

LETTER

Anomalous Hooke's law in disordered graphene

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Anomalous Hooke's law in disordered graphene

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The discovery of graphene, a single monolayer of graphite, has provided an experimental demonstration of stability of 2D crystals. Although thermal fluctuations of such crystals tend to destroy the long-range order in the system, the crystal can be stabilized by strong anharmonicity effects. This competition is the central issue of the crumpling transition, i.e., a transition between flat and crumpled phases. We show that anharmonicity-controlled fluctuations of a graphene membrane around equilibrium flat phase lead to unusual elastic properties. In particular, we demonstrate that stretching ξ of a flake of graphene is a nonlinear function of the applied tension at small tension: $\xi \propto \sigma^{\eta/(2-\eta)}$ and $\xi \propto \sigma^{\eta/(8-\eta)}$ for clean and strongly disordered graphene, respectively. Conventional linear Hooke's law, $\xi \propto \sigma$, is realized at sufficiently large tensions: $\sigma \gg \sigma_*$, where σ_* depends both on temperature and on the disorder strength.

Hooke's law (HL)—introduced by Robert Hooke about 350 years ago—states that the force needed to extend or compress an elastic body by some distance is proportional to that distance. Conventional theory of elasticity predicts that this law is fulfilled for low fields (in the so-called 'elastic range' of tensions) and gets violated at sufficiently large tensions.

The goal of this Letter is to explore stretching of graphene, a famous two-dimensional (2D) material [1–10], as a reaction on applied tension. Measurement of the elasticity of free-standing graphene is accessible to current experimental techniques [11–15]. Remarkably, we find that, for graphene, HL fails even in the limit of the infinitesimally small tension. The underlying physics has a very close relation to the well known problem of thermodynamic stability of 2D crystals [16, 17].

Free-standing graphene is a remarkable example of an elastic crystalline 2D membrane with a high bending rigidity $\kappa \simeq 1$ eV. The most important feature distinguishing such a membrane from conventional 2D semiconductor systems is the existence of specific type of out-of-plane phonon modes—flexural phonons (FP) [18].

In contrast to in-plane phonons with the linear dispersion, the FP are very soft, $\omega_q \propto q^2$, and, consequently, the out-of-plane thermal fluctuations are unusually strong and tend to destroy graphene membrane by driving it into the crumpled phase [18]. The competing effect is the anharmonicity that plays here a key role.

This question was intensively discussed more than two decades ago [18–35] in connection with biological membranes, polymerized layers, and inorganic surfaces. The interest to this topic has been renewed more recently [36–43] after discovery of graphene. It was found [19–25] that the anharmonic coupling of in-plane and out-of-plane phonons stabilizes the membrane for not too high temperatures T . This is connected with a strong renormalization of the bending rigidity [24, 26, 32], $\kappa \rightarrow \kappa_q \propto q^{-\eta}$, for $q \rightarrow 0$, with a certain critical index η . Due to the high bare value of κ , clean graphene remains flat up to all realistic temperatures. The critical exponent η was determined within several approximate analytical schemes [22, 24, 25, 32, 36]. Numerical simulations for a 2D membrane embedded in 3D space yield $\eta = 0.60 \pm 0.10$ [30] and $\eta = 0.72 \pm 0.04$ [35].

In a recent paper [44], we have developed a theory of rippling and crumpling in *disordered* free-standing graphene. We have shown that random fluctuations of the membrane curvature caused by static disorder may strongly affect properties of the membrane. We have derived coupled renormalization-group (RG) equations describing the combined flow of $\varkappa$ and disorder strength b , determined the phase diagram (flat versus crumpled) in the $(\varkappa, b)$ plane, and explored the rippling in the flat phase.

In the present letter, we explore the fate of HL in clean and disordered graphene. We find that linear HL breaks down both for clean and disordered cases, so that deformation of the membrane subjected to a small stretching tension $\sigma > 0$ scales as $\Delta L \propto \sigma^\alpha$, with a non-trivial exponent α . In the opposite case, $\sigma < 0$, $\Delta L < 0$, the membrane undergoes a buckling transition [23]. We obtain the critical index α that turns out to be different for clean and disordered cases. Our findings imply that for sufficiently strong disorder the anomalous elasticity of graphene is fully determined by static random deformations—ripples. The non-linearity of elasticity of graphene found in this work is in agreement with recent experimental findings [14, 15]. Related theoretical results have been recently obtained for clean membranes in the ribbon geometry [45] and by numerical simulations [46].

