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Research on Establishment of Two Degree-of-freedom Head Tracking Model Based on Single Image Sensor

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Abstract. This paper starts with the structural relationship of the head tracking system based on single imaging device, establishes equation set of projection relation, the rationality of the system is determined by analyzing the possibility of the equation solution, projection relationship is established, finally, the mathematical model of the two degree-of-freedom tracking system based on single image sensor is completed from theory.

1. Introduction

The so-called head tracking technology is a technology that can realize real-time measurement of human head motion parameters, the corresponding product is called head position tracker[1]. In military applications, many advanced fighters and helicopters are equipped with helmet sighting device, its role is to use the pilot's head rotation indicate the orientation of the target and achieve rapid aiming, the technology for measuring head parameters is the head tracking technology [2]. The head tracking technology is widely used in the virtual reality field in civil use, and generally requires measurement of six degree-of-freedom parameter. However, for military applications, as long as the parameters of the azimuth and elevation two degree-of-freedom are provided, these parameters are enough to meet the demand, this paper establishes a two-degree-of-freedom head tracking model based on single imaging period, which can simplify tracking system structure while meeting demand, it has certain application value. The derivation process of the whole mathematical model will be described in detail below.

2. Establishment of Projection Relationship

In order to calculate the three-dimensional pose of the measured object through the two-dimensional image information, it is necessary to establish a correct projection model, thereby linking the characteristic point information in the three-dimensional model with the image point coordinate information in the two- dimensional image, obtain more parameters, and make adequate preparation for subsequent calculations [2].

The head tracker designed in this paper applies the structure of single imaging component and three feature points, according to this; we can establish the projection model of the head tracker, as shown in Figure 1.

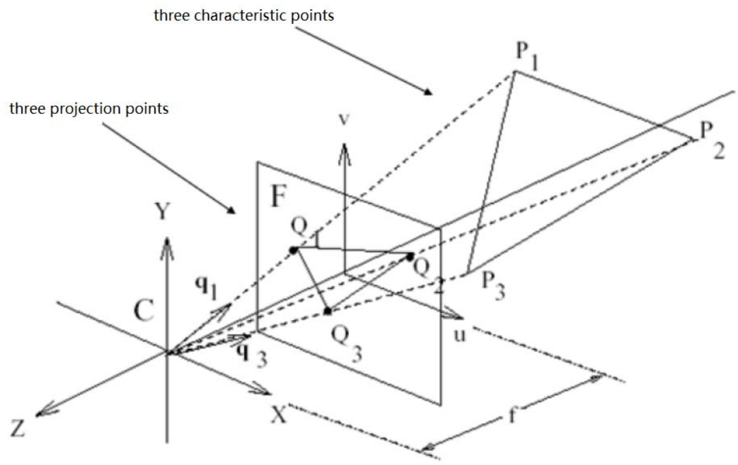

Figure.1 projection model of head tracking system

P_1, P_2, and P_3 are three infrared radiation sources, f is the focal length of the imaging system based on CMOS image sensor, coordinate system XYZ is the camera coordinate system, and C is the central light spot of the camera lens. In order to facilitate the establishment of the projection relationship and subsequent calculations, we make the imaging plane of the image sensor relative to point C and symmetric to the f distance from the front of the camera, it is called as the front imaging plane [3] [4]. For example, the plane F in Fig.2 is the front imaging plane in this system, and Q_1, Q_2, and Q_3 are the images formed by the radiation sources P_1, P_2, and P_3 on the plane F, and their coordinate values are opposite numbers of the actual measured coordinate values of the corresponding points. According to the relevant knowledge of projection imaging, we can regard the triangle constituted by Q1, Q2 and Q3 image point as the triangle composed of the infrared radiation sources P1, P2, and P3 reduced by a certain multiple (s) and then projected, as shown in Fig.2.

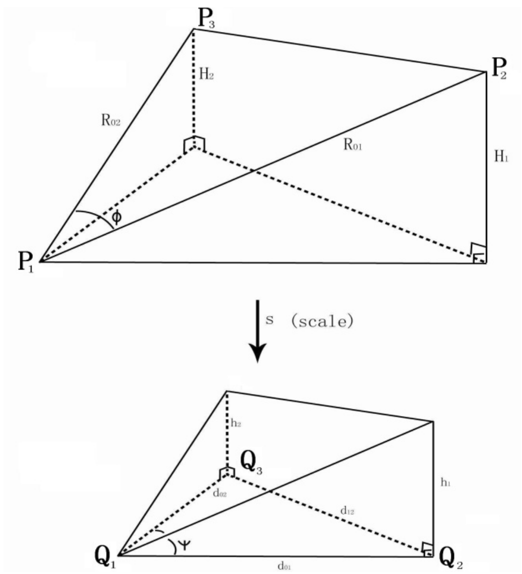

Figure 2. Schematic diagram of the relationship between characteristic points and projection points

