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Analytical design of soliton molecules in fibers

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Abstract

We present an analytical method for designing fiber systems for a highly stable propagation of soliton molecules. This analytical design uses the variational equations of the soliton molecule to determine the parameters of the most suitable fiber system for any desired soliton, thus reducing dramatically the cost of the whole procedure of design, for both the appropriate fiber system and the desired soliton molecule.

Keywords: nonlinear fiber optics, optical communications, dispersion management, soliton molecules, nonlinear Schrodinger equation, collective variables

(Some figures may appear in colour only in the online journal)

A soliton molecule refers to a collective entity made of several pulses located very close to each other, and bound by a certain phase relation [1–10]. During the propagation of a soliton molecule, the temporal position and the phase of each pulse can vibrate around an equilibrium configuration in a manner similar to that of an atom within a molecule. Unlike conventional solitons (i.e., with single pulse) which have been widely used in fiber optic transmission systems [11–15], soliton molecules have been only minimally examined, because for a long time they were considered as having no practical interest. Conventional solitons are mainly present in long-haul transmission systems, where they are used to encode binary data in the OOK (on–off keying) modulation format [15]. The advantage of this modulation format lies in its simplicity, because it uses only two symbols (namely, '1' which is encoded by a soliton, and '0' which is encoded by the absence of light), necessitating a relatively simple detection system [15]. However, this modulation format suffers from a poor spectral efficiency, which is inadequate to meet the increasing demand of transmission capacities. Presently, efforts focus rather on modulation formats of high spectral efficiency, called multi-level formats because of the use of a higher number of symbols to encode binary data. Each symbol corresponds to a 'state' of modulation associated with well defined values for the amplitude and phase of the optical carrier. In recent years, multi-level formats (involving both the phase and the amplitude of the optical carrier) have been permitted to dramatically increase the transmission capacities [16]. However, such modulation formats are not without drawbacks. In fact, increasing the number of symbols beyond four makes the transmission system highly prone to nonlinear effects, which cause distortions in the signal phase. In this context, recent studies suggested that the use of soliton molecules could provide a viable alternative because of the more strong immunity of solitons against nonlinear effects in fibers [6, 8]. In [6, 8] the authors demonstrated experimentally the propagation of two-soliton molecules and three-soliton molecules, in a dispersion-managed (DM) fiber system. Those results suggest that the 'absence of light', the 'DM soliton', the 'two-soliton molecule', the 'three-soliton molecule', and so on, may be considered as the different states of a multi-level modulation format, which could provide the best

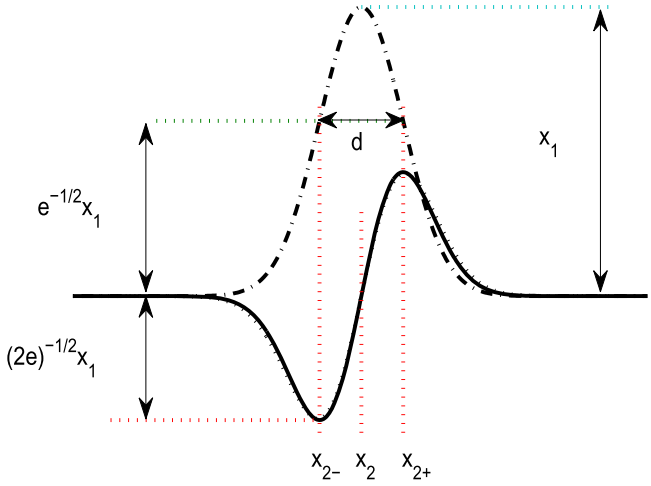

Figure 1. Schematic representation of a conventional DM soliton (dot-dashed curve) and two-soliton molecule (solid line).