We consider a 2D membrane embedded in the d -dimensional space ($d > 2$). The starting point of our analysis is the energy functional

$$E = \int d^2x \left[\frac{\varkappa}{2} (\Delta \mathbf{r})^2 + \frac{\mu}{4} (\partial_\alpha \mathbf{r} \partial_\beta \mathbf{r} - \delta_{\alpha\beta})^2 + \frac{\lambda}{8} (\partial_\gamma \mathbf{r} \partial_\gamma \mathbf{r} - D)^2 \right]$$

which can be obtained from the general gradient expansion of elastic energy [21] by using a certain rescaling of coordinates (see discussion in [44]). Here $\varkappa$ is the bare bending rigidity, while μ and λ are in-plane coupling constants. The d -dimensional vector $\mathbf{r} = \mathbf{r}(\mathbf{x})$ describes a point on the membrane surface and depends on the 2D coordinate $\mathbf{x}$ that parametrizes the membrane. The vector $\mathbf{r}$ can be split into $\mathbf{r} = \xi \mathbf{x} + \mathbf{u} + \mathbf{h}$, where vectors $\mathbf{u} = (u_1, u_2)$, $\mathbf{h} = (h_1, \dots, h_{d_c})$ represent in-plane and out-of-plane displacements, respectively, and $d_c = d - 2$. The stretching factor ξ is equal to unity in the mean-field approximation but gets reduced due to fluctuations. In terms of $\mathbf{u}$, $\mathbf{h}$, and ξ , the energy becomes

$$E = \frac{L^2(\mu + \lambda)(\xi^2 - 1)}{2} \times \left[\xi^2 - 1 + \int \frac{d^2\mathbf{x}}{L^2} \partial_\alpha \mathbf{h} \partial_\alpha \mathbf{h} \right] + E_0, \quad (1)$$

where $\tilde{\mathbf{u}} = \xi \mathbf{u}$ and $E_0 = E_0(\tilde{\mathbf{u}}, \mathbf{h})$ describes the energy of in-plane and out-of-plane fluctuation. We proceed now to include the static disorder. As shown in [44], the relevant disorder is produced by a random curvature. The energy of fluctuations including such

disorder reads [28]

$$E_0(\mathbf{u}, \mathbf{h}) = \int d^Dx \left\{ \frac{\varkappa}{2} (\Delta \mathbf{h} + \boldsymbol{\beta})^2 + \mu u_{ij}^2 + \frac{\lambda}{2} u_{ii}^2 \right\}. \quad (2)$$

Here $u_{\alpha\beta} = (\partial_\alpha u_\beta + \partial_\beta u_\alpha + \partial_\alpha \mathbf{h} \partial_\beta \mathbf{h})/2$ is the strain tensor and $\boldsymbol{\beta} = \boldsymbol{\beta}(\mathbf{x})$ is a random vector with Gaussian distribution $P(\boldsymbol{\beta}) = Z_\beta^{-1} \exp \left[-(1/2b) \int \beta^2(\mathbf{x}) d^Dx \right]$, where b is the disorder strength and Z_β is a normalization factor. For $\boldsymbol{\beta} = 0$, $E_0(\mathbf{u}, \mathbf{h})$ coincides with the conventional expression for elastic energy of nearly flat membrane [18].

The second term in the square brackets in equation (1) describes the coupling between fluctuations and stretching. Such a coupling leads to shrinking of the membrane in the x -plane. As a result, the optimal value of ξ deviates from the mean-field value $\xi = 1$ due to the fluctuations. The size of the membrane with fluctuations R is related to the size L of the membrane without fluctuations as follows: $R = \xi L$. Hence, the ‘projected’ area of the membrane reads $A = \xi^2 L^2$.

For $\sigma = 0$, the equilibrium value of ξ reads⁶ [44]:

$$\xi^2 = 1 - \langle \partial_\alpha \mathbf{h} \partial_\alpha \mathbf{h} \rangle / 2. \quad (3)$$

Here angular brackets denote the Gibbs averaging. Application of tension σ to the membrane leads to the increase of ξ^2 , as compared to equation (3). Below we calculate function $\xi(\sigma)$, both for clean and disordered cases.