So we can get the following equation sets:

$$h_1^2 + d_{01}^2 = (sR_{01})^2 \tag{1}$$

$$h_2^2 + d_{02}^2 = (sR_{02})^2 \tag{2}$$

$$(h_1 - h_2)^2 + d_{12}^2 = (sR_{12})^2 \tag{3}$$

We only want to find the positive real number solution for this equation set; this solution is the reduction factor s in the projection. In general, the positive solution of the equation is given by the following formula:

$$s = \sqrt{\frac{b \pm \sqrt{b^2 - ac}}{a}} \tag{4}$$

Due to the influence of square root number, the equation will have zero, one or two solutions that meet the requirements. Below we need to demonstrate whether these kinds of solutions exist or not, and if so, which solutions are to meet the geometric relationship of the actual projection.

First, let's judge the signs of the coefficients a, b, and c of the equation. In Figure 3.3, set φ as the included angle between $\overrightarrow{m_1} - \overrightarrow{m_0}$ and $\overrightarrow{m_2} - \overrightarrow{m_0}$, and φ is the included angle between $\vec{i_1} - \vec{i_0}$ and $\vec{i_2} - \vec{i_0}$, it is obtained by the cosine theorem:

$$\begin{aligned} a &= 4R_{01}^2 R_{02}^2 - (2R_{01}R_{02}\cos\phi)^2 \\ &= 4(R_{01}R_{02}\sin\phi)^2 \end{aligned} \tag{5}$$

$$b = 2(R_{01}^2 d_{02}^2 + R_{02}^2 d_{01}^2 - 2R_{01}R_{02}d_{01}d_{02}\cos\phi\cos\varphi) \tag{6}$$

$$\begin{aligned} c &= 4d_{01}^2 d_{02}^2 - (2d_{01}d_{02}\cos\varphi)^2 \\ &= 4(d_{01}d_{02}\sin\varphi)^2 \end{aligned} \tag{7}$$

At the same time, ${}^1\!/_2 R_{01}R_{02}\sin\phi$ is equal to the area of the model triangle formed by three luminous diodes, so a is equal to 16 times of the model triangle area. Similarly, c is also equal to 16 times the model triangle area.

Let us assume that the model triangle is normal, namely the model triangle is not a simple point or a simple line. Through this assumption we can get: $a \neq 0$ and $\phi \neq 0$.

It is easy to judge a>0 and c>0 by the formula (5) and (7). It is not difficult to infer b>0 from formula (6):

$$
\begin{aligned}
b &= 2(R_{01}^2 d_{02}^2 + R_{02}^2 d_{01}^2 - 2R_{01}R_{02}d_{01}d_{02}\cos\phi\cos\varphi) \\
&\succ 2(R_{01}^2 d_{02}^2 + R_{02}^2 d_{01}^2 - 2R_{01}R_{02}d_{01}d_{02}) \\
&= 2(R_{01}d_{02} - R_{02}d_{01})^2 \\
&\geq 0
\end{aligned}
$$

Let us judge the symbols of $b^2 - ac$ below, which are obtained by formula (5), (6), and (7):

$$
\begin{aligned}
b^2 = {} & 4(R_{02}^4 d_{01}^4 - 4R_{02}^3 d_{01}^3 R_{01} d_{02}\cos\phi\cos\varphi + 2R_{01}^2 R_{02}^2 d_{01}^2 d_{02}^2 + \\
& 4R_{01}^2 R_{02}^2 d_{01}^2 d_{02}^2 \cos^2\phi\cos^2\varphi - 4R_{01}^3 d_{02}^3 R_{02} d_{01}\cos\phi\cos\varphi + R_{01}^4 d_{02}^4)
\end{aligned}
$$

