advantage to the load applied such that it is possible to lift loads a number of times heavier than what is actually possible with that effort. Over the years, cranes have developed from a basic wire wound over a pulley, to all terrain, highly sophisticated cranes capable of lifting up to a thousand tons. The common uses of cranes are for loading and unloading goods, for construction and on the shop floor to aid in the assembly of heavy components. They can be classified as follows:

The primary part of a crane is its boom. A boom is a cantilever beam, which may either be fixed or hinged at one end. A boom consists of the following parts:[1]

1. The outermost part of the boom is known as mother boom. It consists of the remaining sections as well as the mechanism for extension and retraction.
2. The second section of the boom is known as the middle boom. It is housed inside the mother boom.
3. This is the third section of boom, which is inside the middle boom. The crane hook is suspended directly from this section.

A crane boom may consist of plates welded together, or a lattice structure.[2]The boom taken under consideration here is the former type, which is mostly employed in truck mounted, rough terrain and all terrain cranes. A rectangular cross section is the most conventional one, where four plates are welded together. However, higher the number of welds, higher is the chance of failure. Hence, we intend to minimize the possibility of failures by using only two lines of welding instead of four. To achieve this, an extruded section may be used, with the cross-section being achieved by bending a sheet.[3]

2. Definition of the problem

The cross-section considered for this study is shown in figure 1 given below:

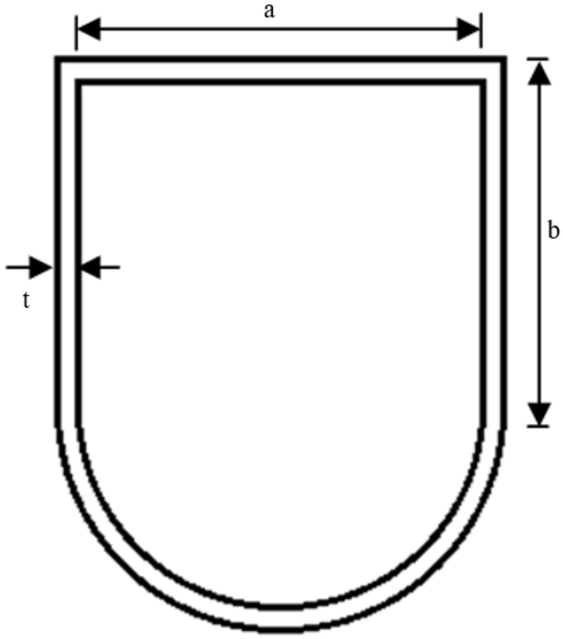

Figure 1. Extruded cross-section with dimensions.

The optimum dimensions of thin-wall extruded cross-section are to be defined. The top surface is a sheet of width 'a', and the two sides comprise of sheets of height 'b'. The fourth side is an arc of radius '$\frac{a}{2}$'. The thickness is taken as 't' for all the sheets. The ratio of thickness and length is a constant, which serves as local stability conditions: [4]

$\frac{t}{a} = \delta_1, \frac{t}{b} = \delta_2.$

Hence, we intend to define optimum dimensions of 'a' and 'b'.

Area of cross section:

$$A = at + 2bt + \left(\frac{\pi}{2}\right)t(a+t)$$
$$= \left(\delta_1 + \frac{\pi}{2}\delta_1\right)a^2 + 2\delta_2 b^2 + \frac{\pi}{2}\delta_1\delta_2 ab$$
$$= k_1 a^2 + k_2 b^2 + k_3 ab \quad (1)$$

where these substitutions are introduced:

$k_1 = \left(\frac{2+\pi}{2}\right)\delta_1,\ k_2 = 2\delta_2, \qquad k_3 = \frac{\pi}{2}\delta_1\delta_2$

3. Objective function and constraint function

If the material and length of structure are kept constant, then the area function can be taken as the objective function (F).

$$F \equiv A = k_1 a^2 + k_2 b^2 + k_3 ab \quad (2)$$

In order to form the constraint function, axial stress and bending are taken into consideration, which are the major loads acting on a boom. Thus the constraint function is represented as a function taking into consideration the failure parameters relevant for a crane boom:[5]

$$\varphi = \frac{N}{A} + \frac{M_x}{W_x} + \frac{M_y}{W_y} - R_l = 0 \tag{3}$$

where, N - Axial force
M_x, M_y - Moments of flexion about x and y axes
W_x, W_y - Section modulus for corresponding axes
R_l - Limiting stress

The section modulus about x and y axes are represented as functions of cross section areas and corresponding sides, as follows:

$$W_x = \alpha_x \left(b + \frac{a}{2}\right) A \, , W_y = \alpha_y A a \tag{4}$$

Thus, the constraint function can be represented as follows:

$$\varphi = \frac{N}{A} + \frac{M_x}{\alpha_x\left(b+\frac{a}{2}\right)A} + \frac{M_y}{\alpha_y A a} - R_l = 0 \tag{5}$$