Clean case ($b = 0$). For $\sigma \neq 0$, the propagator of h -modes calculated in the harmonic approximation is given by (see footnote 6 for technical details)

$$\langle h_{\mathbf{q}}^\alpha h_{-\mathbf{q}'}^\beta \rangle = (2\pi)^2 \delta(\mathbf{q} - \mathbf{q}') \delta_{\alpha\beta} G_{\mathbf{q}}^0, \quad (4)$$

where $G_{\mathbf{q}}^0 = T/(\varkappa q^4 + \sigma q^2)$. Tension σ is given by a derivative of the free energy F with respect to A [25], $\sigma = \partial F / \partial A$, and is related to ξ as (see footnote 6)

$$\sigma = (\mu + \lambda)(\xi^2 - 1 + \langle \partial_\alpha \mathbf{h} \partial_\alpha \mathbf{h} \rangle / 2). \quad (5)$$

Conventional HL can be derived from equation (5) by neglecting the contribution of fluctuations and assuming that ξ is close to unity: $\sigma_{\text{conv}} \approx k_0(\xi - 1)$. Here $k_0 = 2(\mu + \lambda) \approx 400 \text{ N m}^{-1}$ is in-plane stiffness predicted for flat graphene [47, 48] and measured in [11, 12]. The main purpose of the further discussion is to demonstrate that the contribution of fluctuations is of crucial importance, so that this law fails in the limit $\sigma \rightarrow 0$, where stretching turns out to be a nonlinear function of σ .

For large momenta, $q > q_\sigma$, where $q_\sigma = \sqrt{\sigma/\varkappa}$, Green's function is approximately given by $G_{\mathbf{q}}^0 = T/\varkappa q^4$. The strong infrared singularity $G_{\mathbf{q}}^0 \propto 1/q^4$ leads to a logarithmic divergence of $\langle \partial_\alpha \mathbf{h} \partial_\alpha \mathbf{h} \rangle$ and, consequently, in view of equation (3), to the renormalization of ξ [44]. Hence, ξ becomes scale-dependent: $\xi \rightarrow \xi_L$, where $L \sim q^{-1}$. At finite q , the renormalization is stopped because of the term

⁶ See supplementary material for technical details.

σq^2 in the denominator of G_q^0 . To determine the q -dependence of the renormalization of ξ , one should take into account that the bending rigidity is also renormalized for sufficiently small wave vectors $q \ll q_*$ according to the RG equation [24, 26, 32]

$$d\kappa/d\Lambda = \eta\kappa \Rightarrow \kappa_q = \kappa(q_*/q)^\eta. \quad (6)$$

Here $\Lambda = \ln(q_*/q)$, η is the anomalous dimension of the bending rigidity, q_* is the inverse Ginzburg length,

$$q_* \simeq \sqrt{\tilde{\mu} T}/\kappa \quad (7)$$

and $\tilde{\mu} = 3\mu(\mu + \lambda)/[8\pi(2\mu + \lambda)]$, see [44]. Below, we assume that $q_* \gg q_\sigma$. In this case, a competition between the two terms in the denominator of G_q^0 leads to appearance of a new spatial scale $\tilde{q}_\sigma$ determined by the condition $\kappa_q q^2 = \sigma$, yielding $\tilde{q}_\sigma = q_\sigma (q_\sigma/q_*)^{\eta/(2-\eta)}$. Next, we calculate $\langle \partial_\alpha \mathbf{h} \partial_\alpha \mathbf{h} \rangle$ with the use of equation (6) and substitute it in equation (5) (see footnote 6 for details). This yields an equation that determines the dependence of $\xi = \xi_{L \rightarrow \infty}$ on σ

$$\frac{\sigma}{\mu + \lambda} = \xi^2 - 1 + \frac{d_c T}{4\pi} \int_0^{q_{uv}} \frac{q dq}{\kappa_q q^2 + \sigma}, \quad (8)$$

where q_{uv} is the ultraviolet cutoff ($q_{uv} \gg q_*$). In the absence of stress ($\sigma = 0$), equation (8) simplifies. For $d_c \gg 1$, when $\eta = 2/d_c$, one gets [22, 44]

$$\xi^2|_{\sigma=0} \equiv \xi_0^2 = 1 - \kappa_{cr}/\kappa = 1 - T/T_{cr}, \quad (9)$$

where $\kappa_{cr} = d_c^2 T/8\pi$ and $T_{cr} = 4\pi\eta\kappa/d_c$ is the temperature of crumpling transition (CT) for a given value of bare bending rigidity κ . For $T < T_{cr}$, the stretching factor is finite, $\xi_0 > 0$, and the membrane is in the flat phase. For $T > T_{cr}$, the membrane undergoes the CT, so that $\xi \rightarrow 0$ for $L < \infty$. Interestingly, equation (9) predicts a negative expansion coefficient of the membrane, $d\xi_0/dT < 0$.

