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Mathematical modeling of the building thermal state taking into account the heat and energy impact of the environment

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Abstract. There are some difficulties in analysing the actual thermal state of a building under changing conditions of the environment, particularly solar insolation. The methodological approach to overcoming these difficulties and the corresponding mathematical models of the thermal state of a building are proposed. The base of the models is a power circuit in which there are described all components of heat losses and heat revenues. Controlled and uncontrolled heating systems are considered by representing heating devices as a source of effort or as a source of flow. The proposed models as a result of the analysis of the thermal state allow obtaining energy indicators of the building and an objective picture of the thermal comfort in the rooms simultaneously.

1. Introduction

Nowadays there are two problems in the focus of dwelling building engineers, the first problem is a microclimate quality improvement and the second one is an energy efficient increasing. Since the improving of microclimate causes the increasing of energy consumption, these two problems are combined in essence. The searching for these problems' solutions has led to a new conception of ecological building [1] which purpose is to provide high air quality at minimum energy consumption. There are different approaches and software for resolving the particular problem, for example, one of which is specialized program EnergyPlus [2]. The approach considered in the article is common for different kinds of energy circuits and is a prerequisite for the establishment of methodological bases for analyzing the state of the building and its energy supply systems.

According to the system approach the mathematical model of a building thermal state as a single heat energy system consists of three interrelated submodels [3]:

- the mathematical submodel of heat transfer through an enclosure of a building,
- the mathematical submodel of heat accumulation characteristics of a building enclosure and radiation and convective heat exchanging in building rooms,
- the mathematical submodel of heat and energy impact of the environment on a building.

Such approach allows us to divide the complex task of thermal steady-state analyzing into simpler parts and to introduce them into a general model as needed. There are several approaches to build the mathematical submodel of heat transfer. For analyzing of steady-state it is worthwhile to use the submodel of heat transfer in the form of an array of two-terminal resistive components that allows us to represent thermal characteristics of enclosure constructions with varying degrees of detail [4]. The next step is formation an equivalent circuit (a power circuit) of the thermal state in a building which allows using the network approach of the theory of power circuits for further analysis.

According to the theory of power circuits the power circuit can be described by following system of equations [4, 5]

$$\begin{cases} \mathbf{\Pi} \boldsymbol{X} = -\boldsymbol{F}, \\ \mathbf{\Gamma} \boldsymbol{Y} = 0, \\ \boldsymbol{Y} = \boldsymbol{E} - \mathbf{R}\boldsymbol{X}, \end{cases} \tag{1}$$

where $\mathbf{\Pi}, \mathbf{\Gamma}$ are incidence matrix and circle matrix, $\boldsymbol{X}, \boldsymbol{Y}$ are column vectors of flows (serial variables) and efforts (parallel variables) of brunches, $\boldsymbol{E}, \boldsymbol{F}$ are column vectors of effort sources and flow sources and $\mathbf{R}$ is resistance matrix of resistive components.

Such mathematical model of the thermal state in a building allows us to analyze a thermal steady-state for the buildings in which there are heated rooms, unheated premises and spaces with forced cooling. This corresponds to the requirements of the current normative documents in particular [6] and [7], which require the division of a building into separate zones and the calculation energy demands for heating and cooling of each zone.

The mathematical submodel of heat transfer should be complemented with the mathematical submodel of heat and energy impact of the environment and it is the purpose of the article. The mathematical submodel of heat accumulation characteristics of a building has an important role in transient processes analyzing however it is possible to omit it in steady-state analyses.

2. The mathematical model of the thermal state in a building

Applying of the proposed model will be shown on an example of a representative building. The building consists of three floors (height of each floor is 3 m) on each floor there are four flats. There is a basement in the building.

Plan of a typical floor (except a ground floor) and main dimensions are shown in figure 1.