immunity against nonlinear effects in fiber-optic transmission systems. Despite these attractive prospects, it is clear that the road towards possible applications of soliton molecules in fiber-optic transmission systems is still long and strewn with major difficulties. One of them lies in the fact that DM solitons admit only very specific values of temporal width, power peak, energy, and chirp [12]. Indeed, the combined effects of the nonlinearity and the structure of the dispersion map, impose strict constraints on the parameters of the light pulses that can propagate in a highly stable manner, while executing a perfectly periodic breathing [12, 17–19]. Such pulses, also called 'fixed points' of transmission, are essential in the case of long-haul DM systems (of the order of several thousands of Km). A major difficulty in the use of DM solitons lies in the carving of their intensity profile. Indeed, at the practical level, there is currently no device for generating such solitons, due to the complexity of their intensity profiles. Most practical systems use rather pulses with Gaussian profile; which generate ultimately radiation effects highly detrimental to the stability of solitons. However, such radiation effects may be considerably reduced by adjusting at best the Gaussian profile to the exact profile of the fixed point [18]. Such an adjustment can be made only by means of theoretical tools, which are necessary to obtain the exact profile of the fixed point.

Currently, in the case of soliton molecules, this adjustment is performed through a procedure of search of the parameters of the fixed points, known as being the Nijhof method [19]. The procedure starts with a soliton molecule whose profile is arbitrarily chosen, and one lets the molecule propagate within the system over a distance which is also chosen in an arbitrary manner. Then one calculates the average field of pulse over this distance, and one uses it as initial condition for the next step. Thus, one repeats the field-averaging procedure, step by step, until the average field ceases to vary. The final average field then gives the intensity profile of the fixed point. This procedure suffers from two major drawbacks:

(i) First, this method is too time consuming, due to its iterative nature and the necessity to make arbitrary choices on the pulse profile at the beginning of the procedure, and on the propagation distance which is used to calculate the average field at each step of the procedure. Moreover, with such arbitrary choices, there is no guarantee that the method converge to a fixed point.

(ii) The Nijhof method [19] allows to find the fixed point of a given DM system, but does not allow to do the reverse operation; that is, determine the parameters of a DM system that admits any desired light molecule as fixed point.

In the present work, we resolve these two major drawbacks by proposing a method for analytically designing the DM system that admits any desired light molecule as fixed point. We illustrate this method on two-soliton molecules.

One can describe in a realistic way the propagation of light pulses in optical fiber systems by the nonlinear Schrödinger equation (NLSE) which follows [12]

$$\frac{\partial\psi}{\partial z}+\frac{\mathrm{i}\beta_2(z)}{2}\frac{\partial^2\psi}{\partial t^2}-\mathrm{i}\gamma(z)|\psi|^2\psi=0, \qquad (1)$$

where $\psi(z, t)$ is the envelope amplitude of the electric field measured in units of square root of Watts at position z in the fiber system, and at time t in the moving frame. The parameters $\beta_2(z)$ and $\gamma(z)$ designate the chromatic dispersion and the Kerr nonlinearity coefficients, respectively.

Although a soliton molecule is a full entity, its dynamical behavior is rather similar to that of a mechanical system made up of several components interacting in a rather elastic manner. Each component may be considered as a conventional DM soliton, but whose internal dynamic is not necessarily the same as that of the other components of the molecule, since optical fiber is an environment which is always more or less prone to various perturbations. Consequently, the intensity profile of a soliton molecule is necessarily far more complex than the bell-shaped profile of a conventional DM soliton. It is therefore extremely difficult to ascertain the internal dynamics of a soliton molecule by a direct analysis of its intensity profile obtained by solving the NLSE (1). On the other hand, one can get a clear insight of the overall dynamical behavior of the molecule, including its internal dynamics, by means of a relatively small number of dynamical variables, called collective coordinates, provided that those coordinates are appropriately chosen so as to describe all the major degrees of freedom of the internal dynamics of the molecule. The method of analytical design of solitons molecules that we propose in this work, is fundamentally based on a collective coordinate approach. These coordinates are introduced in the theoretical treatment through a trial function, called ansatz function, which is chosen to be of the following form