For $\sigma \neq 0$, we assume for simplicity $d_c = 1$, $\mu \sim \lambda \sim k_0$, (this is the case for graphene) and rewrite equation (8) as follows (see derivation in supplementary material)

$$\frac{2\sigma_*}{k_0} \left[\frac{\sigma}{\sigma_*} + \frac{1}{\alpha} \left(\frac{\sigma}{\sigma_*} \right)^\alpha \right] = \xi^2 - \xi_0^2, \quad (10)$$

where

$$\alpha = \eta/(2 - \eta), \quad \sigma_* = Ck_0 T/T_{cr} \quad (11)$$

and $C \sim 1$ is a numerical coefficient. Equation (10) represents a general form of HL for clean membrane. The lhs of this equation contains two terms: a regular term, proportional to σ , and an irregular one that shows a fractional scaling with σ . Analytical approximations [32], as well as numerical simulations [30, 35] for the physical case $D = 2$, $d = 3$, show that $\eta \simeq 0.7$, yielding $\alpha \simeq 0.54 < 1$. Hence, the irregular term dominates at small σ , and ξ shows an anomalous behavior, while the linear HL, $d\sigma/d\xi = k_0$, is realized for sufficiently large tensions, $\sigma \gg \sigma_*$. For sufficiently low temperatures, $T \ll T_{cr}$, the stretching corresponding to σ_* is small, $\xi_* - \xi_0 \sim T/T_{cr} \ll 1$. For $\sigma > \sigma_*$, the term $(\sigma/\sigma_*)^\alpha$ becomes subleading. (In this case $\tilde{q}_\sigma$ turns out to be larger than q_* , which leads

to additional suppression of this term, $\alpha^{-1}(\sigma/\sigma_*)^\alpha \rightarrow \ln(\sigma/\sigma_*)$ (see footnote 6)). One may introduce two exponents, governing the stretching in the anomalous regime. Far from the transition point ($T < T_{cr}$), one can expand $\xi^2 - \xi_0^2 \approx 2(\xi - \xi_0)\xi_0$, thus finding

$$\xi - \xi_0 \propto \sigma^\alpha, \quad \text{far from CT point.} \quad (12)$$

Exactly at the transition point $T = T_{cr}$, $\xi_0 = 0$ and

$$\xi \propto \sigma^{\alpha/2}, \quad \text{at the CT point.} \quad (13)$$

The above results can be easily generalized to an arbitrary dimensionality of the membrane, $D > 2$, by replacing $d^2\mathbf{q} \rightarrow d^D\mathbf{q}$ in equation (8). This leads to the replacement $\alpha \rightarrow (D - 2 + \eta)/(2 - \eta)$ of the critical index in equations (10), (12) and (13). The latter equation for α was obtained previously in [23, 25] for $\eta = 0$, which corresponds to the⁷ case $d_c = \infty$, and predicted in [26] from scaling considerations. As seen from equation (10), the tension leads to an increase of T_{cr} and, respectively, to a decrease of κ_{cr} . Indeed, setting $\xi = 0$ in equation (10) and assuming that $\sigma \ll \sigma_*$, we find the tension-induced change of the critical temperature, $\delta T_{cr}/T_{cr} = -\delta\kappa_{cr}/\kappa_{cr} \sim (\sigma/k_0)^\alpha$.