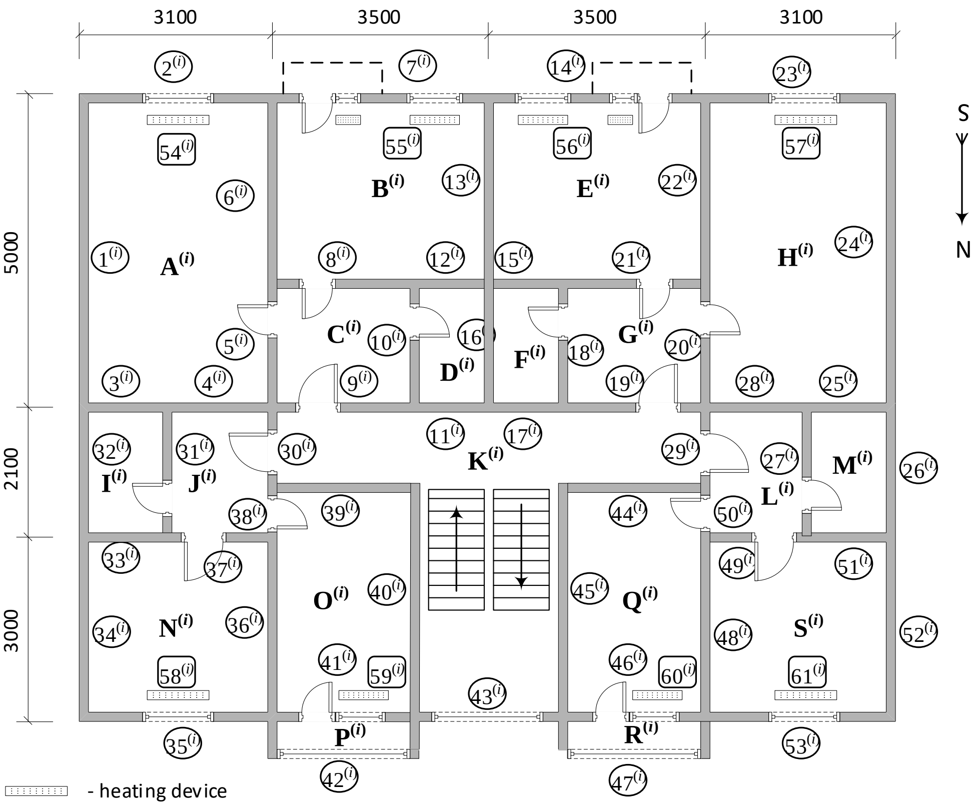

Figure 1. The plan of a typical floor of the building.

The numbers in circles indicate the main enclose constructions and the numbers in rounded rectangles indicate the heating devices. Bold capital letters indicate rooms' names of the building. Index in all cases indicates the number of a floor. The orientation of the building relative to the world is also shown in figure 1.

The analysis of a thermal state in the building is performed from simpler case to more complicated one. At first, an only mathematical model of heat transfer including the heating system is considered. It is necessary for the reason that such kind of model is the basis for determining rated (peak) mode of a heating system.

2.1. The mathematical model of a thermal state in the building that does not take into account heat and energy impact of the environment

According to the [4] each element of an enclose construction should be represented by at least one separate two-terminal component in the submodel of heat transfer. In order to simplify the presentation of the material the whole possible array of resistive components for each room is simplified to a seven-terminal network (figure 2(b)). First six equivalent resistive components $R_{e.\mathrm{s1}}$, $R_{e.\mathrm{s2}}$, $R_{e.\mathrm{s3}}$, $R_{e.\mathrm{s4}}$, $R_{e.\mathrm{c}}$, $R_{e.\mathrm{f}}$ reproduce equivalent thermal characteristics of main enclose constructions (walls S1, S2, S3, S4, celling C and floor F in figure 2(a)) and seventh equivalent resistive component $R_{e.\mathrm{i}}$ reproduces heat losses through air infiltration (ventilation).

