

PAPER • OPEN ACCESS

Computer Aided Design of Civil Structures – Topology Optimization in Statics and Dynamics

To cite this article: R Peter 2018 *IOP Conf. Ser.: Mater. Sci. Eng.* **407** 012171

View the [article online](#) for updates and enhancements.

You may also like

- [Development of distributed compliant mechanism for two distinct applications: displacement amplifying and object holding mechanism](#)
Sachin R Kandharkar, Sujit S Pardeshi, Rohan Soman et al.
- [Topology optimization of structures with stress constraints: Aeronautical applications](#)
J Paris, S Martínez, F Navarrina et al.
- [Topology optimization of magnetorheological fluid layers in sandwich plates for semi-active vibration control](#)
Xiaopeng Zhang and Zhan Kang



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

Computer Aided Design of Civil Structures – Topology Optimization in Statics and Dynamics

R Peter

Vienna University of Technology, Karlsplatz 13/E2063, A-1040 Vienna, Austria

*peter.rosko@tuwien.ac.at

Abstract. The objective of the paper is structural topology optimization in statics and dynamics. The topology is defined on the base of the material distribution in elements of the structure. The optimal topology design is defined as an optimization problem. The objective is the maximum stiffness of the structure in statics and maximal natural frequency in dynamics of the structure. The constraints of the optimization problem are related to the mass of the structure and to the loading and boundary conditions of the structure. The evolutionary method - genetic algorithm is applied for optimization problem solution. Examples on optimal topology of structures in statics and dynamics are presented. The obtained results are comparable with results based on experience of civil engineers. The structural topology optimization does not use experience from structural evolution and it is applicable for design of new structures – prototypes.

1. Introduction

The aim of engineer design is a structure which optimally satisfies all technical, economical and manufacturing requirements. Topology optimization of structures is a relatively new field of research. The concept for topology optimization was proposed by M.P. Bendsøe and N. Kikuchi in 1988 [1]. The interest of scientific community was great from the beginning and persists to this day. It is surprising when the computer - a machine, can do the same design of structure which resembles the design of engineers after centuries of experience. By new types of structures with special loading cases often absents the experience of engineers. This is the reason to use the topology optimization of the structure. Intensive research in the last decades resulted effectively optimized topologies of structures applied in practice.

Most of the problems of the optimal topology were solved in linear statics. Scientific discussions on structural optimization and requirements formulated by the practice are motivation to invest research work into the field of optimal topology of civil structures.

2. Methods

The structural optimization can be distinguished into three levels: dimension optimization, shape optimization and topology optimization (includes material optimization). The order of levels reflects the historical development. The reverse order corresponds with the sequence applied in practice-oriented design.



2.1. Optimization procedures

The optimal topology design is defined as an optimization problem. The optimization methods serve as a tool to minimize or maximize the objective function(s) satisfying the prescribed constraints. The optimization is generally an iterative process. The appropriate optimization method depends on mathematical properties of the objective and constraint functions. Generally, two groups of methods are available:

The first group of methods is the group of gradient based methods. Very intensive research of mathematical programming started in sixties. The researchers often use the Svanberg's Method of Moving Asymptotes (MMA) from [2] very successfully.

The second group contains evolutionary algorithms (e.g. GA - genetic algorithm [3]), does not need any special properties of objective functions. This is the advantage. The disadvantage is that optimization process is more expensive. The powerful hardware enables successfully to use evolutionary procedures in the last years.

2.2. Dimension optimization

Designers commonly do the dimension optimization of structures in statics step by step. The dimension optimization can be automated using mathematical programming methods and evolutionary procedures.

2.3. Shape optimization

The expansion of the shape optimization occurred when the discretization methods in mechanics reached a sufficient level of development. The shape optimization is the most frequently applied on the finite element model of the structure.

2.4. Topology optimization

Last decades the optimal topology replaces the experience gained during many years. Nowadays is the topology optimization a modern tool for design of new types of structures.

Topology does not have a long history. Key works in the short history of the structural topology optimization have been done by M.P. Bendsøe and N. Kikuchi [1], G.I.N. Rozvany [4] and O. Sigmund [5]. Although the excellent work of other scientists is acknowledged. The best overview of the topology optimization can be found in the papers of H.A. Eschenauer and N. Olhoff [6], G.I.N. Rozvany [7], M.P. Bendsøe, E. Lund, N. Olhoff and O. Sigmund [8] and in the books from M.P. Bendsøe [9], M.P. Bendsøe and O. Sigmund [10]. The first applications of the optimal topology of structures were realized in statics. The model of the structure was established on the base of the finite element method. Recently the topology optimization has been extended in probability design, dynamics and other fields of physics [11].