Disordered case. The derivation of perturbative RG equations for disordered graphene is performed by using replica trick within RPA scheme, in analogy with the case $\sigma = 0$ studied in [44]. Technical details of calculations are presented in supplementary material. First, we find $\langle \partial_\alpha \mathbf{h} \partial_\alpha \mathbf{h} \rangle$ in the harmonic approximation:

$$\overline{\langle \partial_\alpha \mathbf{h} \partial_\alpha \mathbf{h} \rangle} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{d_c T}{\kappa q^2 + \sigma} \left[1 + f \frac{\kappa q^2}{\kappa q^2 + \sigma} \right]. \quad (14)$$

Here the overbar denotes the disorder averaging and $f = b\kappa/T$ is a dimensionless parameter characterising the ratio of disorder to thermal fluctuations. For fixed κ and f , the integral in equation (14) logarithmically diverges for $\kappa q^2 \gg \sigma$ and saturates for $\kappa q^2 \ll \sigma$. In view of equation (3), we conclude that ξ is renormalized:

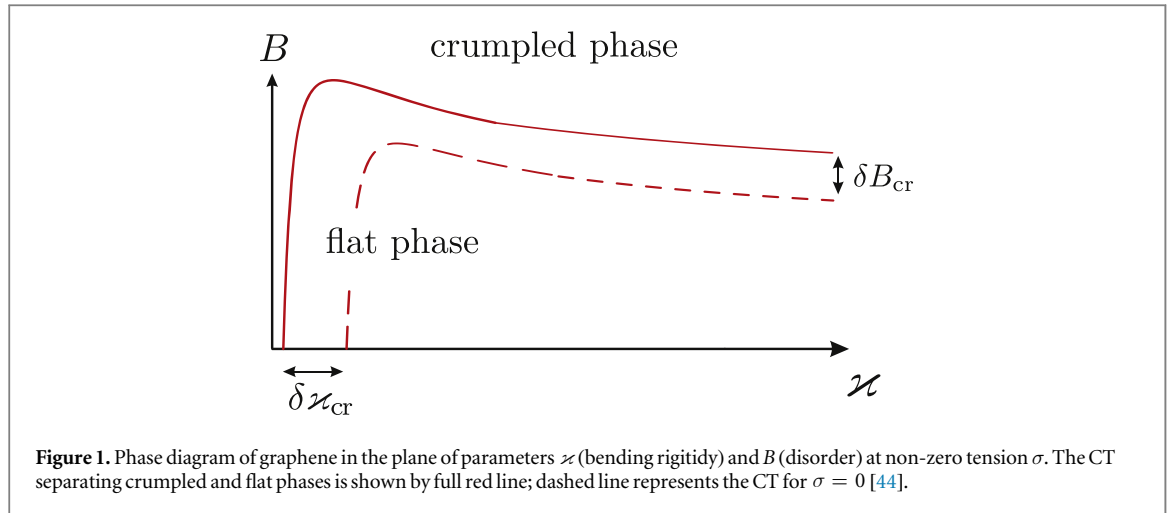
$$\frac{d\xi^2}{d\Lambda} \approx -\frac{d_c T}{4\pi \kappa} (1 + f), \quad \text{for } q \gg \tilde{q}_\sigma \quad (15)$$

and $d\xi^2/d\Lambda = 0$ for $q \ll \tilde{q}_\sigma$. The Ginzburg scale q_* is also affected by disorder [44]: $q_* \sim \sqrt{\tilde{\mu} T(1 + 2f)}/\kappa$. For strong disorder or low temperatures, $f \gg 1$, we find that

$$q_* \sim \sqrt{\tilde{\mu} b/\kappa} \quad (16)$$

is independent of temperature, while for weak disorder ($f \ll 1$), we recover equation (7), $q_* \propto T^{1/2}$. Below we show that $\tilde{q}_\sigma$ is also modified by sufficiently strong disorder.

⁷ In these papers, HL was written in terms of tension $\sigma' = L^{-2} \partial F / \partial \xi = 2\xi\sigma$.



In the harmonic approximation, κ and f are constants. However, they become scale-dependent due to the coupling between in-plane and out-of-plane fluctuations: $\kappa \rightarrow \kappa_q$ and $f \rightarrow f_q$. For $q \gg \tilde{q}_\sigma$, corresponding RG equations were derived in [44] (see also equation (S38) of supplementary material). For strong disorder, $f \gg 1$, the RG equations look: $d\kappa/d\Lambda = \eta\kappa/4$ and $df/d\Lambda = -3\eta/4$. The first equation yields $\kappa_q = \kappa(q_*/q)^{\eta/4}$. Equating $\kappa_q q^2$ to σ , we find:

$$\tilde{q}_\sigma = q_\sigma (q_\sigma/q_*)^{\eta/(8-\eta)}, \quad (17)$$

where q_* is given by equation (16). Since κ changes faster than f , one can set $f = \text{const}$ in equation (15). Using equation (5), we find that the equation that determines the dependence of ξ on σ for a strongly disordered membrane is given by equation (10) with $\xi_0^2 = 1 - B$

$$\alpha = \eta/(8 - \eta) \simeq 0.1, \quad \text{and} \quad \sigma_* = C'k_0B, \quad (18)$$

where $B = bd_c^2/2\pi$ and $C' \sim 1$ is a numerical coefficient. Note that the temperature drops out from the Hooke's law for disordered membrane. For $\sigma = 0$, the CT ($\xi = 0$) corresponds to $B = B_{\text{cr}} = 1$, in agreement with previous study (see figure 5 of [44]). For $B \ll B_{\text{cr}}$ the stretching corresponding to σ_* reads $\xi_* - \xi_0 \sim B/\alpha$. The tension enhances the critical value of disorder: $\delta B_{\text{cr}} \equiv B_{\text{cr}} - 1 \sim \alpha^{-1}(\sigma/k_0)^\alpha$. The anomalous stress-strain relations have the form (12) and (13) for $B < B_{\text{cr}}$ and $B = B_{\text{cr}}$, respectively, with appropriate replacement of α : the 'clean' value (11) is replaced by the considerably smaller 'dirty' value (18). Hence, stretching of a strongly disordered membrane is a nonlinear function of a weak tension, just as in the clean case. However, the corresponding power-law exponents differ from that of a clean system. As in the clean case, the conventional HL is restored for $\sigma \gg \sigma_*$.

The RG flow for ξ stops at $q \sim \tilde{q}_\sigma$. Thus, if $\xi > 0$ at this scale, the system is in the flat phase. Conversely, if ξ becomes zero before $\tilde{q}_\sigma$ is reached, the membrane crumples. The phase diagram in the parameter plane (κ, B), as obtained by numerical solution of RG

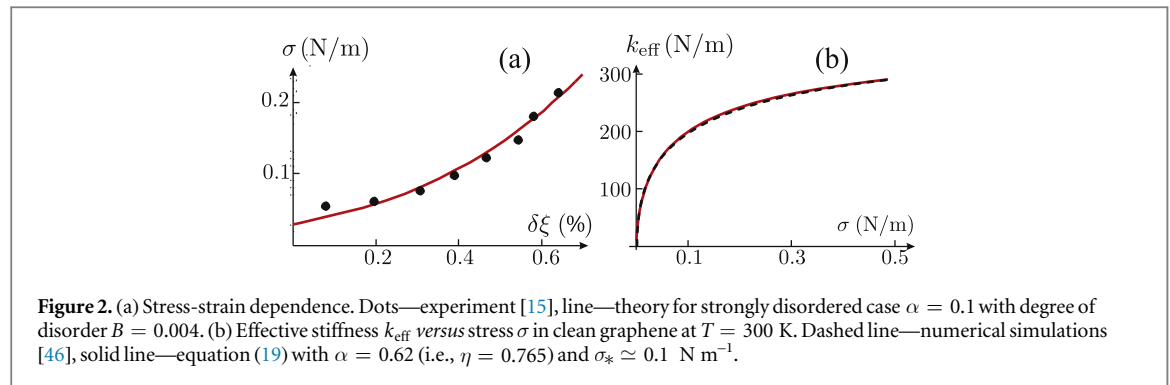
equations, is shown in figure 1. The tension shifts the line separating the flat and crumpled phases; this shift is characterized by $\delta\kappa_{\text{cr}}$ and δB_{cr} . Interestingly, the RG flows for κ and f do not stop at the point $q \sim \tilde{q}_\sigma$ (see footnote 6). However, for smaller q , such that $\kappa_q q^2 \ll \sigma$, the scaling of κ is irrelevant for the CT and the membrane remains flat.

Both in the clean and disordered case, it is convenient to introduce the effective stiffness

$$k_{\text{eff}} = \partial\sigma/\partial\xi \simeq k_0 \frac{(\sigma/\sigma_*)^{1-\alpha}}{1 + (\sigma/\sigma_*)^{1-\alpha}}. \quad (19)$$

It is strongly reduced for a weak strain ($\sigma \ll \sigma_*$), vanishing at the point of the buckling transition ($\sigma = 0$).