$$\psi(z, t)=x_1\phi(\xi)\mathrm{e}^{\frac{\mathrm{i}}{2}x_3^2x_4\xi^2+\mathrm{i}x_3x_5\xi+\mathrm{i}x_6}, \qquad (2)$$

where $\xi=(t-x_2)/x_3$ and the function ϕ is as yet an unknown function, which will be chosen to represent at best the exact profile of the soliton molecule. Here, the coordinates x_2, x_4, x_5 and x_6, represent respectively the position of the center-of-mass, the chirp, the frequency, and the phase constant of the soliton molecule as a whole entity. x_1 and x_3 are coordinates which contribute to structuring the intensity profile of the molecule, as we illustrate in the following two cases:

i. The ansatz (2) can be used to represent the profile of the conventional DM soliton, by simply setting $\phi = \exp(-\xi^2)$. In this case, x_1 represents the soliton's amplitude, as shown in the dot-dashed curve of figure 1, while the temporal width of the soliton (full width at half maximum of the intensity profile) is given by $\Delta t_{\rm FWHM} = x_3\sqrt{2\ln 2}$.

ii. In the case of a two-soliton molecule, ansatz (2) can be used with a function ϕ given by [1, 9]

$$\phi = \xi\exp(-\xi^2). \tag{3}$$

The solid curve in figure 1 illustrates the temporal profile of this molecule, which consists of two pulses with π phase shift. Here, it should be noted that the coordinate x_1 does no longer represent exactly the solitons amplitude, which is now given by $a = x_1/\sqrt{2e}$. The second relevant parameter for characterizing this molecule is the separation between the positions of the two pulses, given by $d = x_{2+} - x_{2-} = x_3\sqrt{2}$, as illustrated in figure 1.

By inserting the ansatz (2)–(3) in the NLSE (1), and applying the projection operator method on the resulting equation [20], we obtain the following set of ordinary differential equations, which describe the spatial evolution of the collective coordinates of the soliton molecule

$$\dot{x}_1 = 0.5\beta_2(z)x_1x_4, \tag{4a}$$
$$\dot{x}_2 = -\beta_2(z)x_5, \tag{4b}$$
$$\dot{x}_3 = -\beta_2(z)x_3x_4, \tag{4c}$$
$$\dot{x}_4 = -\beta_2(z)(\alpha_1x_3^{-4} - x_4^2) - \alpha_2\gamma(z)x_1^2x_3^{-2}, \tag{4d}$$
$$\dot{x}_5 = 0, \tag{4e}$$
$$\dot{x}_6 = \beta_2(z)(\alpha_3x_3^{-2} - 0.5x_5^2) - \alpha_4\gamma(z)x_1^2. \tag{4f}$$

Here, α_1, α_2, α_3 and α_4 are coefficients that depend on the particular choice of the function ϕ, which is closely related to the particular type of soliton molecule considered. Their values will be specified in what follows. Then, from the set of equations (4*a*)–(4*f*), we obtain the equation of evolution of the collective coordinate x_3

$$\dot{x}_3^2 = -\alpha_1\beta_2^2x_3^{-2} - 2\alpha_2\beta_2\gamma E_0x_3^{-1} + 2c, \tag{5}$$

where $E_0 \propto x_1(0)^2x_3(0) = x_1(z)^2x_3(z)$ is the energy of the molecule. The constant of integration c is determined by imposing that the middles of the fiber segments of the dispersion map (of normal and anomalous dispersion) correspond to chirp-free points such that

$$c_\pm = 0.5\alpha_1\beta_{2\pm}^2x_{3\pm}^{-2} + \alpha_2\beta_{2\pm}^2\gamma_\pm\ E_0x_{3\pm}^{-1}, \tag{6}$$

for the fiber section with normal dispersion (+) and for the fiber section with anomalous dispersion (−). x_{3+} and x_{3-} represent the minimum separation between the two pulses (of the molecule) within the normal and the anomalous dispersion fibers, respectively.