3. Results and Discussion

The topology is defined on the base of the material distribution in elements of the structure usually with help of material density ρ . An important issue in the model problem is the relation between density and stiffness. In the so-called SIMP-model [8] is the stiffness proportional to density in the power p where $p > 1$.

3.1. Mathematical problem definition

Typical topology problem definition in statics is as follows:

Optimization problem formulation for maximum stiffness (= min compliance):

$$\min_{\rho} C(\rho) \quad (1)$$

$$\sum_{e=1}^N v_e \rho_e \leq V, \quad 0 < \rho_{\min} \leq \rho_e \leq 1, \quad e = 1, \dots, N \quad (2)$$

$$C(\rho) = f^T u \quad (3)$$

$$K(\rho)u = f \quad (4)$$

$$K(\rho) = \sum_{e=1}^N \rho_e^p K_e \quad (5)$$

with

ρ density, V volume, K stiffness matrix of the structure, K_e global stiffness matrix of element e , u displacements, f loading-forces, with stochastic loading $f_j = f_j(d_j)$ where d_j Gauss deviation is.

The main task in dynamics of structures is to leave the resonance interval. This problem occurs in design of structures by earthquake excitation, wind excitation and machine excitation. This leads to optimization problem formulation for maximum natural frequency:

$$\max_{\rho} \{ \omega_1 = \omega_{\min} \} \quad (6)$$

$$(K(\rho) - \omega_i^2 M) \Phi_i = 0, \quad i = 1, \dots, N \quad \text{where } K(\rho) = \sum_{e=1}^N \rho_e^p K_e \quad (7)$$

$$\sum_{e=1}^N v_e \rho_e \leq V, \quad 0 < \rho_{\min} \leq \rho_e \leq 1, \quad e = 1, \dots, N \quad (8)$$

with

ω_i natural frequency, Φ_i i modes

The topology optimization problem of minimizing the dynamic compliance subject to periodic forces is defined as follows:

$$\min_{\rho} C(\rho) = (f^T u)^2 \quad (9)$$

$$(K(\rho) - \Omega_j^2 M)u = f_j \quad \text{where } K(\rho) = \sum_{e=1}^N \rho_e^p K_e \quad (10)$$

$$\sum_{e=1}^N v_e \rho_e \leq V, \quad 0 < \rho_{\min} \leq \rho_e \leq 1, \quad e = 1, \dots, N \quad (11)$$

The force $f_j = f_j(\Omega_j, \bar{m}_j, d_j)$ is weighted function of frequency Ω_j , \bar{m}_j and Gauss deviation.

The probability based constraint in this case is included in the problem definition.

3.2. Methods of solution

The structures are modelled on the base of finite element method. The finite element mesh is regular and fine. Density of material is defined as variable for each finite element. The initial area has simple geometry and allows wide variability of solutions. The mass is in the initial area constantly distributed (constant density). The genetic algorithm [3] is used for the optimization problem (see chapter 3.1.).

3.3. Examples-Results

3.3.1. Structural optimization in statics: Bridge example

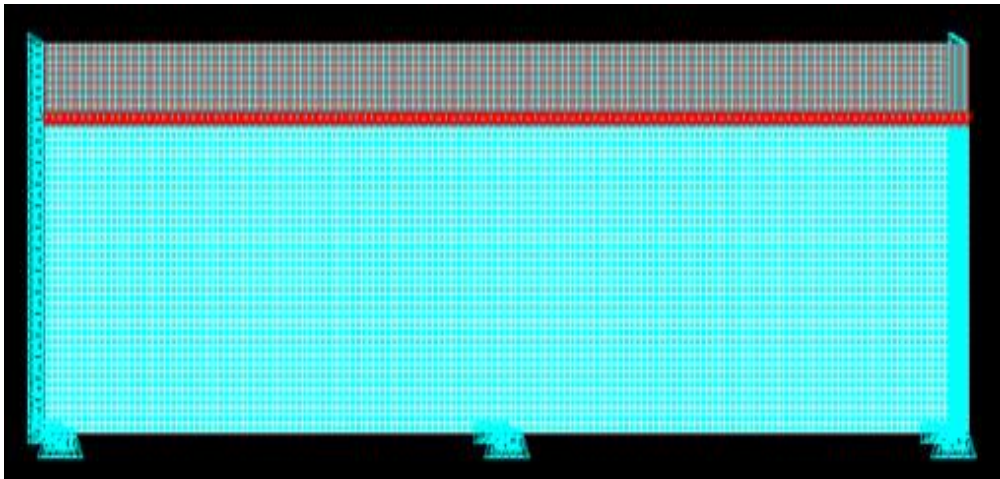


Figure 1. Initial structure: down: 3 supports, top: uniformly distributed load.