Coefficients α_x and α_y have analytical values:

$$\alpha_x = \frac{W_x}{\left(b + \frac{a}{2}\right)A} = \frac{I_x}{\left(b + \frac{a}{2}\right)Ay_{max}}$$
$$= \frac{t[\left(\frac{2\pi+1}{16}\right)a^3 + \frac{2}{3}b^3 + 7344.04t^3 - 68.005a^2t - 206.05at^2 - 9626.067bt^2]}{69.376t\left(b + \frac{a}{2}\right)A}$$

$$\alpha_y = \frac{W_y}{Aa} = \frac{I_y}{Aax_{max}}$$
$$= \frac{t(\frac{2}{3}bt^2 + \frac{1}{2}a^2b + abt + \frac{9}{16}a^3 + \frac{1}{8}a^2t + \frac{1}{4}at^2 + \frac{1}{8}t^3)}{aA(\frac{a}{2})}$$

Substituting the appropriate ratios of $\frac{t}{a}$ and $\frac{t}{b}$ as δ_1 and δ_2 respectively, and plugging in the values of δ_1 and δ_2 as 0.014 and 0.022 [5] respectively, we get:

$$\alpha_x = (6.431 \times 10^{-7})(\frac{42749.726}{\varepsilon^3} + 242954.325\varepsilon^3 + 7344.04 - \frac{140506.1983}{\varepsilon^2} - \frac{9365.91}{\varepsilon} - 687576.214\varepsilon)$$

$$\alpha_y = (1.4195 \times 10^{-6})(47.619\varepsilon + \frac{52826}{\varepsilon^3} + \frac{47209.44}{\varepsilon^2} + \frac{2077.48}{\varepsilon} + 0.125)$$

For recommended range of ε = 0.65 to 0.80, hence the ratios of maximum and minimum values of α_x and α_y are as follows:

$$\frac{\alpha_{x_{max}}}{\alpha_{x_{min}}} = 1.00343, \qquad \frac{\alpha_{y_{max}}}{\alpha_{y_{min}}} = 1.71143 \tag{6}$$

4. Mathematical modelling

The given parameters can be represented by a vector as: $\vec{x} = (L, N, M_x, M_y, R_l)$

And vector of variables is: $\vec{y} = (a, b)$

To determine optimum parameters a_0 and b_0, the Lagrange multipliers method is used. In order to minimize or maximize the function $A = A(a, b)$ at a certain point, it is necessary to satisfy equations:

$$\frac{\partial\varphi}{\partial a} = 0,\ \frac{\partial\varphi}{\partial b} = 0 \tag{7}$$

The Lagrange function can be represented as:[6]

$$\varphi(a, b, \lambda) = A(a, b) + \lambda\varphi(a, b) \tag{8}$$

where λ is the unknown Lagrange's multiplier. So the equation (8) can also be written as:

$$\frac{\partial A}{\partial a} + \lambda\frac{\partial\varphi}{\partial a} = 0, \frac{\partial A}{\partial b} + \lambda\frac{\partial\varphi}{\partial b} = 0 \tag{9}$$

Combining the two equations, the multiplier λ can be eliminated as follows:[7]

$$\frac{\partial A}{\partial a} + \lambda\frac{\partial\varphi}{\partial a} = 0$$

$$or, \qquad \lambda = \frac{-\left(\frac{\partial A}{\partial a}\right)}{\left(\frac{\partial\varphi}{\partial a}\right)}$$

$$\frac{\partial A}{\partial b} + \lambda\frac{\partial\varphi}{\partial b} = 0$$

$$or,\ \left(\frac{\partial A}{\partial b}\right)\left(\frac{\partial\varphi}{\partial a}\right) = \left(\frac{\partial A}{\partial a}\right)\left(\frac{\partial\varphi}{\partial b}\right) \tag{10}$$

5. Optimum parameters

Substituting equation (5) into equation (10) results into:

$$\frac{\partial A}{\partial b}\left[\frac{M_x}{2\alpha_x A\left(b+\frac{a}{2}\right)^2} + \frac{M_y}{\alpha_x A a^2}\right] = \frac{\partial A}{\partial a}\left[\frac{M_x}{\alpha_x A\left(b+\frac{a}{2}\right)^2}\right] \tag{11}$$

After substitution of expression (1) in expression (11), optimum relation of the sides *a* and *b* are as follows:

$$\xi_o = \frac{b_o}{a_o} = \frac{-4k_2 \pm [16{k_2}^2 - 16k_2(2k_3 - 4k_1)]^{1/2}}{8k_2} = 0.53 \tag{12}$$

The values of moment of flexion, axial force and other relevant parameters have been obtained from [8,9]

-$M_x = 550\ [kNm]$, $M_y = \psi M_x$, where $\psi = 0.40 - 0.75$;
-Axial force is N=115 [kN];
-Limiting stress is $R_l = 196\Delta$ [MPa], where coefficients of stress variation is

$$\Delta = 0.75 - 1.25$$

-Bending stress σ_b =115.11 [MPa]