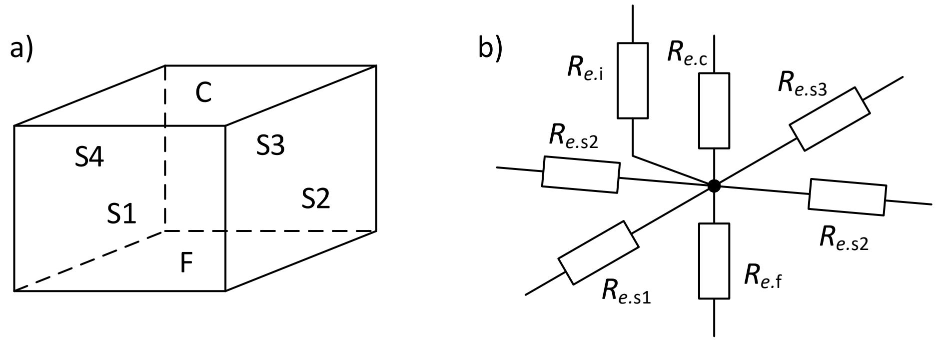

Figure 2. A separate room of the building (a) and its equivalent circuit (power circuit) as a part of an energy circuit of a building (b).

There are two possible systems of flow and effort dimensions [4]. The approach in which flow X is in W and effort Y is in K is accepted. Therefore resistive components dimension is K/W and their values should be defined as

$$R = R_{\mathrm{ht}} / S, \qquad (2)$$

where R_{ht} is heat transfer resistance in $\mathrm{m^2 \cdot K/W}$ and S is square in $\mathrm{m^2}$. There are such values of R_{ht}: outside wall – 2.4 $\mathrm{m^2 \cdot K/W}$, side wall – 3.2 $\mathrm{m^2 \cdot K/W}$, interior flat room – 0.6 $\mathrm{m^2 \cdot K/W}$, interior wall – 0.2 $\mathrm{m^2 \cdot K/W}$, celling – 0.76 $\mathrm{m^2 \cdot K/W}$, roof – 3.2 $\mathrm{m^2 \cdot K/W}$, floor – 2.8 $\mathrm{m^2 \cdot K/W}$, window – 0.82 $\mathrm{m^2 \cdot K/W}$, flat door – 0.6 $\mathrm{m^2 \cdot K/W}$, interior door – 0.24 $\mathrm{m^2 \cdot K/W}$, balcony door – 0.94 $\mathrm{m^2 \cdot K/W}$.

In order to reproduce the thermal state of a building in the model, should take into account the heat introduction by heating devices.

A building in relation to the thermal state is considered as an uninsulated system that is separated from the universe by an imaginary boundary. The part of the universe that we are interested in is usually called "the system" and everything else is called "the surroundings". The first law of thermodynamics for the system can be expressed as the sum of the energy flows across its boundary is equal to the change in energy content of the system ΔE, i.e.

$$\Delta E = E_{\mathrm{in}} - E_{\mathrm{out}}, \qquad (3)$$

where E_{in} and E_{out} are the energy flows in and out respectively.

Energy can flow across the boundary as work done either to the system or by the system, as heat transfer either in or out of the system or it can be carried in a flow of matter across the boundary. Hence,

$$\Delta E = (W_{\text{in}} - W_{\text{out}}) + (Q_{\text{in}} - Q_{\text{out}}) + (E_{\text{mass.in}} - E_{\text{mass.out}}), \tag{4}$$

where W, Q and E_{mass} are the energy carried by work, heat transfer and matter moving in and out of the system, as indicated by the subscripts.

Regardless of how heat energy is introduced into rooms, all sources of the power circuit are divided on two kinds. If a source provides a constant temperature in the room it should be represented as a source of effort E and if a source provides a constant amount of thermal energy regardless of the room temperature it should be represented as a source of flow F. In general, an introduction of heat energy into building rooms is shown in figure 3.