$$
ac = 16R_{01}^2 R_{02}^2 d_{01}^2 d_{02}^2 \sin^2\phi\sin^2\varphi
$$

$$
\begin{aligned}
b^2 - ac = {} & 4(R_{02}^4 d_{01}^4 - 4R_{02}^3 d_{01}^3 R_{01} d_{02}\cos\phi\cos\varphi + \\
& (2 + 4\cos^2\phi\cos^2\varphi - 4\sin^2\phi\sin^2\varphi)R_{01}^2 R_{02}^2 d_{01}^2 d_{02}^2 - \\
& 4R_{01}^3 d_{02}^3 R_{02}\, d_{01}\cos\phi\cos\varphi + R_{01}^4 d_{02}^4) \\
= {} & 4(R_{01}d_{02})^4 (t^4 - 4\cos\phi\cos\varphi t^3 + (2 + 4\cos^2\phi\cos\varphi + \\
& 4\sin^2\phi\sin^2\varphi)t^2 - 4\cos\phi\cos\varphi t + 1) \\
= {} & 4(R_{01}d_{02})^4 (t^4 - 2(\cos(\phi+\varphi) + \cos(\phi-\varphi))t^3 + \\
& (2 + 4(\cos\phi+\varphi)\cos(\phi-\varphi))t^2 - \\
& 2(\cos(\phi+\varphi) + \cos(\phi-\varphi))t + 1) \\
& = 4(R_{01}d_{02})^4 (t^2 - 2\cos(\phi+\varphi)t + 1)(t^2 - 2\cos(\phi-\varphi)t + 1) \\
\geq {} & 4(R_{01}d_{02})^4 (t^2 - 2t + 1)(t^2 - 2t + 1) \\
= {} & 4(R_{01}d_{02})^4 (t-1)^4 \\
\geq {} & 0
\end{aligned}
$$

Among them, $t = \dfrac{R_{02}d_{01}}{R_{01}d_{02}}$. So we can get $\dfrac{b - \sqrt{b^2 - ac}}{a} \geq 0$, so s=$\dfrac{b - \sqrt{b^2 - ac}}{a}$, there may be one or two positive real solutions. Below we will prove that only one real number solution accord with the actual requirements through the corresponding relationship and constraints of the actual geometric projection model.

In Figure 2, according to the cosine theorem

$$
(sR_{01})^2 - d_{01}^2 = h_1^2 \geq 0 \tag{8}
$$

$$
(sR_{02})^2 - d_{02}^2 = h_2^2 \geq 0 \tag{9}
$$

It can be concluded that from formula (3.11) and (3.12):

$$
\frac{d_{01}}{R_{01}} \leq s\,,\ \frac{d_{02}}{R_{02}} \leq s \tag{10}
$$

In other words, all s values must meet the requirements of formula (10).

Set $s_1 = \sqrt{\dfrac{b - \sqrt{b^2 - ac}}{a}}$, $s_2 = \sqrt{\dfrac{b + \sqrt{b^2 - ac}}{a}}$

Let's discuss the relationship between s_1, s_2 and $\frac{d_{01}}{R_{01}}$, $\frac{d_{02}}{R_{02}}$, and find out which solution meets our requirements.

Set $f = (\frac{d_{01}}{R_{01}})^2$ or $f = (\frac{d_{02}}{R_{02}})^2$, it can be obtained by formula (5), (6), (7):

$$\begin{aligned} &af^2 - 2bf + c \\ &\quad = 4(R_{01}R_{02}\sin\phi)^2 f^2 - 2(2(R_{01}^2 d_{02}^2 + R_{02}^2 d_{01}^2 \\ &\quad\quad - 2R_{01}R_{02}d_{01}d_{02}\cos\phi\cos\varphi)) f + 4(d_{01}d_{01}\sin\varphi)^2 \\ &\quad = 4(R_{01}^2 R_{02}^2(1-\cos^2\phi) f^2 - (R_{01}^2 d_{02}^2 + R_{02}^2 d_{01}^2 \\ &\quad\quad - 2RR_{01}R_{02}d_{01}d_{02}\cos\phi\cos\varphi) f + d_{01}^2 d_{02}^2(1-\cos^2\varphi)) \end{aligned} \tag{11}$$

When $f = (\frac{d_{01}}{R_{01}})^2$, formula (11) can be further inferred:

$$\begin{aligned} &4(-\frac{R_{02}^2 d_{01}^4}{R_{01}^2}\cos^2\phi + 2\frac{R_{02}d_{01}^3 d_{02}}{R_{01}}\cos\phi\cos\varphi - d_{01}^2 d_{02}^2\cos^2\varphi) \\ &\quad = -4R_{02}^2 d_{01}^2(\frac{d_{01}}{R_{01}}\cos\phi - \frac{d_{02}}{R_{02}}\cos\varphi)^2 \end{aligned} \tag{12}$$

When $f = (\frac{d_{02}}{R_{02}})^2$, formula (11) can be further inferred:

$$\begin{aligned} &4(-\frac{R_{01}^2 d_{02}^4}{R_{02}^2}\cos^2\phi + 2\frac{R_{01}d_{02}^3 d_{01}}{R_{02}}\cos\phi\cos\varphi - d_{01}^2 d_{02}^2\cos^2\varphi) \\ &\quad = -4R_{01}^2 d_{02}^2(\frac{d_{02}}{R_{02}}\cos\phi - \frac{d_{01}}{R_{01}}\cos\varphi)^2 \end{aligned} \tag{13}$$