Following the procedure of designing the conventional DM soliton systems [13], here we derive the expression for the maximum temporal separation between solitons $x_{3\max}$ from the continuity of chirp when the soliton molecule propagates from one fiber section to another fiber section. Then, from equations (5) and (4*d*) we obtain the following expression for the maximum separation

$$x_{3\max} = \alpha_2\beta_{2+}\beta_{2-}(\gamma_+\beta_{2-} - \gamma_-\beta_{2+})/(c_+\beta_{2-}^2 - c_-\beta_{2+}^2). \tag{7}$$

To derive the expression for the length of the fiber sections, we integrate equation (5) with respect to z. We find the length of normal L_+ and anomalous L_- fiber sections, to be

$$L_\pm = 2[g(\beta_{2\pm}, \gamma_\pm, c_\pm, x_{3\max}) - g(\beta_{2\pm}, \gamma_\pm, c_\pm, x_{3\pm})], \tag{8}$$

where the function $g(\beta_2, \gamma, c, x_3)$ is found to take the following forms, depending on both $c = c_-$ (c_+) and $\Delta = -8\alpha_1\beta_2^2c - (2\alpha_2\beta_2\gamma E_0)^2$

(i) For $c > 0$ and $\Delta < 0$

$$g(\beta_2, \gamma, c, x_3) = \sqrt{R}\,(2c)^{-1} + \eta_0\ln(2c\sqrt{R} + \eta_1), \tag{9}$$

(ii) For $c > 0$ and $\Delta = 0$

$$g(\beta_2, \gamma, c, x_3) = \sqrt{R}\,(2c)^{-1} + \eta_0\ln(\eta_1). \tag{10}$$

(iii) For $c < 0$ and $\Delta < 0$

$$g(\beta_2, \gamma, c, x_3) = \sqrt{R}\,(2c)^{-1} + \eta_0\arcsin\frac{\eta_1}{\sqrt{-\Delta}} \tag{11}$$

with $\eta_0 = \frac{\alpha_2\gamma\beta_2E_0}{2c\sqrt{c}}$, $\eta_1 = 4cx_3 - 2\alpha_2\gamma\beta_2E_0$ and $R = R(x_3) = 2cx_3^2 - 2\alpha_2\beta_2\gamma E_0x_3 - \alpha_1\beta_2^2$. It is worth noting that in [13], where the conventional DM soliton is considered, only the case (9) were considered. The different cases enumerated in relations (9), (10) and (11), constitute the generalization of the procedure reported in [13], so as to treat not only conventional DM solitons but also soliton molecules. Thus, to design the dispersion map $(L_-/2,\ L_+,\ L_-/2)$ of the appropriate DM system for any desired soliton molecule, we proceed as follows. We consider the beginning point of the dispersion map as the midpoint of the anomalous dispersion fiber, and use the following set of parameters: x_{3-}, E_0, $\beta_{2\pm}$, $\gamma_\pm$ and $x_{3\max}$ as input data. Then, these four parameters are used to calculate the constants of integration c_- and c_+ from equations (6) and (7) respectively. Next, the minimum pulse-to-pulse separation in the normally dispersive fiber segment is determined from equation (6). Finally, the lengths L_- and L_+ required for periodic evolution of the desired soliton molecule, are evaluated by using the formula (8).

From a practical viewpoint, our procedure of analytical design requires that we begin by specifying the fiber parameters and those of the desired soliton molecule. For illustrative purposes, let us consider a DM system having the following parameters: $\beta_{2-} = -5.2\ \mathrm{ps^2\,km^{-1}}$, $\beta_{2+} = 4.3\ \mathrm{ps^2\,km^{-1}}$, and $\gamma_\pm = 1.4\ \mathrm{W^{-1}\,km^{-1}}$. Suppose we wish to transmit through this system, a soliton molecule having the following parameters: $E_0 = 0.177$ pJ, $x_{3-} = 10$ ps, $x_{3\max} = 15.2$ ps (which corresponds to a breathing factor $T_e = x_{3\max}/x_{3-} = 1.52$, a peak power $P_0 = 10$ mW, and temporal width $\Delta t_{\rm FWHM} = 11.77$ ps, for each pulse of the molecule). Then, our analytical formula (8) immediately gives us the lengths of the two segments of fibers of the dispersion