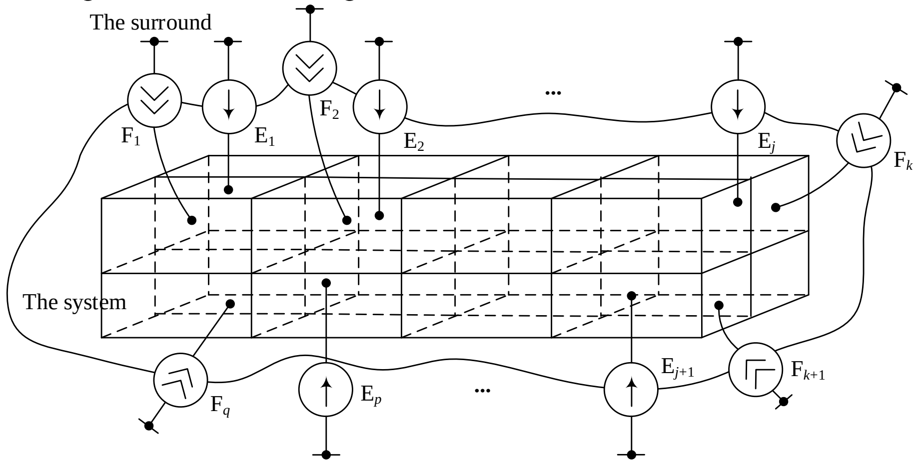

Figure 3. Introduction of heat energy into building rooms.

In a power circuit each source of effort is represented by a separate branch. The part of the power circuit of the building for the southern facade of a typical floor is shown in figure 4.

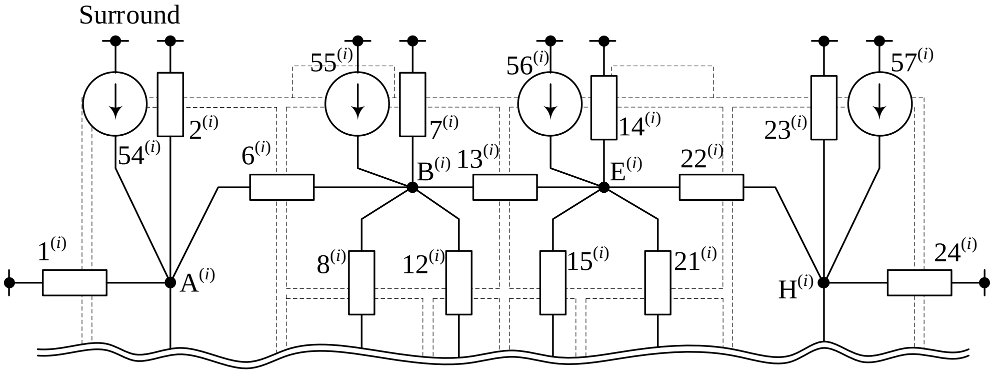

Figure 4. The part of the power circuit of the building for the southern facade of a typical floor.

Each room in the power circuit corresponds to the separate node (the nodes' names in figure 4 correspond to the rooms' names in figure 1). The numbering of resistive components (passive branches) in figure 4 corresponds to the numbering of enclosure constructions in figure 1 and the numbering of sources of flow corresponds to the numbering of heating devices.

The next stage is to construct a directed graph (digraph) for the power circuit and to choose a graph tree. In the case of a linear mathematical model of heat transfer (linear power circuit), there are no additional recommendations how to form digraph tree. In the case of a nonlinear mathematical model of heat transfer (nonlinear power circuit), the graph tree should be formed from edges whose resistance is the largest. This provides the highest convergence rate of numerical methods. Thus it is proposed to introduce edges into a graph tree in such sequence:

- the edges that correspond to outside enclosure constructions,
- the edges that correspond to interior flat constructions,
- the edges that correspond to interior constructions.

In figure 5 it is shown a directed graph for a part of the power circuit of the building that corresponds to a typical floor. The edges of the digraph tree are highlighted by thick lines.