Let us now discuss characteristic values of parameters for the case of graphene. In [44] we estimated the amplitude of the static disorder as $b = 0.03$ based on experimental measurements of parameters of ripples [50]. Taking the bare value of the bending rigidity for graphene, $\kappa \simeq 1$ eV, we find $f \simeq 1$ at room temperature. This implies that at $T \simeq 300$ K the system is in the crossover regime between the clean and disordered limits. In this regime, the exponent α takes a non-universal value between the clean ($\alpha \simeq 0.5$) and disordered ($\alpha \simeq 0.1$) values. For low low temperatures, $T \ll 300$ K, we predict then the disordered value $\alpha \simeq 0.1$, while for elevated temperatures the clean value $\alpha \simeq 0.5$ should be reached. (In fact, α flows as a function of σ , tending to the clean value $\simeq 0.5$ for smallest strains. This flow is, however, logarithmically slow and may be difficult to observe experimentally.) Clearly, the crossover temperature may vary depending on sample preparation (degree of disorder). For clean samples, we estimate the crossover tension and stretching at $T \simeq 300$ K from equation (11), yielding $\sigma_* \simeq 1$ N/m and $\xi_* - \xi_0 \simeq 0.003$ (for $\eta = 0.7$ and $C = 1$). For disorder-dominated samples with the above disorder strength $b = 0.03$, we get $B \simeq 0.005$, which yields, according to (18), an estimate $\sigma_* \simeq 2$ N m⁻¹ and $\xi_* - \xi_0 \simeq 0.05$ (for $C' = 1$).



Our results compare well with a recent detailed experimental study of graphene elasticity [15]. It was found there that the room-temperature in-plane stiffness of graphene is reduced compared to its value k_0 for ‘ideal’ graphene (no disorder, $T = 0$) by a large factor (up to ~ 40) at low stretching. When temperature was lowered down to 10 K, the stiffness showed a sizeable increase, still remaining much smaller than 400 N m^{-1} . These data are in agreement with our conclusion that ripples and FP strongly weaken the in-plane stiffness, yielding comparable contributions at room temperature. In figure 2(a) we compare our theory with the strain-stress dependence presented in figure 2(c) of [15]. We use equation (10) describing both clean and disorder case with the appropriate choice of α and σ_* , considering α and σ_* as fitting parameters. The solid lines in figure 2(a) show dependence of σ on $\delta\xi = \xi(\sigma) - \xi(\sigma_0)$, where σ_0 is built-in stress extracted from experimental data [15]. The best fit is achieved for $\alpha \approx 0.1$ and $\sigma_* \approx 1.68 \text{ N m}^{-1}$. The obtained value of α implies that the sample is in the disorder-dominated regime, with the degree of disorder $B \approx 0.004$ (we set the numerical coefficient $C' = 1$ here). This estimate is in agreement with the value $B = 0.005$ [44] obtained from the transmission-electron-microscopy data of [49, 50]. For a more detailed comparison with experiment, measurements of strain-stress curves in a wide range of temperatures would be of great interest.

Our results also compare very well with numerical simulations of [46] which were performed for clean graphene. In particular, the scaling of k_{eff} for $f \ll 1$ is in an excellent agreement with the large-sample data (number of atoms 37888). For comparison, we used the empirical formula (11) of [46] which perfectly fits numerical data obtained there (see footnote 6 for details). As seen from figure 2(b), the numerical data are very well described by our equation (19) with $\alpha \approx 0.62$ ($\eta \approx 0.76$), as expected in the clean limit. The comparison to the results of numerical simulations allows us to determine the numerical coefficient C in equation (11), which turns out to be $C \approx 0.093$. The corresponding crossover stress and strain values read $\sigma_* \approx 0.1 \text{ N m}^{-1}$ and $\xi_* - \xi_0 \approx 0.0006$, respectively.

To conclude, the theory of anomalous Hooke’s law has been developed, for both clean and disordered graphene. In both cases, scaling of the deformation with the external force obeys a fractal power law in the limit of weak forces. This behavior is dominated by thermal fluctuations for clean graphene, while for strongly disordered graphene it is governed by static ripples. Remarkably, the same coupling between longitudinal and transverse modes that enhances the bending rigidity, thus rescuing the flat phase of the membrane, leads simultaneously to a dramatic softening of the in-plane elasticity.

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