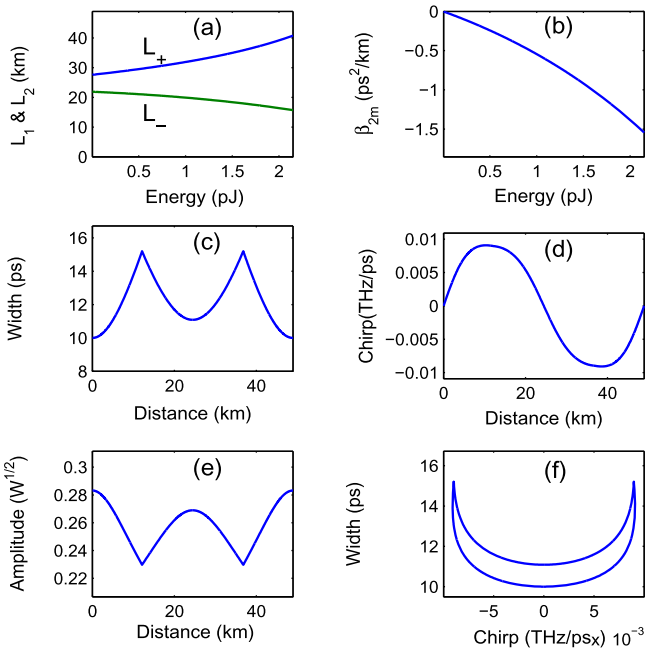

Figure 2. Illustration of the analytical design of the DM system.

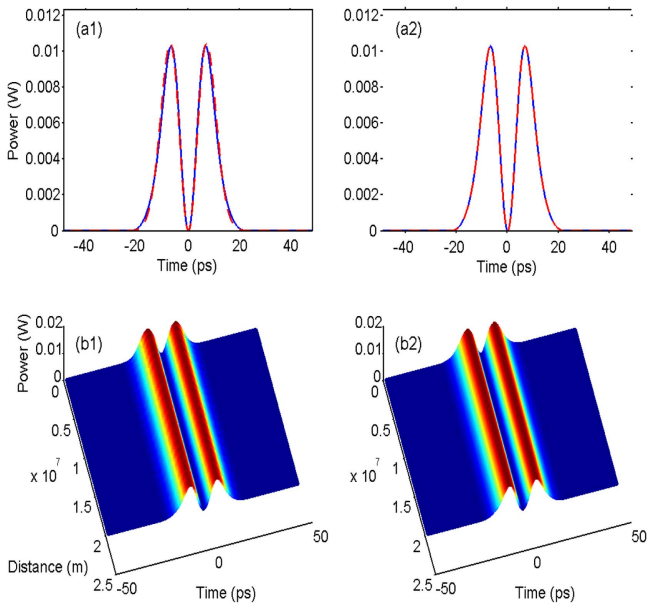

Figure 3. 20 000 km propagation of a bisoliton molecule obtained by numerically solving the NLSE. (a1)-(b1) uses the analytically design solution as input condition. (a2)-(b2) uses the exact soliton as input condition. $\beta_{2m} = -0.079$ ps^2 km^{-1}.

map: $L_- = 22.602$ km, et $L_+ = 26.462$ km. The resulting average dispersion is then $\beta_{2m} = -0.079$ ps^2 km^{-1}. Here, it is worth emphasizing that the fundamental property of a DM soliton is that it propagates along the fiber, while executing a perfectly periodic internal dynamic. It is therefore essential to check that our analytical designed system of soliton molecules, possesses this fundamental property, i.e., check whether the soliton molecule defined by the profile (2)–(3) is able to propagate along the system while performing a perfectly periodic internal dynamic.