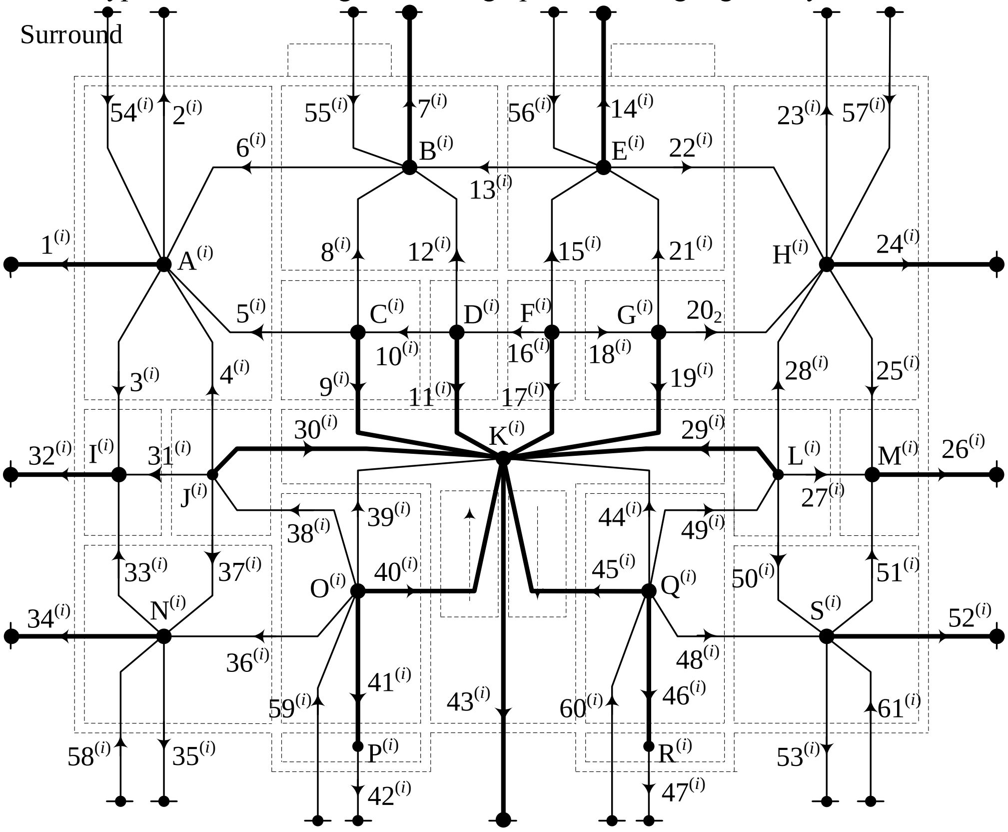

Figure 5. The directed graph of a typical floor of the building.

The digraph of the whole building is non-planar and consists of the digraphs of all floors that are connected with each other by edges which correspond to the inter-floor ceilings. These edges should be introduced into the set of graph chords.

The calculation of the mathematical model obtained in a form of equations set (1) could be performed by any of known formalized methods. The mesh-flows method (analog of the mesh-current method in the theory of electrical circuits) was applied in the work. As a result, there are obtained the main parameters of thermal steady-state such as temperatures and heat flows for separate rooms.

The calculated parameters of thermal state for rooms of the second and third (upper) floors are shown in table 1. The values of calculated temperatures (T) are given for all rooms and the calculated heat removal from heating devices (Q) are given for heated rooms. The calculations are performed at an ambient temperature –1.5ºC that corresponds to the average temperature for the day part of the representative day in January for a latitude of Lviv [7]. The obtained results give a detailed picture of the thermal state and they are the basis for constructing an energy balance of a building according to current normative documents of Ukraine [6, 7] and international documents [8].

In addition, it is possible to obtain temperatures of internal and external surfaces for enclosure constructions. For example, for the room A (figure 1) there are such temperatures of inner surfaces for the walls: wall 1 – 20.1ºC, wall 2 – 19.8ºC, wall 3 – 20.8ºC, wall 4 – 20.9ºC, wall 5 – 20.9ºC and wall 6 – 21ºC. It allows us to better estimate the thermal comfort in rooms of the building.

Table 1. The calculated parameters of thermal state for rooms of the second and third (upper) floors of the building (not taking into account the heat and energy impact of the environment).