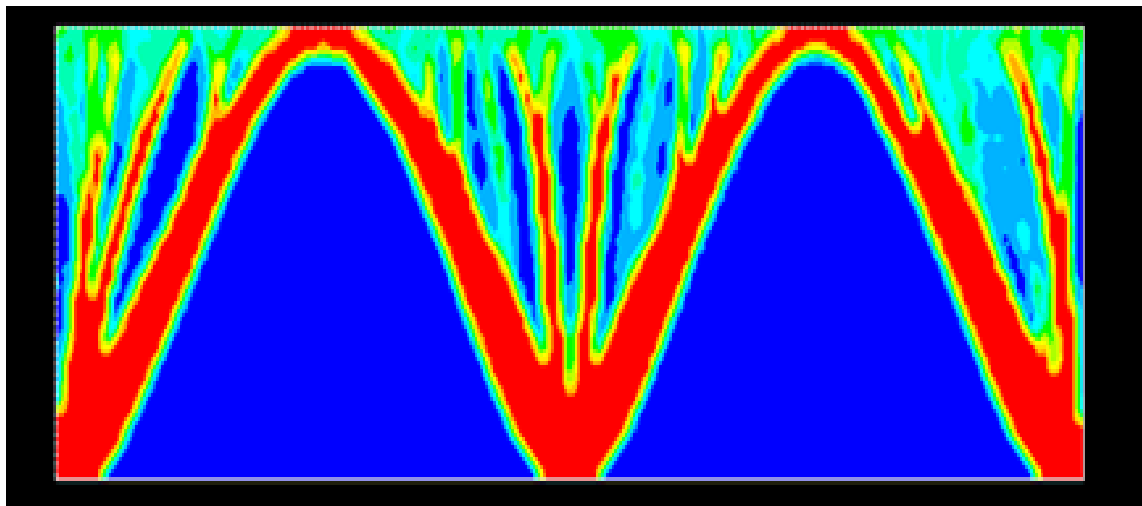


Figure 2. Result: Optimal density distribution – max. stiffness.

The initial plane structure is supported by three supports at the bottom. The loading on the top is uniformly distributed – see figure 1. The optimization problem formulation is given in (1) - (5). The resulting image shows the mass distribution for the maximum stiffness – see figure 2.

3.3.2. Structural optimization in statics: Cantilever example

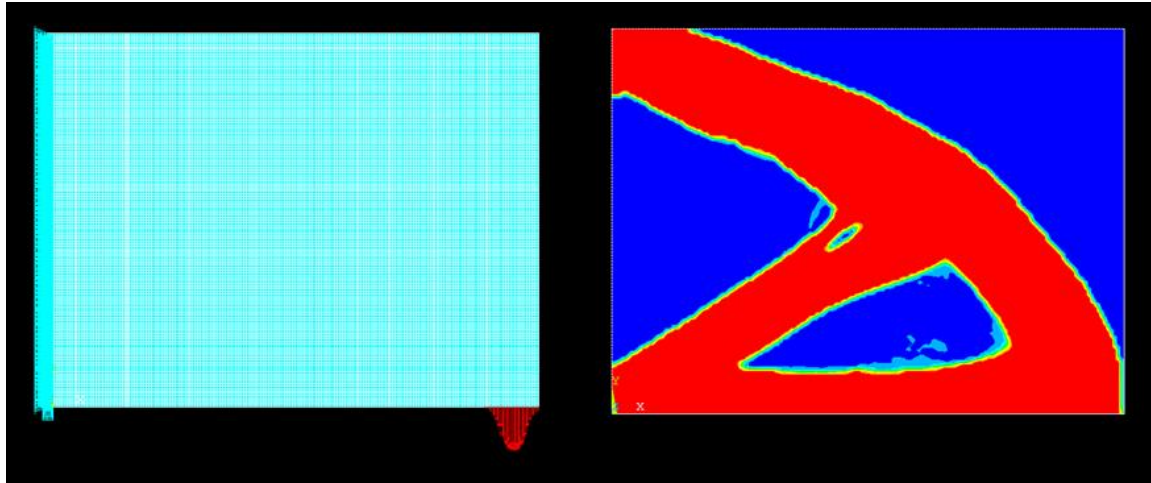


Figure 3. Initial structure: left supported down: vertical load distribution.

Figure 4. Result: Optimal density distribution-max. stiffness

The initial plane structure is supported on the left side. The Gauss-distributed load is located at the bottom on the right side – see figure 3. The optimization problem formulation is given in (1) - (5). The resulting image shows the mass distribution for the maximum stiffness – see Figure 4.