Figures 2(a) and (b) illustrate the results of analytical design that we obtained in the following range of energy: 0.0036 pJ $< E_0 <$ 2.15 pJ. In particular, figures 2(c), (d), (e), and (f), obtained for $\beta_{2m} = -0.079$ ps^2 km^{-1}, show a perfectly periodic evolution of the soliton parameters over the map length $L_{\text{map}} = L_- + L_+$ designed analytically.

The results of figure 2, demonstrate that the soliton molecules designed analytically, are endowed with the fundamental property related to the periodic character of the internal dynamics of DM solitons. However this property does not indicate whether or not the soliton molecule is able to propagate in a stable manner, over a considerable distance in a system subject to perturbations (e.g., the photon noise). In this regard, it is worth noting that, as a general rule, the temporal profile of the exact fixed point of a DM line is endowed with very small side lobes at the leading and trailing edges of the soliton. But those side lobes, which are generally visible only on the log scale of the intensity profile of the soliton, play no significant role in the soliton dynamics, and are impossible to synthesize at the practical level. The exact fixed point can be obtained only numerically via the Nijhof method [19]. The fixed point obtained by our analytical procedure, whose temporal profile is given by the Hermite Gaussian ansatz (2)–(3), with the system parameters given by our analytical formula (8), is not rigorously the exact fixed point of the system, because the Hermite Gaussian ansatz has a completely smooth profile. We will thus refer to the fixed point designed analytically as being the *proximity fixed point* (PFP). The PFP has the dual advantage, of being much easier to calculate and much easier to carve at the practical level (when compared with the exact fixed point). In this context, it is useful to examine the stability of our PFP in a situation more realistic than the one represented in figures 2(c), (d), (e), and (f), i.e., in a situation where the propagation is not limited to a single period of the dispersion map. A realistic situation consists in injecting the soliton in an ultra-long deployed line (containing a large number of periods of dispersion map), in a perturbed environment (e.g., under the effects of the numerical noise or a photon noise).

So, we injected the soliton molecule whose parameters are represented in figures 2(c), (d), (e) and (f), in an ultra-long DM system designed analytically with the energy $E_0 = 0.177$ pJ and the average dispersion $\beta_{2m} = -0.079$ ps^2 km^{-1}. We numerically solved the NLSE (1) by means of the usual split-step Fourier method [12, 15]. The simulation contains necessarily a numerical noise (generated by rounding errors of numerical computation and truncation errors of the split-step Fourier method). Figures 3(a1) and (b1) show the simulation of propagation of the PFP over 2×10^4 Km. Figure 3(a1) shows the profile of the soliton molecule at the fiber input (dashed curve), and at the fiber output (solid line). Figure 3(b1) shows the evolution of the soliton'(s) temporal profile (recorded after each period of the dispersion map), as a function of the propagation distance. Figures 3(a1) and (b1) illustrate remarkably the very high stability of our analytically designed PFP, in an environment perturbed by the numerical noise. Furthermore, it is important to compare the dynamical behavior of our PFP (figures 3(a1)-(b1)), with that of the exact fixed point given by the Nijhof method [19], which is

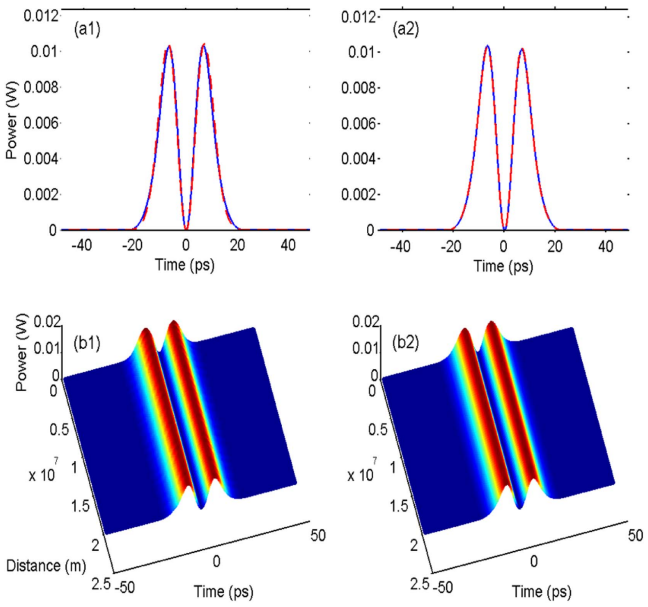

Figure 4. Same numerical simulation as in figure 3, but in the presence of photons noise.