Room	$A^{(2)}/A^{(3)}$ Heated	$B^{(2)}/B^{(3)}$ heated	$C^{(2)}/C^{(3)}$ unheated	$D^{(2)}/D^{(3)}$ unheated	$E^{(2)}/E^{(3)}$ heated	$F^{(2)}/F^{(3)}$ unheated	$G^{(2)}/G^{(3)}$ unheated	$H^{(2)}/H^{(3)}$ heated	$I^{(2)}/I^{(3)}$ unheated	$J^{(2)}/J^{(3)}$ unheated
T, °C	21/21	21/21	20.6/18.9	20.4/18.1	21/21	20.4/18.1	20.6/18.9	21/21	17.8/16.2	19.8/18.6
Q, W	281/399	337/427	–	–	337/427	–	–	281/399	–	–
Room	$K^{(2)}$ unheated	$L^{(2)}/L^{(3)}$ unheated	$M^{(2)}/M^{(3)}$ unheated	$N^{(2)}/N^{(3)}$ heated	$O^{(2)}/O^{(3)}$ heated	$P^{(2)}/P^{(3)}$ unheated	$Q^{(2)}/Q^{(3)}$ heated	$R^{(2)}/R^{(3)}$ unheated	$S^{(2)}/S^{(3)}$ heated	Surround
T, °C	18.4	19.8/18.6	17.8/16.2	21/21	21/21	9.5/8.9	21/21	9.5/8.9	21/21	–1.5
Q, W	–	–	–	245/318	139/230	–	139/230	–	245/318	9146[a]

[a] the heat extraction from the heating system of the whole building.

Further development of the mathematical model of a thermal steady-state is to complement it by a mathematical submodel of heat and energy impact of the environment. There some features for cases of controlled and uncontrolled heating systems.

2.2. The mathematical model of a thermal state in the building that take into account heat and energy impact of the environment (for the case of a controlled heating system)

The most important component of heat end energy impact of the environment is a solar insolation which brings a certain amount of heat into rooms through the translucent elements of enclosure and heats the external surfaces of opaque enclosure (figure 6(a)).

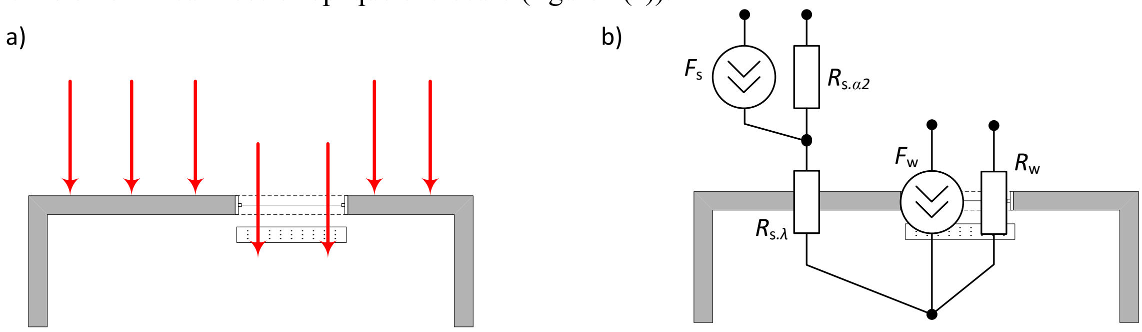

Figure 6. Heat and energy impact of the environment (a) and its power circuit (b).

The equivalent resistance of main enclosure construction $R_{e.s}$ (figure 2(b)) has to be divided on a resistance of translucent elements (R_s) and a resistance of opaque elements (R_w). Moreover, the heat transfer resistance for opaque constructions of an enclosure has to be divided into two parts $R_{s.\lambda}$ (includes the resistance of heat transfer between the wall surface and the air inside and the resistance of thermal conductivity) and $R_{s.\alpha 2}$ (the resistance of heat transfer between the wall surface and the air outside) (figure 6(b)). The source of effort E_h corresponds to the heating device and the sources of flow F_s and F_w correspond to a solar insolation.