3.3.3. Structural optimization in dynamics: Plate example in dynamics.

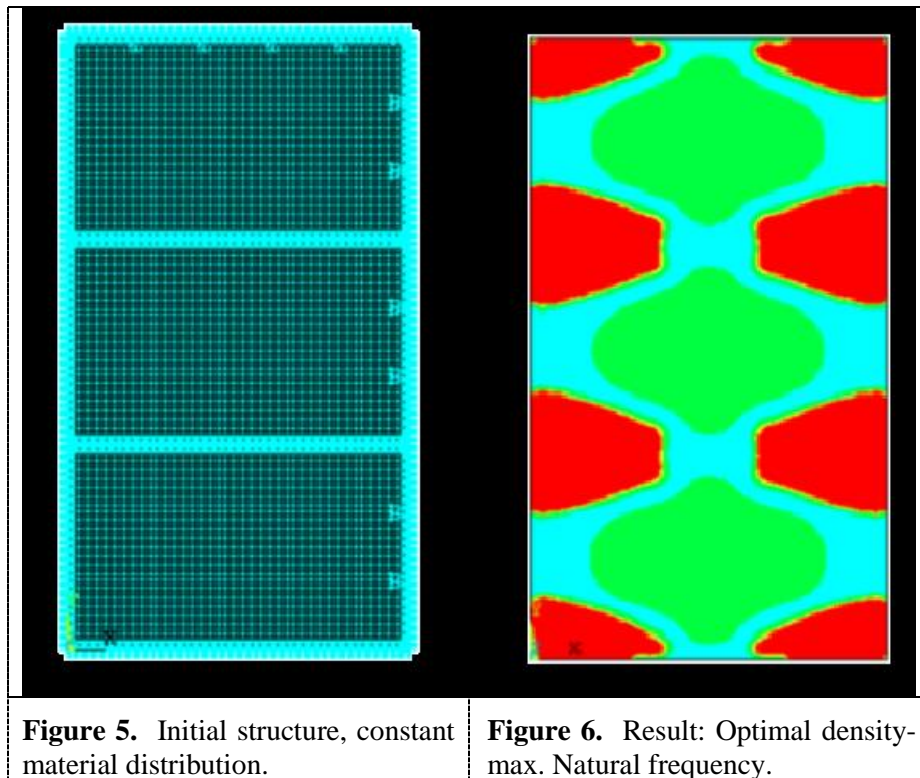


Figure 5. Initial structure, constant material distribution.

Figure 6. Result: Optimal density-max. Natural frequency.

The plate shown in plan view with line supports is loaded by vertical uniformly distributed dead load – see figure 5. The optimization problem formulation is given in (6) - (8). The result of the optimization procedure: optimal mass distribution for maximum natural frequency is shown in Figure 6.

All results are compatible with previous research and agree with results published in [8], [9] and [10].

4. Conclusions

The paper provides the overview in the research area and presents examples on optimal topology of structures in statics and dynamics. The topology design of structures is defined as mathematic problem. The problem definitions are stated. Results of the examples represent the solution of material distribution of optimal designed structures. The results of simple examples are comparable with results of designer's experiences. The structural topology optimization does not use experience from structural evolution and it is applicable for design of new structures – prototypes.

References

- [1] Bendsøe M P and Kikuchi N 1988 Generating optimal topologies in structural design using a homogenization method *Computer Methods in Applied Mechanics and Engineering* **71** (2) pp 197-224
- [2] Svanberg K 1987 The method of moving asymptotes – A new method for structural optimization *International Journal for numerical Methods in Engineering* **24** pp 359-373
- [3] Goldberg D E 1989 Genetic algorithms in search, optimization and machine learning, (New York: Addison Wesley Publishing Company)
- [4] Rozvany G I N, Zhou M and Birker T 1992 Generalized shape optimization without homogenization *Structural optimization* **4** pp 250-252
- [5] Sigmund O 1994 Design of material structures using topology optimization PhD thesis (Lyngby: Department of Solid Mechanics Technical University of Denmark), *DCAMM Special Report* No. S69
- [6] Eschenauer H A and Olhoff N 2001 Topology optimization of continuum structures: A review *Appl Mech Rev* **54** (4) pp 331-390
- [7] Rozvany G I N 2001 Aims, scope, methods, history and unified terminology of computer-aided topology optimization in structural mechanics, *Structural and Multidisciplinary Optimization* **21** pp 90-108
- [8] Bendsøe M P, Lund E, Olhoff N and Sigmund O 2005 Topology optimization – broadening the areas of application *Control and Cybernetics* **34** (1) pp 7-35
- [9] Bendsøe M P and Sigmund O 2004 *Topology optimization, Theory, Methods and Applications*, (Berlin Heidelberg: Springer Verlag)
- [10] Bendsøe M P 1995 *Optimization of Structural Topology, Shape and Material*, (Berlin Heidelberg: Springer Verlag)
- [11] Maute K and Fragopol D M 2003 Reliability-based design of MEMS mechanism by topology optimization *Computers&Structures* **81** pp 813-824