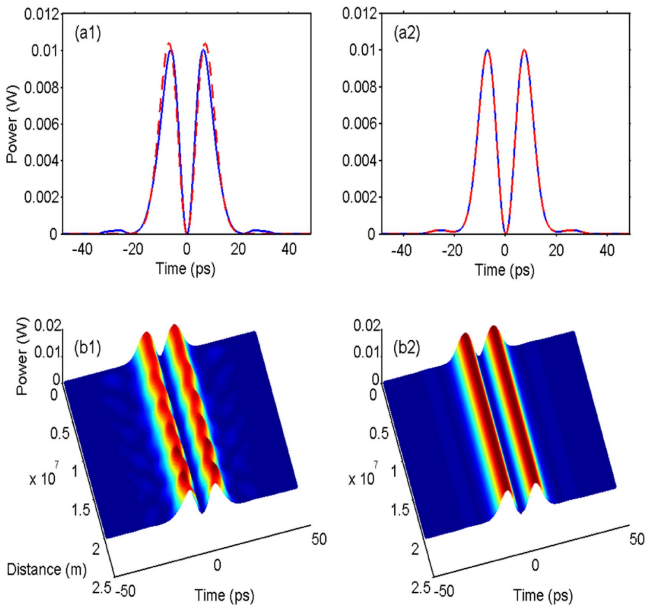

Figure 5. Same numerical simulation as in figure 3 , but with $\beta_{2m} = 6.46 \text{ x } 10^{-4}\,\text{ps}^2\,\text{km}^{-1}$.

displayed in figures 3(a2) and (b2). As can be seen in figures 3, the dynamical behavior of our PFP agrees extremely well with that of the exact fixed point.

On the other hand, we have examined the robustness of this soliton molecule in an even more strongly perturbed environment. To this end, we carried out the same simulation as in figure 3, but by systematically adding a photon noise after each period of the dispersion map. We arbitrarily chose a noise power comparable to the one that may be generated by an amplifier of gain G which fully compensates the losses induced by a standard Telecom fiber (having a loss coefficient of $0.2\,\text{dB km}^{-1}$). In other words, we arbitrarily chose the maximum noise power to be $P_{\text{noise}} = h\nu n_{\text{sp}}(G-1)\Delta t$ where G represents the amplifier gain, n_{sp} is the spontaneous emission factor, h is Planck'(s) constant, Δt is the temporal window of the pulse, ν is the photon frequency. The results of our simulations are represented in figures 4, which show that the effects of this photons noise manifest themselves essentially in the same way for the two types of pulses considered. Indeed, whatever be the input pulse, PFP (figures 4(a1) and (b1)) or exact fixed point (figures 4(a2) and (b2)), the cumulative effect of the photons noise causes a slight distortion of the soliton'(s) profile, with a slight asymmetry between the two pulses of the molecule (and a small difference of peak power between the two pulses), which is clearly visible in figures 4(a1) and (a2). Despite these slight distortions of pulse profile, the PFP shows a remarkable robustness.

In principle, the PFP should have a dynamical behavior all the more close to that of the exact fixed point, as the difference between their respective temporal profiles, $q = \psi_{\text{exact}} - \psi_{\text{PFP}}$, is small. The amplitude of the field q, which we call *residual field*, depends essentially on the structure of the dispersion map of the system (and specifically, on the average dispersion of the system). We can thus predict that the PFP will have a less good stability if the residual field is strong. To verify this point, we considered an other value of the average dispersion, for which the residual field energy is larger than in the case of figures 3 and 4. To this end, we performed analytical design for the average dispersion $\beta_{2m} = 6.46 \text{ x } 10^{-4}\,\text{ps}^2\,\text{km}^{-1}$, and the numerical simulation of propagation of the PFP having initially the profile defined by the Hermite–Gaussian ansatz (2).