The dimension of flow X corresponds to the power hence flow of a source of flow F_w which brings a certain amount of heat into rooms should be defined as

$$F_w = k_{sh} I_{sol} g_{gl} S_w, \tag{5}$$

where k_{sh} is shadow coefficient, I_{sol} is solar insolation per unit area (43 W in January for a southern facade for a latitude of L'viv and 30 W for a horizontal surface [7]), g_{gl} is transmission coefficient of solar insolation through translucent elements of the enclosure (0.75 for triple-glazed window [7]) and S_w is square of translucent elements.

Flow of a source flow F_s which reproduces the heating of the external surface of opaque enclosure should be defined as

$$F_s = k_{sh} I_{sol} S_s, \tag{6}$$

where S_s is square of opaque elements.

The part of the power circuit of the building including heat and energy impact of the environment for the southern facade of a typical floor is shown in figure 7.

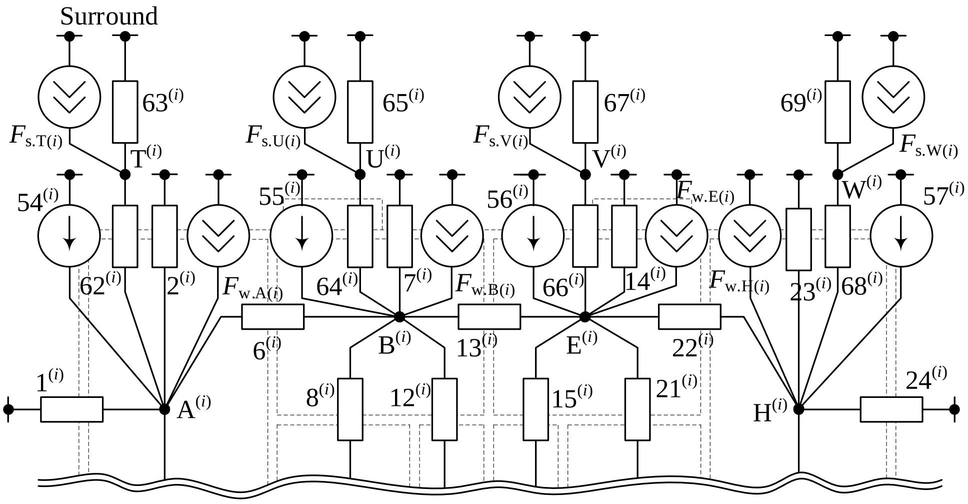

Figure 7. The part of the power circuit of the building including heat and energy impact of the environment for the southern facade of a typical floor.

Following given rules, the power circuit and the mathematical model of a thermal steady-state for the whole building is formed. The calculated parameters of thermal state for rooms of the second and third (upper) floors are shown in table 2.

Table 2. The calculated parameters of thermal state for rooms of the second and third (upper) floors of the building (for the case of a controlled heating system).

Room	$A^{(2)}/A^{(3)}$ heated	$B^{(2)}/B^{(3)}$ heated	$C^{(2)}/C^{(3)}$ unheated	$D^{(2)}/D^{(3)}$ unheated	$E^{(2)}/E^{(3)}$ heated	$F^{(2)}/F^{(3)}$ unheated	$G^{(2)}/G^{(3)}$ unheated	$H^{(2)}/H^{(3)}$ Heated	$I^{(2)}/I^{(3)}$ unheated	$J^{(2)}/J^{(3)}$ unheated
T, ºC	21/21	21/21	20.6/19.1	20.4/18.3	21/21	20.4/18.3	20.6/19.1	21/21	17.8/16.4	19.8/18.8
Q, W	238/343	290/370	–	–	290/370	–	–	238/343	–	–
Room	$K^{(2)}$ unheated	$L^{(2)}/L^{(3)}$ unheated	$M^{(2)}/M^{(3)}$ unheated	$N^{(2)}/N^{(3)}$ heated	$O^{(2)}/O^{(3)}$ heated	$P^{(2)}/P^{(3)}$ unheated	$Q^{(2)}/Q^{(3)}$ heated	$R^{(2)}/R^{(3)}$ unheated	$S^{(2)}/S^{(3)}$ heated	Surround
T, ºC	18.4	19.8/18.8	17.8/16.4	21/21	21/21	9.4/9.1	21/21	9.4/9.1	21/21	–1.5
Q, W	–	–	–	245/310	139/219	–	139/219	–	245/310	8172