Substituting the above values in (5), we get the optimum length of a, i.e. 'a_0':

$$a_0 = 0.4\ m$$

Substituting this value in ξ_o, we get $b_0 = 0.212\ m$

From the formulae, it is evident that the area of cross section is directly proportional to moment of flexion, i.e. $A_0 = f(\psi)$, for trapezium and box-rectangular cross-sections. The area of cross section is found to be inversely proportional to the limiting stress. The variation has been depicted in the following figures 2 and 3:

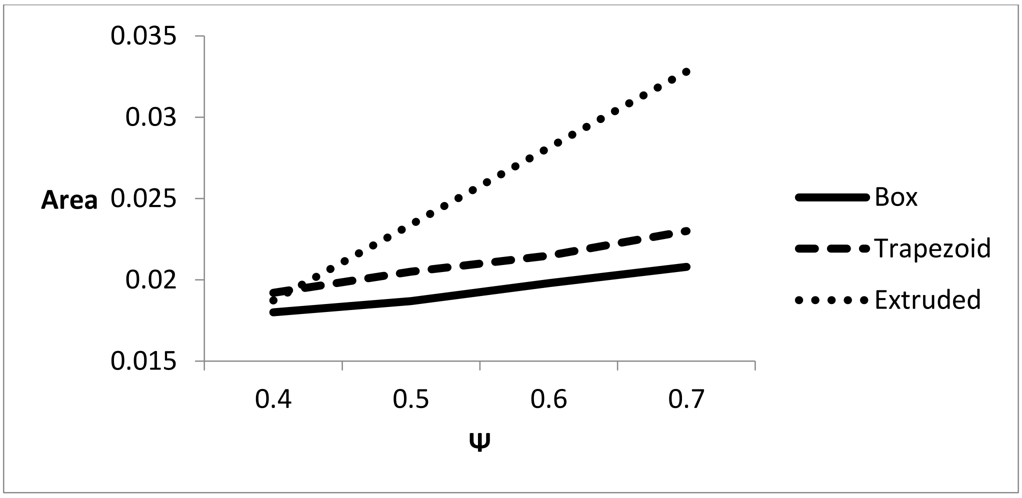

Figure 2. Comparison of Surface Area of Box Rectangular, Trapezoid and Extruded Cross-Sections of Boom with respect to φ .[10]

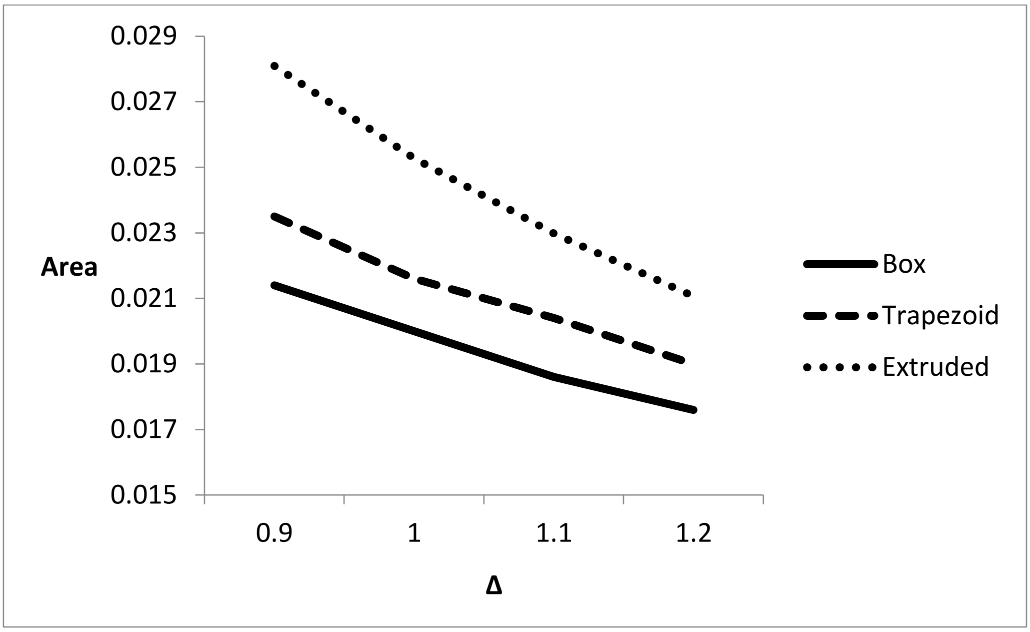

Figure 3. Comparison of Surface Area of Box Rectangular, Trapezoid and Extruded Cross-Sections of Boom with respect to Δ.[10]

6. Conclusion

In this paper, the optimum dimensions of a hollow extruded cross section for constructing the boom of a mobile crane are defined. Simple formulae are derived in the analytical form, which can be put to practical use when designing a crane. Comparison between the box section, trapezoidal section and the extruded section has been performed from the point of view of minimizing area. By plotting the Area vs Moment of flexion curves for all three cross sections, it is seen that the extruded section is more economical than trapezoidal section for low values of moment of flexion. However, for greater values of moment of flexion, the extruded section proves to be the least economical.

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