The results, which are displayed in figures 5(a1) and (b1), show that here also, our PFP executes a highly stable propagation, comparable to that of the exact fixed point (figure 5(a2) and (b2)). By the way, we note the presence of pedestals in the profile of our PFP after 20 000 km, which we attribute to the increased level of the initial residual field. But these pedestals have no significant harm on the stability of our soliton molecule over thousands of km.

By the way, it should be noted that, a fundamental property of DM solitons (which makes a great qualitative difference when compared with conventional solitons), is that they can exist in the normal dispersion regime. In this respect, the results of figure 5, which correspond to a positive value of the average dispersion, demonstrate that the soliton molecules designed analytically are also endowed with this fundamental property of DM solitons.

To conclude, in this work we have presented a fully analytical method for designing DM fiber systems which enable a highly stable propagation of soliton molecules. This analytical method has two main virtues: it represents only an extremely small fraction of the amount of calculations required when the conventional numerical procedure is used. It uses a more realistic intensity profile (at the practical level) than the complex profile of the exact soliton molecule, but which is close enough to the exact profile for achieving essentially the same level of stability. As a final note, we would like to stress that although the analytical formulas obtained in this work correspond to a lossless system, these

formulas may be used to design the practical version of the system including losses and periodic amplification, by following the procedure described in [14].

References

[1] Paré C and Bélanger P-A 1999 *Opt. Commun.* **168** 103–9
[2] Feng B F and Malomed B A 2004 *Opt. Commun.* **229** 173–85
[3] Stratmann M, Pagel T and Mitschke F 2005 *Phys. Rev. Lett.* **95** 143902
[4] Gabitov I, Indik R, Mollenauer L, Shkarayev M and Stepanov M 2007 *Opt. Lett.* **32** 605–7
[5] Hause A, Hartwig H, Seifert B, Stolz H, Böhm M and Mitschke F 2007 *Phys. Rev.* A **75** 063836
[6] Rohrmann P, Hause A and Mitschke F 2012 *Sci. Rep.* **2** 866
[7] Xiaohui L *et al* 2012 *Opt. Commun.* **285** 1356–61
[8] Rohrmann P, Hause A and Mitschke F 2013 *Phys. Rev.* A **87** 043834
[9] Alamoudi S M, Al Khawaja U and Baizakov B B 2014 *Phys. Rev.* A **89** 053817
[10] Xiaohui L, Yishan W, Wei Z and Wei Z 2014 *Laser Phys. Lett.* **11** 075103
[11] Lakoba T I, Yang J, Kaup D J and Malomed B A 1998 *Opt. Commun.* **149** 366–75
[12] Zakharov V E and Wabnitz S 1998 (Berlin: Springer)
[13] Nakkeeran K, Moubissi A B, Tchofo Dinda P and Wabnitz S 2001 *Opt. Lett.* **20** 1544–6
[14] Moubissi A B, Nakkeeran K, Tchofo Dinda P and Wabnitz S 2002 *IEEE Photonics Technol. Lett.* **14** 1279–81
[15] Agrawal G P 2010 *Fiber-Optics Communication Systems* 4th edn (New York: Wiley)
[16] Rahman T *et al* 2014 *J. Lightwave Technol.* **33** 1794–804
[17] Turitsyn S K, Bale B G and Fedoruk M P 2012 *Phys. Rep.* **521** 1–69
[18] Ngabireng C M and Tchofo Dinda P 2005 *Opt. Lett.* **30** 595–7
[19] Nijhof J H B, Forysiak W and Doran N J 2000 *IEEE J. Sel. Top. Quantum Electron.* **6** 330–6
[20] Tchofo Dinda P, Moubissi A B and Nakkeeran K 2001 *Phys. Rev.* E **64** 016608