For a controlled heating system the temperatures in the heated rooms are the same as in table 1 but heat removal from heating devices has decreased. Decreasing of heat consumption for the whole building equals 10.6%.

2.3. The mathematical model of a thermal state in the building that take into account heat and energy impact of the environment (for the case of an uncontrolled heating system)

In the case of an uncontrolled heating system the heat removal from the heating devices does not change (or these changes could be neglected). To reproduce this in the mathematical model of a thermal state in the building, the heating devices should be represented by the sources of flow instead of the sources of effort. It means that the sources of effort $54^{(i)}$, $55^{(i)}$, $56^{(i)}$ i $57^{(i)}$ in figure 7 should be replaced by the sources of flow whose flows are equals to powers of heating devices in table 1.

Following given rules, the power circuit and the mathematical model of a thermal state for the whole building are formed. The calculated parameters of thermal state for rooms of the second and third (upper) floors are shown in table 3.

Table 3. The calculated parameters of thermal state for rooms of the second and third (upper) floors of the building (for the case of an uncontrolled heating system).

Room	$A^{(2)}/A^{(3)}$ heated	$B^{(2)}/B^{(3)}$ heated	$C^{(2)}/C^{(3)}$ unheated	$D^{(2)}/D^{(3)}$ unheated	$E^{(2)}/E^{(3)}$ heated	$F^{(2)}/F^{(3)}$ unheated	$G^{(2)}/G^{(3)}$ unheated	$H^{(2)}/H^{(3)}$ Heated	$I^{(2)}/I^{(3)}$ unheated	$J^{(2)}/J^{(3)}$ unheated
T, ºC	25.2/24.8	25.8/25.0	24.6/24.0	24.4/23.8	25.8/25.0	24.4/23.8	24.6/24.0	25.2/24.8	19.4/19.8	20.4/20.8
Q, W	281/399	337/427	–	–	337/427	–	–	281/399	–	–
Room	$K^{(2)}$ unheated	$L^{(2)}/L^{(3)}$ unheated	$M^{(2)}/M^{(3)}$ unheated	$N^{(2)}/N^{(3)}$ heated	$O^{(2)}/O^{(3)}$ heated	$P^{(2)}/P^{(3)}$ unheated	$Q^{(2)}/Q^{(3)}$ heated	$R^{(2)}/R^{(3)}$ unheated	$S^{(2)}/S^{(3)}$ heated	Surround
T, ºC	21.1/21.6	20.4/20.9	19.4/19.9	21.1/21.5	21.2/21.7	9.6/10.4	21.2/21.7	9.6/10.4	21.1/21.5	–1.5
Q, W	–	–	–	245/318	139/230	–	139/230	–	245/318	9146

In the case of an uncontrolled heating system, the heat removal from the heating system does not change, but the environmental impact is manifested in changing the temperatures in the rooms.

3. Conclusions

The mathematical models of a thermal state in a building both without and with taking into account heat and energy impact of the environment are proposed in the paper. The first model is relevant for the analysis of the rated (peak) mode of a heating system and the second model is relevant for the analysis of building thermal states in different conditions.

The proposed models as a result of the thermal steady-state analysis allow obtaining energy indicators of the building and an objective picture of the thermal comfort in the rooms simultaneously.

The addition of a mathematical model of thermal state with a mathematical submodel of heat and energy impact of the environment requires the changes in the mathematical submodel of the heat transfer of the building. The solar insolation could be adequately reproduced by the sources of flow in the power circuit.

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