From formula (12) and (13), we can whether $f = (\frac{d_{01}}{R_{01}})^2$ or $f = (\frac{d_{02}}{R_{02}})^2$, the following conclusions can be drawn

$$af^2 - 2bf + c \le 0$$

It can be obtained the derivation of above formula:

$$af^2 - 2bf + c \le 0$$

$$\frac{1}{a}((af - b)^2 - (b^2 - ac)) \le 0$$

It can be obtained from a>0:

$$af^2 - 2bf + c \leq 0$$

$$\frac{1}{a}((af-b)^2 - (b^2 - ac)) \leq 0$$

So:

$$f \geq \frac{b - \sqrt{b^2 - ac}}{a}$$

Or

$$f \leq \frac{b + \sqrt{b^2 - ac}}{a}$$

Namely:

$$s_1 \leq \frac{d_{01}}{R_{01}}, \quad s_2 \geq \frac{d_{02}}{R_{02}} \tag{14}$$

Finally, we consider formula (11) and (14) and see that S has only a single positive real solution, $s_2 = \sqrt{\frac{b + \sqrt{b^2 - ac}}{a}}$ can meet the requirements of the actual projection system, thus establishing the projection relationship between the infrared radiation source and the image point.

3. Establishment of Two Degree-of-freedom Model

The so-called two degree-of-freedom parameter refers to two parameters of the azimuth and pitch angle of sight line. These two parameters are currently required for the helmet sighting device aiming at the target; it is enough for indicating the orientation of target [5].

It is not difficult to see from the projection relationship established in the previous paragraph that the plane P1P2P3 constituted by the infrared radiation source in Fig.2 is parallel to the Q1M2M3 after reduced by s times spatially, therefore, only the azimuth and pitch angle of the plane Q1M2M3 are evaluated, The corresponding posture of the head can be obtained.

In order to solve the attitude angle, the coordinate system is established as follows: the focus of the rectangular diagonal line of the CMOS image sensor as origin of the coordinate system, the imaging plane is the XOY plane, and the straight line which is perpendicular to the imaging plane and passes the O point is the Z axis, according to the projection relationship established in the previous paragraph, it is not difficult to find the coordinates of Q1, M2, M3:

$$Q_1(x_1, y_1, 0) \qquad M_2(x_2, y_2, h_1) \qquad M_3(x_3, y_3, h_2)$$

Among them, x1, x2, x3, y1, y2, y3 are the opposite numbers of the corresponding coordinate components of the three image points obtained by the CMOS image sensor, and h1 and h2 can be obtained by the formula (1) and (2):

$$h_1 = \sqrt{(sR_{01})^2 - d_{01}^2} \tag{15}$$

$$h_2 = \sqrt{(sR_{02})^2 - d_{02}^2} \tag{16}$$

So:

$$\overrightarrow{Q_1M_2} = \{x_1 - x_2, y_1 - y_2, -h_1\} \tag{17}$$

$$\overrightarrow{Q_1M_3} = \{x_1 - x_3, y_1 - y_3, -h_2\} \tag{18}$$

The normal vector $\overrightarrow{N}$ of the plane Q1M2M3 can be obtained from the formula (17) and (18),

$$\overrightarrow{N} = \overrightarrow{Q_1M_2} \times \overrightarrow{Q_1M_3} = \left\{ \begin{vmatrix} y_1 - y_2 & -h_1 \\ y_1 - y_3 & -h_2 \end{vmatrix}, -\begin{vmatrix} x_1 - x_2 & -h_1 \\ x_1 - x_3 & -h_2 \end{vmatrix}, \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_1 - x_3 & y_1 - y_3 \end{vmatrix} \right\}$$

$$\overrightarrow{N} = \{N_x, N_y, N_z\}$$

$$= \left\{ \begin{vmatrix} y_1 - y_2 & -h_1 \\ y_1 - y_3 & -h_2 \end{vmatrix}, -\begin{vmatrix} x_1 - x_2 & -h_1 \\ x_1 - x_3 & -h_2 \end{vmatrix}, \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_1 - x_3 & y_1 - y_3 \end{vmatrix} \right\}$$

According to the definition of azimuth and pitch angle, it can be obtained.
Azimuth

$$\alpha = actg(N_y / N_x)$$

$$\beta = actg\left(N_x \Big/ \sqrt{N_y^2 + N_z^2} \right)$$

4. Conclusion

This paper is based on the structure of the head tracking system of single projection device, the correctness of the projection relationship is determined by establishing projection equation set and discussing the possibilities of the solution, finally, two extremely important parameters in the head tracking are obtained: azimuth and pitch angle, the two degree-of-freedom head tracking model based on single imaging period is established, the tracking system structure can be simplified under the condition that the requirements are met, and it has a certain application value.

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